Natural Logic
Homework 3: Backround assumptions on verbs

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Exercise 1

If $\Gamma \not\models$ All x are y,
prove that there is a model $\mathcal{M}$ of Γ
with just one point where All x are y is false.

Exercise 2

Here is main point behind the algorithm for deciding the logic and for building counter-models:
If $\Gamma \vdash$ All x y, then there some proof tree for this
all of whose terms are either x, or y, or subterms of one of the leaves.
This is actually not so easy to show.

Your task Find a simple example of a proof tree in this logic with the property that some term t in it is not one of the terms in the conclusion, and also not a subterm of any leaf.

Suppose that we want to work with background assumptions that are not expressible in our logic so far.
For example, consider the following purported inference:

$$\frac{\text{All bicyclists are football players} \quad \text{All football fans love all football players}}{\text{All football fans like all bicyclists}} \qquad (1)$$

The way in which we are doing things so far, there is no relation between like and love in our models.
And so we cannot derive (1).
The point again is that we cannot add a premise such as
"loving involves liking"
because this is not expressible in the language.
For this reason, we expand our overall framework.

In addition to our assumptions Γ, we can adopt a set Δ of background assumptions of the form $r \sqsubseteq s$ for $r, s \in \mathbf{R}$. (Δ is the Greek letter Delta, ad $\mathbf{R}$ is our set of verbs.)
In this case above, we might have

$$\Delta \quad = \quad \{\text{loves} \sqsubseteq \text{likes}\}.$$

Definition

a model respecting a set of assumptions A model $\mathcal{M}$ respects Δ if whenever $r \sqsubseteq s$ belongs to Δ, then $[\![r]\!] \subseteq [\![s]\!]$ in $\mathcal{M}$.
In the proof theory, we define $\Gamma; \Delta \vdash \varphi$ by adding to the proof system in of the logic the following rules:

$$\frac{\text{All } x \ (r \text{ all } y)}{\text{All } x \ (s \text{ all } y)} \ r \sqsubseteq s$$

We have one rule for each inequality assertion $r \sqsubseteq s$ in Δ.

Exercise 3

Now derive (1). That is, show that

$$\Gamma; \Delta \vdash \text{All fans (like all bicyclists)},$$

where Γ is

All bicyclists football-players
All football-fans (love all football-players)