

Q520: Eighth Homework: Logic.
 Due: Thursday, April 17

$\frac{}{All\ X\ are\ X}$	$\frac{All\ X\ are\ Z\ \ All\ Z\ are\ Y}{All\ X\ are\ Y}$
$\frac{Some\ X\ are\ Y}{Some\ X\ are\ X}$	$\frac{All\ Y\ are\ Z\ \ Some\ X\ are\ Y}{Some\ Z\ are\ X}$
$\frac{}{J\ is\ J}$	$\frac{J\ is\ M\ \ M\ is\ F}{F\ is\ J}$
$\frac{J\ is\ an\ X\ \ J\ is\ a\ Y}{Some\ X\ are\ Y}$	$\frac{All\ X\ are\ Y\ \ J\ is\ an\ X}{J\ is\ a\ Y}$
$\frac{M\ is\ an\ X\ \ J\ is\ M}{J\ is\ an\ X}$	

Figure 1: A complete set of rules for *All*, *Some*, and proper names.

1. Let Γ be the following set of sentences:

- (a) Mary is a poet.
- (b) Every poet is an artist.
- (c) Susan is a vegetarian.
- (d) Susan is Mary.

Using the logical system above, show that $\Gamma \vdash \textit{Some vegetarian is an artist}$.

2. Show that

$$\{\textit{Some } X \textit{ are } Y, \textit{Some } Y \textit{ are } Z\} \not\models \textit{Some } X \textit{ are } Z.$$

[Recall that to show an assertion of the form $\Gamma \not\models S$, all you have to do is come up with a context where all the sentences in Γ are true, and S is false.]

3. Suppose we want to add sentences to our system of the form *No* X are Y .

- (a) In terms of our semantics, when do you think such a sentence should be true?
- (b) Find three interesting sound rules having to do with *No*, either by itself, or in combination with *All* and *Some*. [Hint: what should it mean to say *No* X are X ?
- (c) Give an example of an interesting deduction $\Gamma \vdash S$ in your system, in parallel to what we did in problem 1 above.

4. The word *most* is tricky to handle, and “most” courses on logic would completely skip it. We’ll take *most* to mean *strictly more than half*. In other words, in a context,

$$\llbracket \textit{Most } X \textit{ are } Y \rrbracket = \textit{true} \quad \text{iff} \quad \llbracket X \cap Y \rrbracket > \frac{1}{2} \llbracket X \rrbracket.$$

So if there are four senators in a room and exactly two of them are doctors, then *most senators are doctors* should come out false in that context. Similarly, if $\llbracket X \rrbracket = \emptyset$ in a context, then *Most X are X* should come out false, since 0 is not $> \frac{1}{2}0$. Please note that in this problem, we assume that all sets are *finite*, because we want to get close to everyday reasoning, not mathematics. This means that if you know about “infinite arithmetic”, you should forget about it for this problem.

Below, you’ll find eight rules. Decide which are sound and which are not. More precisely, decide which represent “good reasoning”, using our assumption about the meanings of *most*, *some*, *all*. For the ones that are sound, give a short reason based on the formal semantics. For the ones that are not, give a model where the assumption(s) is(are) true and the conclusion false.

$$\begin{array}{cccc} \frac{\textit{Most X are Y}}{\textit{Most X are X}} & \frac{\textit{Most X are Y}}{\textit{All X are Y}} & \frac{\textit{All X are Y}}{\textit{Most X are Y}} & \frac{\textit{Most X are Y}}{\textit{Most Y are Y}} \\ \frac{\textit{Most X are Y}}{\textit{Some X are Y}} & \frac{\textit{Some X are X}}{\textit{Most X are X}} & \frac{\textit{Most X are Y}}{\textit{Some Y are Z}} & \frac{\textit{Most X are Z}}{\textit{Some Y are Z}} \\ \frac{\textit{Most X are Z}}{\textit{Most X are Y}} & \frac{\textit{All Z are Y}}{\textit{Most X are Y}} & \frac{\textit{All Y are Z}}{\textit{Most X are Y}} & \end{array}$$

5. This is a more challenging problem based on the last one. Decide which of the following is sound, and which is not.

$$\frac{\textit{All X are Z} \quad \textit{All Y are Z} \quad \textit{Most Z are Y} \quad \textit{Most Y are X}}{\textit{Most X are Y}}$$

$$\frac{\textit{All U are X} \quad \textit{Most X are V} \quad \textit{All V are Y} \quad \textit{Most Y are U}}{\textit{Some U are V}}$$

[Hint: one is sound, and one is not.]