

Q520: Homework
Due: Thursday, March 27

1. Let X and Y be random variables on a finite probability space. We know what $H(X)$ and $H(Y)$ mean, and we also know what $H(X, Y)$ means. (This last entropy is just the entropy of the single random variable X, Y that gives pairs of values. For example, in class and in my notes we computed $H(Fl, Age)$ on a particular space.)

Your problem: show that if X and Y are independent, then $H(X, Y) = H(X) + H(Y)$.

The point is to write a clear proof. It will be a few lines of summations, and at one or more points you will need to use the assumption that X and Y are independent.

2. Two vectors are *orthogonal* if their dot product is zero.

Let S be any set of vectors in a vector space. Let S^\perp be the set of vectors in the space which are orthogonal to all elements of S . Show that S^\perp is a subspace.

[The point for you should be to review the meaning of “subspace” and to get a clear, concise proof. For a start, you might work the special case where S has just two elements, say v and w .]

3. For this and the next problems, you will need to read and work through the web site

<http://www.uwlax.edu/faculty/will/svd/index.html>.

Read the first six sections, up to and including the one called ‘Projections’. Do the problems at the bottom of the sections. (All of the problems in this tutorial are short, and for the most part they are straightforward calculations or estimations. I don’t think that any involve proofs.) Your answers can therefore be very short.