NATURAL LOGIC

Larry Moss, Indiana University

EASLLC, 2014
What does semantics look like when we make inference the primary object of study rather than the secondary one?
What does semantics look like when we make inference the primary object of study rather than the secondary one?

What is the relation of logic and semantics?

What does logic for natural language look like when it is done with a minimum of translation?

Can we build formal tools to simultaneously
— handle inference in interesting fragments
— remain on the “good side” of various logical borders: decidability, complexity.

And will this be of any interest in semantics?
A fairly standard view of these matters

- Why does logic enter into semantics in the first place?

We want to account for natural language inferences such as

- Frege’s favorite food was baozi
- Frege ate baozi at least once
Why does logic enter into semantics in the first place? We want to account for natural language inferences such as

\[
\begin{align*}
\text{Frege's favorite food was baozi} \\
\text{Frege ate baozi at least once}
\end{align*}
\]

The hypothesis and conclusion would be rendered in some logical system or other. There would be a background theory (≈ common sense), and then the inference would be modeled either as a semantic fact:

\[
\text{Common sense} + \text{Frege's favorite food was baozi} \models \text{Frege ate baozi at least once}
\]

or a via a formal deduction:

\[
\text{Common sense} + \text{Frege's favorite food was baozi} \vdash \text{Frege ate baozi at least once}
\]
Why does logic enter into semantics in the first place?

We want to account for natural language inferences such as

Frege’s favorite food was baozi

Frege ate baozi at least once

The hypothesis and conclusion would be rendered in some logical system or other. There would be a background theory (≈ common sense), and then the inference would be modeled either as a semantic fact:

Common sense + Frege’s favorite food was baozi \models\ Frege ate baozi at least once

or a via a formal deduction:

Common sense + Frege’s favorite food was baozi \vdash\ Frege ate baozi at least once

As an aside: We are going to be vitally interested in the difference between the proof-theoretic concept \vdash\ and the model-theoretic concept \models\ in this course!
Why does logic enter into semantics in the first place? We want to account for natural language inferences such as

\[
\begin{align*}
\text{Frege's favorite food was baozi} & \\
\text{Frege ate baozi at least once}
\end{align*}
\]

The hypothesis and conclusion would be rendered in some logical system or other. There would be a background theory (≈ common sense), and then the inference would be modeled either as a semantic fact:

\[
\text{Common sense+Frege's favorite food was baozi} \models \text{Frege ate baozi at least once}
\]

or a via a formal deduction:

\[
\text{Common sense+Frege's favorite food was baozi} \vdash \text{Frege ate baozi at least once}
\]

Either way, it's all in one and the same language.
To carry out this program, it would be advisable to take as expressive a logical system as possible.

First-order logic (FOL) is a good starting point, but for many phenomena we’ll need to go further.

In this regard, FOL is vastly superior to traditional (term) logic.

Various properties of FOL are interest in this discussion, but only secondarily so.
What does undecidability have to do with it?

**Theorem (Church 1936)**

There is no algorithm, which given a finite set \( \Gamma \) of sentences in FOL and another sentence \( \varphi \), decides whether or not \( \Gamma \models \varphi \).

The same goes for the proof-theoretic notion \( \Gamma \vdash \varphi \), since this comes to the same thing, by the Completeness Theorem of FOL.
What does undecidability have to do with it?

**Theorem (Church 1936)**

There is no algorithm, which given a finite set $\Gamma$ of sentences in FOL and another sentence $\varphi$, decides whether or not $\Gamma \models \varphi$.

The same goes for the proof-theoretic notion $\Gamma \vdash \varphi$, since this comes to the same thing, by the Completeness Theorem of FOL.

This is not really relevant to the semantic enterprise, since we are not taking $\Gamma \models \varphi$ to model what humans actually do.

And as for $\Gamma \vdash \varphi$, we don’t care if FOL gives us more power than we actually need.
What does undecidability have to do with it?

**Theorem (Church 1936)**

There is no algorithm, which given a finite set $\Gamma$ of sentences in FOL and another sentence $\varphi$, decides whether or not $\Gamma \models \varphi$.

The same goes for the proof-theoretic notion $\Gamma \vdash \varphi$, since this comes to the same thing, by the Completeness Theorem of FOL.

FOL in is important also connection with the foundations of mathematics.

Undecidability is forced on us in that domain: By Gödel’s Incompleteness Theorem, the set of true sentences of mathematics is not decidable.
One can easily object to the whole enterprise of using FOL in connection with NL inference, on the grounds that FOL cannot handle:

- vague words
- intentions of speakers
- ellipsis
- anaphora
- poetic language

In other words, FOL is too small for the job.
The point is that for “everyday inference”,
a small fragment of FOL should be sufficient.

Also, there is a long tradition in linguistics of dissatisfaction with
models which are “complete r.e.”
and in favor of ones with much less expressive power.

This was once decisive in syntax: the Peters-Ritchie Theorem.
This course: FOL is also too big!

The point is that for “everyday inference”, a small fragment of FOL should be sufficient.

Also, there is a long tradition in linguistics of dissatisfaction with models which are “complete r.e.” and in favor of ones with much less expressive power.

This was once decisive in syntax: the Peters-Ritchie Theorem.

**You decide**

Consider three activities:

A mathematics: prove the Pythagorean Theorem $a^2 + b^2 = c^2$.

B syntax: parse *John knows his mother saw him at her house*.

C semantics: tell whether a hearer of *Nian the Horrible Monster* should infer that the monster liked the color red.

A: mathematics  B: syntax

Where would you put C: semantics?
Following a lot of work in subjects like

linguistics
artificial intelligence
cognitive science

we aim to formalize using the lightest tools possible.

(This contrasts with what is done in foundations of mathematics, where one typically uses whatever tools are needed to get the job done.)
Another way to make this point

In logic, we have a trade-off between expressivity and tractability:

- propositional logic is not very expressive, but it is decidable and has a complete proof system.
- first-order logic is yet more expressive, it is undecidable but has a complete proof system.
- second-order logic is very expressive, it is undecidable and it is not even clear what a complete proof system would be.
Program

Show that significant parts of NL inference can be carried out in decidable logical systems, preferably in “light” systems.

Raise the question of how much semantics can be done in decidable logics.

To axiomatize as much as possible, because the resulting logical systems are likely to be interesting.

To ask how much of language could have been done if the traditional logicians had the mathematical tools to go further than they were able to.
Examples of inferences which we will see in this course

These are the basic data that the course will account for.

First, a few examples from the classical syllogistic:

\[
\begin{align*}
\text{All men are mortal} & \quad \text{Socrates is a man} \\
& \quad \text{Socrates is mortal} \\
\end{align*}
\]

(1)

\[
\begin{align*}
\text{All auctioneers are curmudgeons} & \quad \text{No bartenders are curmudgeons} \\
& \quad \text{No auctioneers are bartenders} \\
\end{align*}
\]

(2)

Our first unit is on the basics of syllogistic logic.
Our “syntax” of sentences will give us

- All X are Y
- Some X are Y
- No X are Y

We adopt the evident semantics.

We craft a logical system which has formal proofs using our syntax of sentences and nothing else.

After this, we want to extend the idea of syllogistic logic.
Syllogistic Logic of All and Some

Syntax: *All p are q*, *Some p are q*

Semantics: A model $\mathcal{M}$ is a set $M$, and for each noun $p$ we have an interpretation $\llbracket p \rrbracket \subseteq M$.

\[
\mathcal{M} \models All\ p\ are\ q \quad \text{iff} \quad \llbracket p \rrbracket \subseteq \llbracket q \rrbracket \\
\mathcal{M} \models Some\ p\ are\ q \quad \text{iff} \quad \llbracket p \rrbracket \cap \llbracket q \rrbracket \neq \emptyset
\]

Proof system:

\[
\begin{align*}
& \quad \underline{All\ p\ are\ p} & \quad All\ p\ are\ q \\
& \underline{All\ p\ are\ n} \quad All\ n\ are\ q & \quad All\ p\ are\ q \\
& \quad Some\ p\ are\ q & \quad Some\ p\ are\ q \\
& \quad Some\ q\ are\ p & \quad Some\ p\ are\ q \\
& \quad Some\ p\ are\ n & \quad Some\ p\ are\ q \\
& \quad Some\ p\ are\ p & \quad Some\ p\ are\ n
\end{align*}
\]
If $\Gamma$ is a set of formulas, we write $\mathcal{M} \models \Gamma$ if for all $\varphi \in \Gamma$, $\mathcal{M} \models \varphi$.

$\Gamma \models \varphi$ means that every $\mathcal{M} \models \Gamma$ also has $\mathcal{M} \models \varphi$.

A proof tree over $\Gamma$ is a finite tree $\mathcal{T}$ whose nodes are labeled with sentences and each node is either an element of $\Gamma$, or comes from its parent(s) by an application of one of the rules.

$\Gamma \vdash S$ means that there is a proof tree $\mathcal{T}$ for over $\Gamma$ whose root is labeled $S$. 
English:
If there is an \( n \), and if all \( ns \) are \( ps \) and also \( qs \), then some \( p \) are \( q \).

Semantic assertion:
\[ \text{Some } n \text{ are } n, \text{ All } n \text{ are } p, \text{ All } n \text{ are } q \models \text{Some } p \text{ are } q. \]

Proof-theoretic assertion:
\[ \text{Some } n \text{ are } n, \text{ All } n \text{ are } p, \text{ All } n \text{ are } q \vdash \text{Some } p \text{ are } q. \]
English:
If there is an $n$, and if all $ns$ are $ps$ and also $qs$, then some $p$ are $q$.
This is something we could check against human intuition and performance.

Semantic assertion:
Some $n$ are $n$, All $n$ are $p$, All $n$ are $q$ $\models$ Some $p$ are $q$.
The reasoning here would be a mathematical proof.

Proof-theoretic assertion:
Some $n$ are $n$, All $n$ are $p$, All $n$ are $q$ $\vdash$ Some $p$ are $q$.
The proof tree is

$$
\begin{array}{c}
\text{All } n \text{ are } p & \text{Some } n \text{ are } n \\
\hline
\text{Some } n \text{ are } p \\
\text{All } n \text{ are } q & \text{Some } p \text{ are } n \\
\hline
\text{Some } p \text{ are } q
\end{array}
$$
**Example of a conclusion which doesn’t follow**

\[
\begin{align*}
\text{All frogs are reptiles.} \\
\text{All frogs are animals.} \\
\hline \\
\text{All reptiles are animals.}
\end{align*}
\]

We can take \( U = \{1, 2, 3, 4, 5, 6\} \).
\( [F] = \{1, 2\} \),
\( [R] = \{1, 2, 3, 4\} \).
\( [A] = \{1, 2, 4, 5, 6\} \).

In this context, the assumptions are true but the conclusion is false. So the argument is invalid.

\[
\text{All frogs are reptiles, All frogs are animals \( \not\Rightarrow \) All reptiles are animals.}
\]
The connection

**Completeness Theorem**

\[ \Gamma \models \varphi \iff \Gamma \vdash \varphi \]

References to related work:

All the logical systems in this course are complete.
If you follow most of the details, you’ll learn a lot of technical material.
Let us add complemented atoms $\overline{p}$ on top of $S$, with interpretation is via set complement.

So we have

- All $p$ are $q$
- Some $p$ are $q$
- All $p$ are $\overline{q} \equiv$ No $p$ are $q$
- Some $p$ are $\overline{q} \equiv$ Some $p$ aren’t $q$
The logical system

All $p$ are $p$

Some $p$ are $q$

All $p$ are $n$

All $n$ are $q$

All $q$ are $\bar{q}$

All $q$ are $p$

All $\bar{q}$ are $\bar{q}$

All $\bar{q}$ are $p$

All $\bar{q}$ are $q$

All $n$ are $p$

Some $n$ are $q$

Some $p$ are $\bar{p}$

Ex falso quodlibet
Let $\Gamma$ be

\[
\{\text{All } q \text{ are } p, \text{All } \neg q \text{ are } p, \text{All } q \text{ are } n, \text{Some } p \text{ are } \neg n\}
\]

Here is a proof tree showing $\Gamma \vdash \text{Some } p \text{ are } \neg q$:

\[
\begin{align*}
\text{All } q \text{ are } p & \quad \text{All } q \text{ are } n \quad \text{Some } m \text{ are } \neg n \\
\text{All } \neg p \text{ are } \neg q & \quad \text{All } \neg n \text{ are } \neg q \quad \text{Some } \neg n \text{ are } \neg n \\
\text{All } \neg p \text{ are } p & \quad \text{Some } \neg n \text{ are } \neg q \\
\text{All } \neg q \text{ are } p & \quad \text{Some } \neg q \text{ are } \neg q \\
\text{Some } \neg q \text{ are } p & \\
\text{Some } p \text{ are } \neg q
\end{align*}
\]
The rules below seem much less intuitive than the others:

\[
\begin{align*}
\text{All } q \text{ are } \overline{q} & \quad \text{Zero} \\
\text{All } q \text{ are } p & \\
\text{All } \overline{q} \text{ are } q & \quad \text{One} \\
\text{All } p \text{ are } q &
\end{align*}
\]

Typically, the logics in this subject contain a few strange rules.

Sometimes this appears to me to be a nuisance, sometimes a delight.
Ex Falso Quodlibet vs. Reductio ad Absurdum

Some $p$ are $\bar{p}$

\[
\varphi
\]

Ex falso quodlibet

To state Reduction ad Absurdum, we need to define negations:

\[
\begin{array}{c|c}
\text{sentence } \varphi & \text{negation } \bar{\varphi} \\
\hline
\text{All } p \text{ are } q & \text{Some } p \text{ are } \bar{q} \\
\text{Some } p \text{ are } q & \text{All } p \text{ are } \bar{q}
\end{array}
\]

\[
\frac{\alpha}{\bot} \quad \bot \quad [\varphi] \quad \frac{\bar{\varphi}}{\bot} \quad \frac{\varphi}{\text{RAA}}
\]

\[
\alpha \quad \bar{\alpha} \quad \bot \quad \bot \quad \varphi
\]
More examples

Next, some examples from the extended syllogistic logics:

Some dog sees some cat
Some cat is seen by some dog

(3)

Bao is seen and heard by every student  Amina is a student
Amina sees Bao

(4)

All skunks are mammals
All who fear all who respect all skunks fear all who respect all mammals

(5)
More on the “skunks and mammals” example

\[
\begin{align*}
\text{All skunks are mammals} \\
\text{All who fear all who respect all skunks fear all who respect all mammals}
\end{align*}
\]
We first note the following antitonicity principle:

\[
\text{All skunks are mammals} \quad \Rightarrow \quad \text{All who respect all mammals respect all skunks}
\]

This and related principles are going to occupy us for an entire lecture.
And we can apply antitonicity twice!

- All skunks are mammals
- All who respect all mammals respect all skunks
- All who fear all who respect all skunks fear all who respect all mammals
Examples of inferences which we will see in this course

Every giraffe is taller than every gnu
Some gnu is taller than every lion
Some lion is taller than some zebra
Every giraffe is taller than some zebra

More students than professors run
More professors than deans run
More students than deans run

At most as many xenophobics as yodelers are zookeepers
At most as many zookeepers as alcoholics are yodelers
At most as many yodelers as xenophobics are alcoholics
At most as many zookeepers as alcoholics are xenophobics
Example

Assume:

1. There are at least as many non-y as y
2. There are at least as many non-z as z
3. All x are z
4. All non-y are z

Then prove from these that All x are non-y.

Here is a formal proof in the logical system which we’ll see:

\[
\forall(x, z) \quad \forall(y, z) \quad \exists(y, y) \quad \exists(z, z) \\
\text{(HALF)}
\]

\[
\forall(y, z) \quad \exists(y, z) \\
\text{(CARD MIX)}
\]

\[
\forall(x, z) \quad \forall(z, y) \\
\text{(BARBARA)}
\]
The map

This course will mention many fragments, and to help keep things straight, I’ll draw a lot of maps.

The Aristotle boundary is the dividing line between fragments which are formulated syllogistically and those which are not. Reductio proofs are ok. Infinitely many rules are not.

Individual variables take us beyond the Aristotle boundary.

The Turing boundary is the dividing line between decidable and undecidable fragments.

The Peano-Frege boundary divides the fragments which are formulated in first-order logic from those which are not.
**Map of Some Natural Logics**

- **Peano-Frege**
- **Aristotle**
- **Church-Turing**

### Symbols and Notations
- $\mathcal{R}^\dagger$: transitive
- $\mathcal{R}^\dagger$ (tr, opp): transitive with opposites
- $\mathcal{R}$: relational syllogistic
- $S$: all/some/no $p$ are $q$
- $\geq$: adds $|p| \geq |q|$}

### Logics
- **FOL**: first-order logic
  - $FO^2 + \text{trans}$
  - $RC^\dagger(tr)$
  - $RC^\dagger(tr, \text{opp})$
  - $\mathcal{R}^\dagger$

- **2 variable FO logic**
  - $\dagger$ adds full $N$-negation
  - $RC(tr) +$ opposites
  - $RC +$ (transitive) comparative adjs
  - $\mathcal{R} +$ relative clauses
  - $S +$ full $N$-negation
  - $S \geq$ adds $|p| \geq |q|$
I have arranged the course material in a number of units:

- overview + examples
- syllogistic logics
- logics and the sizes of sets
- logics with individual variables
- polarity/monotonicity
- another direction: proof-theoretic semantics
- answers to homework + any further topics you ask about
1. John is a man [Hyp]
2. Any woman is a mystery to any man [Hyp]
3. Jane is a woman
   ____________________________
4. Any woman is a mystery to any man [R, 2]
5. Jane is a mystery to any man [Any Elim, 4]
6. John is a man [R, 1]
7. Jane is a mystery to John [Any Elim, 6]
8. Any woman is a mystery to John [Any intro, 3, 7]
Example of where we would want derivations with variables

- All xenophobics see all astronauts
- All yodelers see all zookeepers
- All non-yodelers see all non-astronauts
- All wardens are xenophobics

\[
\text{All wardens see all zookeepers}
\]
<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>Proof Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>All xenophobics see all astronauts</td>
<td>Hyp</td>
</tr>
<tr>
<td>2</td>
<td>All yodelers see all zookeepers</td>
<td>Hyp</td>
</tr>
<tr>
<td>3</td>
<td>All non-yodelers see all non-astronauts</td>
<td>Hyp</td>
</tr>
<tr>
<td>4</td>
<td>All wardens are xenophobics</td>
<td>Hyp</td>
</tr>
<tr>
<td>5</td>
<td>Jane is a warden</td>
<td>Hyp</td>
</tr>
<tr>
<td>6</td>
<td>All wardens are xenophobics</td>
<td>R, 4</td>
</tr>
<tr>
<td>7</td>
<td>Jane is a xenophobic</td>
<td>All Eliim, 6</td>
</tr>
<tr>
<td>8</td>
<td>All xenophobics see all astronauts</td>
<td>R, 2</td>
</tr>
<tr>
<td>9</td>
<td>Jane sees all astronauts</td>
<td>All Eliim, 8</td>
</tr>
<tr>
<td>10</td>
<td>Jane is a yodeler</td>
<td>Hyp</td>
</tr>
<tr>
<td>11</td>
<td>Jane sees all zookeepers</td>
<td>Easy from 2</td>
</tr>
<tr>
<td>12</td>
<td>Jane is not a yodeler</td>
<td>Hyp</td>
</tr>
<tr>
<td>13</td>
<td>Jane sees all zookeepers</td>
<td>See below</td>
</tr>
<tr>
<td>14</td>
<td>Jane sees all zookeepers</td>
<td>Cases 10-11, 12-13</td>
</tr>
<tr>
<td>15</td>
<td>All wardens see all zookeepers</td>
<td>All Intro</td>
</tr>
</tbody>
</table>
1. Jane is not a yodeler

2. Jane sees all astronauts

3. All non-yodelers see all non-astronauts

4. Jane sees all non-astronauts

5. Bob
   - Bob is a zookeeper

6. Bob is astronaut

7. Jane sees Bob

8. Bob is not astronaut

9. Jane sees Bob

10. Jane sees Bob

11. Jane sees all zookeepers
Assume that

\[ \text{waltz} \leq \text{dance} \leq \text{move} \]

and

\[ \text{grizzly} \leq \text{bear} \leq \text{animal} \]
Assume that

\[ \text{waltz} \leq \text{dance} \leq \text{move} \]

and

\[ \text{grizzly} \leq \text{bear} \leq \text{animal} \]

Assume that we’re talking about a situation where

some bears dance.

Which one would be true?

- some grizzlies dance
- some animals dance
Assume that

\[ \text{waltz} \leq \text{dance} \leq \text{move} \]

and

\[ \text{grizzly} \leq \text{bear} \leq \text{animal} \]

Assume that we’re talking about a situation where

some bears dance.

\[ \Rightarrow \text{some grizzlies dance} \quad \text{false} \]
\[ \Rightarrow \text{some animals dance} \quad \text{true} \]
Assume that
\[
\text{waltz} \leq \text{dance} \leq \text{move}
\]

and
\[
\text{grizzly} \leq \text{bear} \leq \text{animal}
\]

Assume that we’re talking about a situation where

\[\text{some bears dance.}\]

We write

\[\text{some bears} \uparrow \text{dance}\]

Now think about these two

\[\text{some bears waltz}\]
\[\text{some bears move}\]
Assume that

\[
\text{waltz} \leq \text{dance} \leq \text{move}
\]

and

\[
\text{grizzly} \leq \text{bear} \leq \text{animal}
\]

Assume that we’re talking about a situation where

\[
\text{some bears dance}.
\]

We write

\[
\text{some bears}^\uparrow \text{ dance}^\uparrow
\]
Assume that

\[ \text{waltz} \leq \text{dance} \leq \text{move} \]

and

\[ \text{grizzly} \leq \text{bear} \leq \text{animal} \]

Put the arrows on the words bears and dance.

1. Some bears dance.
2. No bears dance.
3. Not every bear dances.
4. John sees every bear.
5. Mary sees no bear.
6. Most bears dance.
7. Any bear in Hawaii would prefer to live in Alaska.
8. If any bear dances, Harry will be happy.
9. If you play loud enough music, any bear will start to dance.
10. Doreen didn’t see any bears dance in her dorm room.
The answers are:

1. Some bears\(\uparrow\) dance\(\uparrow\).
2. No bears\(\downarrow\) dance\(\downarrow\).
3. Not every bear\(\uparrow\) dances\(\downarrow\).
4. John sees every bear\(\downarrow\).
5. Mary sees no bear\(\downarrow\).
6. Most bears dance\(\uparrow\). (No arrow goes on bears in this one.)
7. Any bear\(\downarrow\) in Hawaii would prefer to live in Alaska.
8. If any bear\(\downarrow\) dances\(\downarrow\), Harry will be happy.
9. If you play loud enough music, any bear\(\downarrow\) will start to dance\(\uparrow\).
10. Doreen didn’t see any bears\(\downarrow\) dance\(\downarrow\) in her dorm room.
The answers are:

1. Some bears↑ dance↑.
2. No bears↓ dance↓.
3. Not every bear↑ dances↓.
4. John sees every bear↓.
5. Mary sees no bear↓.
6. Most bears dance↑. (No arrow goes on bears in this one.)
7. Any bear↓ in Hawaii would prefer to live in Alaska.
8. If any bear↓ dances↓, Harry will be happy.
9. If you play loud enough music, any bear↓ will start to dance↑.
10. Doreen didn’t see any bears↓ dance↓ in her dorm room.

The last examples are important because we want to account for shallow inferences in more syntactically realistic fragments.
An example which we’ll see later in detail

The idea is to meld inference with categorial grammar

**Example**

Let $\Gamma$ contain the following inequalities

\[
\begin{align*}
\text{cat} : p & \leq \text{animal} : p \\
\text{every} : p \rightarrow (p \rightarrow t) & \leq \text{most} : p \rightarrow (p \rightarrow t)
\end{align*}
\]

Below is a derivation from $\Gamma$:

\[
\begin{align*}
\text{cat} \leq \text{animal} & \quad \text{(ANTI)} \\
\text{every [animal]} \leq \text{every cat} & \quad \text{(POINT)} \\
\text{every animal} \leq \text{most cat} & \quad \text{(TRANS)} \\
\text{every animal vomits} \leq \text{most cat vomits}
\end{align*}
\]
I have arranged the course material in a number of units:

- overview + examples
- syllogistic logics
- logics and the sizes of sets
- logics with individual variables
- polarity/monotonicity
- another direction: proof-theoretic semantics
- answers to homework + any further topics you ask about
Problem 1
The semantics of *All X are Y* in a model is that 

\[ [X] \subseteq [Y] \].

What should the semantics be for

*All X which are Y are Z?*

Problem 2
Is the following valid or not? And why?

*All skunks are mammals*

\[ \text{All who respect all skunks respect all mammals} \]  (9)

Problem 3
Is the following a sound inference?

*Everyone who likes every logician is a logician*  *There is a duck*

\[ \text{There is a logician} \]

If so, explain what the duck is doing.
If not, give a counter-model.
Problem 4

Is the following valid or not? And why?

Everyone likes everyone who likes Pat
Pat likes every clarinetist
Everyone likes everyone who likes everyone who likes every clarinetist
Objections to keep in mind

If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways.

W. V. O. Quine, *Word and Object*

The logical systems that one would get from looking at inference involving surface sentences would contain many copies of similar-looking rules.

They would miss a lot of generalizations.
If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways.

W. V. O. Quine, *Word and Object*

---

The systems would contain ‘rules’ that are more like complex deduction patterns that need to be framed as rules only because one lacks the machinery to break them down into more manageable sub-deductions. Moreover, those complex rules would be unilluminating.
If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways.

W. V. O. Quine, *Word and Object*

- The systems would lack variables, and thus they would be tedious and inelegant.
If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways.

W. V. O. Quine, *Word and Object*

- It would be impossible to handle standard topics such as quantifier-scope ambiguities.
Similar rules can be succinctly grouped into meta-rules: van Benthem showed how to do this in a general way in the monotonicity calculus.

It’s true that the systems have some complex “rules”. Perhaps this could be turned into an advantage, but aiming for a theory of “shallow inference”

The systems can have variables in some form. But they lead to an interesting issue!

Quantifier-scope ambiguities can be handled. But all the other similar phenomena aren’t yet touched.