Natural Logic
Homework 4: Preparation for Monotonicity

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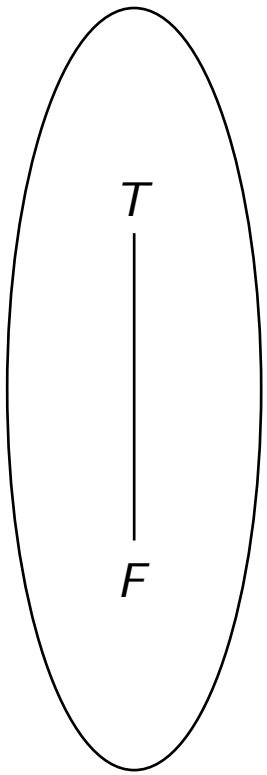
A comment on the All-logic

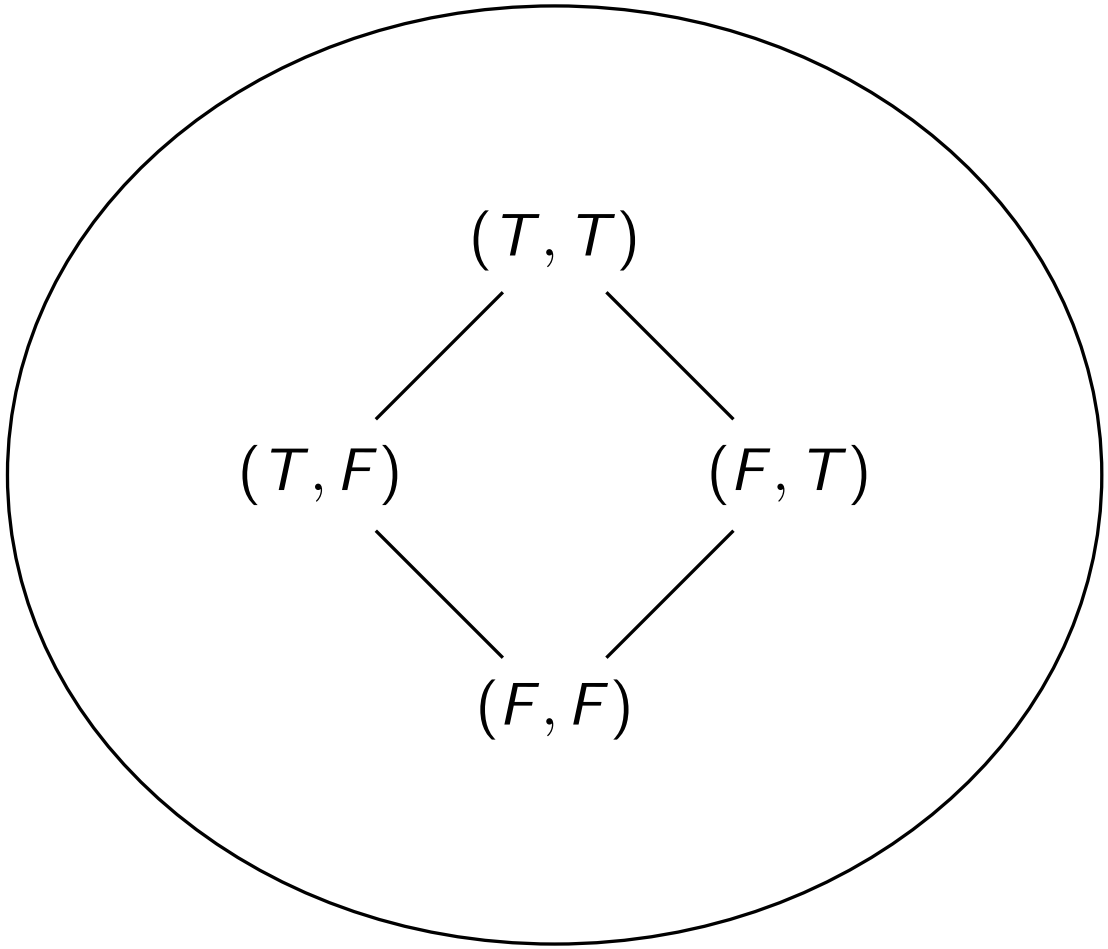
To do this homework, you'll want to review monotone and antitone functions from my initial lecture of the course.
The main poset here is the tiny preorder which I'll call $\mathbb{2}$.



$\mathbb{2}$

Notice that $F < T$.

But we also want a second preorder, $\mathbb{2} \times \mathbb{2}$:



$\mathbb{2} \times \mathbb{2}$

Exercise 1: functions on boolean values

Is the function $\neg : \mathbb{2} \to \mathbb{2}$ monotone or antitone?

Is the conjunction function $\wedge : \mathbb{2} \times \mathbb{2} \to \mathbb{2}$ monotone?
Is it antitone?
(This is the function that has $T \wedge T = T$, $T \wedge F = F$, etc.)

Is the disjunction function $\vee : \mathbb{2} \times \mathbb{2} \to \mathbb{2}$ monotone?
Is it antitone?

Is the implication function $\to : \mathbb{2} \times \mathbb{2} \to \mathbb{2}$ monotone?
Is it antitone?

Exercise 2: functions sets

There are four functions from $\mathbb{2}$ to $\mathbb{2}$:

$$\begin{array}{ll} h(T) = T & h(F) = T \\ i(T) = T & i(F) = F \\ j(T) = F & j(F) = T \\ k(T) = F & k(F) = F \end{array}$$

We want to order these functions by

$$f \leq g \text{ if for all } x \in \mathbb{2},\ f(x) \leq g(x) \text{ in } \mathbb{2}$$

Your task: Draw the picture of this order.

Exercise 3: functions sets, but with a different order

Again, we have the same four functions from $\mathbb{2}$ to $\mathbb{2}$: We want to order these functions by

$$f \preceq g \text{ if for all } x \in \mathbb{2},\ g(x) \leq f(x) \text{ in } \mathbb{2}$$

Note that this is different!
Your task: Draw the picture of this order.

Exercise 4: opposite preorders

For any preorder $\mathbb{P}$, we define a new preorder $\mathbb{P}^{op}$.
It has the same points as $\mathbb{P}$,
but $p \leq q$ in $\mathbb{P}^{op}$ if and only if $q \leq p$ in $\mathbb{P}$.

Draw $\mathbb{2}^{op}$.
For all $\mathbb{P}$, what is $(\mathbb{P}^{op})^{op}$?

Exercise 5: functions preorders

If $\mathbb{P}$ and $\mathbb{Q}$ are preorders,
we define $[\mathbb{P}, \mathbb{Q}]$ to be the set
of all monotone functions $f : \mathbb{P} \to \mathbb{Q}$,
ordered as above: $f \leq g$ iff for all $x \in \mathbb{P}$, $f(x) \leq g(x)$.

One of the following is true, one is false:

1. $[\mathbb{P}^{op}, \mathbb{Q}] = [\mathbb{P}, \mathbb{Q}^{op}]$
2. $[\mathbb{P}^{op}, \mathbb{Q}] = [\mathbb{P}, \mathbb{Q}^{op}]^{op}$

Which is which?