# Extending a natural language proof theory: On ordinary comparatives

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# Goals

- Provide a natural language proof theory for MacCartny's NatLog (MacCartney (2009a,b)) that is
  - sound and complete via representation.
- Provide a semantics for ordinary comparatives
  - John is taller than Mary

  that can be plausibly integrated into such a proof theory (or one like it) (see Moss (2011)).

# Natural language proof

$$\dfrac{\dfrac{\text{couch} \equiv \text{sofa} \in \Gamma}{\Gamma \vdash \text{couch} \equiv \text{sofa}}\ \text{Refl} \qquad \dfrac{}{\Gamma \vdash \text{sofa} \wedge \overline{\text{sofa}}}\ \wedge_1}{\Gamma \vdash \text{couch} \wedge \overline{\text{sofa}}}\ \equiv, \wedge$$

# Synthetic logic

## Definition 1: Syntax of $\mathcal{S}$

Let $\Phi$ be a countable set of proposition letters $p_1, \ldots p_n$ for $n < \omega$ which I will refer to as the set of proper terms. Then,

1. If $\varphi$ is a proper term, then so is $\overline{\varphi}$;
2. If $\varphi$ and $\psi$ are proper terms, then

$$\begin{array}{lll} \varphi \equiv \psi, & \varphi \sqsubset \psi, & \varphi \sqsupset \psi, \\ \varphi \,{}^{\wedge}\, \psi, & \varphi \downharpoonleft\!\upharpoonright \psi, & \varphi \smile \psi \end{array}$$

   are synthetic terms.

# Lexical (MacCartney) relations

- **Equivalence**
  - couch ≡ sofa
  - ‘Couch’ and ‘sofa’ are synonyms
- **Forward entailment**
  - crow ⊏ bird
  - ‘Crow’ is a hyponym of ‘bird’
- **Reverse entailment**
  - crow ⊐ bird
  - ‘Bird’ is a hypernym of ‘crow’
- **Negation**
  - man ^ $\overline{\text{man}}$
  - ‘Man’ and ‘non-man’ are antonyms
- **Alternation**
  - cat ^ dog
  - ‘Cat’ and ‘dog’ are alternates
- **Covers**
  - animal ^ $\overline{\text{human}}$
  - ‘Animal’ and ‘non-human’ are covers

## Definition 2: Synthetic models

Let a synthetic model $\mathbb{M} = \langle D, [\![\cdot]\!] \rangle$, where $D$ is a non-empty set and $[\![\cdot]\!]$ is an interpretation function taking proper terms $\varphi$ to their denotations in $D$ such that

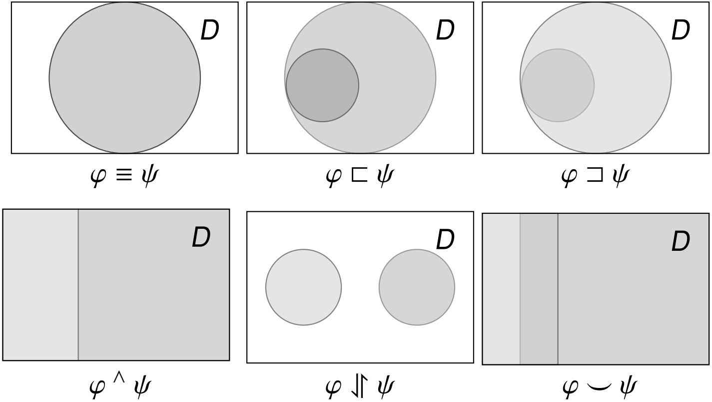
1. $[\![\overline{\varphi}]\!] = D - [\![\varphi]\!]$;
2. $[\![\overline{\overline{\varphi}}]\!] = [\![\varphi]\!]$;
3. $[\![\varphi]\!] \neq [\![\overline{\varphi}]\!]$; and
4. $$[\![\varphi]\!] \neq \begin{cases} \varnothing & \text{or} \\ D & \end{cases}$$

## Definition 3: Synthetic semantics

Let $\varphi$ and $\psi$ be proper terms and $R$ a MacCartney relation. Define the denotation of the synthetic term $\varphi R \psi$, written $[\![\varphi R \psi]\!]$, as

1. **Equivalence**
   - $\mathbb{M} \models \varphi \equiv \psi \Leftrightarrow [\![\varphi]\!] = [\![\psi]\!]$
2. **Forward entailment**
   - $\mathbb{M} \models \varphi \sqsubset \psi \Leftrightarrow [\![\varphi]\!] \subset [\![\psi]\!]$
3. **Reverse entailment**
   - $\mathbb{M} \models \varphi \sqsupset \psi \Leftrightarrow [\![\varphi]\!] \supset [\![\psi]\!]$
4. **Negation**
   - $\mathbb{M} \models \varphi \,^\wedge\, \psi \Leftrightarrow ([\![\varphi]\!] \cap [\![\psi]\!] = \varnothing) \wedge ([\![\varphi]\!] \cup [\![\psi]\!] = D)$
5. **Alternation**
   - $\mathbb{M} \models \varphi \mid \psi \Leftrightarrow ([\![\varphi]\!] \cap [\![\psi]\!] = \varnothing) \wedge ([\![\varphi]\!] \cup [\![\psi]\!] \neq D)$
6. **Covers**
   - $\mathbb{M} \models \varphi \smile \psi \Leftrightarrow ([\![\varphi]\!] \cap [\![\psi]\!] \neq \varnothing) \wedge ([\![\varphi]\!] \cup [\![\psi]\!] = D)$

# MacCartney relations



**Theorem 1: Mutual exclusivity**

If $\mathbb{M}$ is a synthetic model then

$$\mathbb{M} \models \varphi R \psi \Rightarrow \mathbb{M} \not\models \varphi S \psi$$

for $R \neq S$.

**Definition 4: Entailment**

Let $\Gamma$ be a set of synthetic terms. $\Gamma \models \varphi R \psi$ just in case

$$\mathbb{M} \models \Gamma \Rightarrow \mathbb{M} \models \varphi R \psi$$

| R,S | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\wedge$ | $\downharpoonleft\!\upharpoonright$ | $\smile$ |
|---|---|---|---|---|---|---|
| $\equiv$ | $\equiv$ | $\sqsubset$ | $\sqsupset$ | $\wedge$ | $\downharpoonleft\!\upharpoonright$ | $\smile$ |
| $\sqsubset$ | $\sqsubset$ | $\sqsubset$ | $\cdot$ | $\downharpoonleft\!\upharpoonright$ | $\downharpoonleft\!\upharpoonright$ | $\cdot$ |
| $\sqsupset$ | $\sqsupset$ | $\cdot$ | $\sqsupset$ | $\smile$ | $\cdot$ | $\smile$ |
| $\wedge$ | $\wedge$ | $\smile$ | $\downharpoonleft\!\upharpoonright$ | $\equiv$ | $\sqsupset$ | $\sqsubset$ |
| $\downharpoonleft\!\upharpoonright$ | $\downharpoonleft\!\upharpoonright$ | $\cdot$ | $\downharpoonleft\!\upharpoonright$ | $\sqsubset$ | $\cdot$ | $\sqsubset$ |
| $\smile$ | $\smile$ | $\smile$ | $\cdot$ | $\sqsupset$ | $\sqsupset$ | $\cdot$ |

## Definition 5: M-rules

Let Γ be a set of synthetic formulas. Then,

$$\frac{\Gamma \vdash \varphi R \psi \qquad \Gamma \vdash \psi S \vartheta}{\Gamma \vdash \varphi T \vartheta}\; R,S$$

are rules of the calculus.

# Sample M-rule

**$\sqsubset, \sqsubset$-rule**

$$\frac{\Gamma \vdash \varphi \sqsubset \psi \qquad \Gamma \vdash \psi \sqsubset \vartheta}{\Gamma \vdash \varphi \sqsubset \vartheta}\ {\scriptstyle \sqsubset,\sqsubset}$$

## Definition 6: D-rules

Let Γ be a set of synthetic terms. Then,

$$\frac{}{\Gamma \vdash \varphi \equiv \varphi}\,\equiv_1 \qquad \frac{\Gamma \vdash \varphi \equiv \psi}{\Gamma \vdash \psi \equiv \varphi}\,\equiv_2 \qquad \frac{}{\Gamma \vdash \varphi \wedge \overline{\varphi}}\,\wedge_1 \qquad \frac{\Gamma \vdash \varphi \wedge \psi}{\Gamma \vdash \psi \wedge \varphi}\,\wedge_2$$

$$\frac{\Gamma \vdash \varphi \sqsubset \psi}{\Gamma \vdash \psi \sqsupset \varphi}\,\sqsubset_1 \qquad \frac{\Gamma \vdash \varphi \sqsupset \psi}{\Gamma \vdash \psi \sqsubset \varphi}\,\sqsupset_1 \qquad \frac{\Gamma \vdash \varphi \downharpoonleft\!\upharpoonright \psi}{\Gamma \vdash \psi \downharpoonleft\!\upharpoonright \varphi}\,\downharpoonleft\!\upharpoonright_1 \qquad \frac{\Gamma \vdash \varphi \smile \psi}{\Gamma \vdash \psi \smile \varphi}\,\smile_1$$

are rules of the calculus.

## Theorem 2: Complementation

1. $\Gamma, \varphi \equiv \psi \vdash \varphi \wedge \overline{\psi}$
2. $\Gamma, \varphi \wedge \psi \vdash \varphi \equiv \overline{\psi}$
3. $\Gamma \vdash \varphi \equiv \overline{\overline{\varphi}}$ (double negation)
4. $\Gamma, \varphi \sqsubset \psi \vdash \overline{\psi} \sqsubset \overline{\varphi}$ (contraposition)
5. $\Gamma, \varphi \sqsupset \psi \vdash \overline{\psi} \sqsupset \overline{\varphi}$
6. $\Gamma, \varphi \downharpoonleft\!\upharpoonright \psi \vdash \varphi \sqsubset \overline{\psi}$
7. $\Gamma, \varphi \smile \psi \vdash \varphi \sqsupset \overline{\psi}$

# Natural language proof

**Theorem 2.1**

$$\Gamma, \varphi \equiv \psi \vdash \varphi \wedge \overline{\psi}$$

$$\dfrac{\dfrac{\text{couch} \equiv \text{sofa} \in \Gamma}{\Gamma \vdash \text{couch} \equiv \text{sofa}}\ \text{Refl} \qquad \dfrac{}{\Gamma \vdash \text{sofa} \wedge \overline{\text{sofa}}}\ \wedge_1}{\Gamma \vdash \text{couch} \wedge \overline{\text{sofa}}}\ \equiv, \wedge$$

### Definition 7: Explosion

Let $\Gamma$ be a set of synthetic terms. Then,

$$\frac{\Gamma \vdash \varphi R \psi \qquad \Gamma \vdash \varphi S \psi \quad \text{for } R \neq S}{\Gamma \vdash \varphi' T \psi' \text{ for all } \varphi' T \psi'}\text{Exp}$$

is a rule of the calculus.

**Definition 8: Consistency**

$\Gamma$ is consistent if, and only if $\Gamma \nvdash \varphi R \psi$ for some synthetic term $\varphi R \psi$.

# Inconsistency

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{animal} \sqsubset \text{life} \in \Gamma}{\Gamma \vdash \text{animal} \sqsubset \text{life}}\,\text{Refl}}{\Gamma \vdash \text{life} \sqsupset \text{animal}}\,\sqsubset_1 \qquad \dfrac{\text{animal} \smile \overline{\text{human}} \in \Gamma}{\Gamma \vdash \text{animal} \smile \overline{\text{human}}}\,\text{Refl}}{\Gamma \vdash \text{life} \smile \overline{\text{human}}}\,\sqsupset, \smile \qquad \dfrac{\text{life} \sqsupset \overline{\text{human}} \in \Gamma}{\Gamma \vdash \text{life} \sqsupset \overline{\text{human}}}\,\text{Refl}}{\Gamma \vdash \varphi' T \psi' \text{ for all } \varphi' T \psi'}\,\text{Exp}$$

# Representation

**Theorem 3: Completeness**

Let Γ be a consistent of synthetic terms. Then,

$$\Gamma \vdash \varphi R \psi \Leftrightarrow \Gamma \models \varphi R \psi$$

- Every consistent Γ induces an order on the set of proper terms Φ;
- That ordered set can be transformed into an orthoposet;
- Every orthoposet can be represented as a system of sets (Zierler and Schlessinger (1965), Calude *et al* (1999) Moss (2007));
- The system of sets will function as a synthetic model.

# Ordinary comparatives

# Ordinary comparatives

- **Ordinary Comparatives**
  1. John is taller than Mary
- **Negation**:
  1. John is no taller than Mary
  2. John isn't taller than Mary
- **Coordination**:
  1. John is taller than Mary or Sue is
  2. John is taller than Mary is or Sue is
- **Quantification**:
  1. John is taller than everyone else
  2. John is taller than exactly two woman
- **Temporal expressions**:
  1. John is taller than he was yesterday
  2. John will be taller than he is today
- **Modal expressions**:
  1. John is taller than he was yesterday
  2. John will be taller than he is today

# Previous analyses

1. **Degrees** (Seuren 1973)

$$\exists d\, (\mathbf{tall}\,(john, d) \wedge \neg\, (\mathbf{tall}\,(mary, d)))$$

where $d$ is a variable ranging over degrees.

2. **Precisifications** (Kamp 1975)

$$\exists p\, (p \models \mathbf{tall}\,(john) \wedge \neg\, (p \models \mathbf{tall}\,(mary)))$$

where $p$ is a variable ranging over precisifications of a base model $\mathbb{M}$.

3. **Comparison classes** (Klein 1980)

$$\exists c\, (c \models \mathbf{tall}\,(john) \wedge \neg\, (c \models \mathbf{tall}\,(mary)))$$

where $c$ is a variable ranging over comparison classes.

4. **‘Measure’ functions** (Kennedy 1997)

$$\mathbf{tall}\,(john) > \mathbf{tall}\,(mary)$$

where $\mathbf{tall}\,(john)$ and $\mathbf{tall}\,(mary)$ correspond to the degrees of John’s and Mary’s (maximal) height respectively.

# Derived degrees

- Authors have tried to derive 'degrees' as equivalence classes of more basic ontology such as individuals (Cresswell 1976; Bale 2008; van Rooij 2010; Bale 2011; van Rooij 2011).
- Natural to view entities $e$ as being an ordered set of 'atoms' $e = (A, \leq)$.
- Minimally extend Frege's definition of having the same amount as to formalize Carnap's (1967) example of arranging rods by length.
- Let $\delta$ be a function taking an individual $e$, world $w$, and time $t$, to the set of triples $(e', w', t')$ such that $e$ is order isomorphic ($\cong$), i.e., has the same height, to $e'$ at world $w'$ and time $t'$.
- $\delta$ induces a linear order $>$ over the set $I \times W \times T$, where each equivalence class of entity, world, time triples corresponds to a 'degree'.
- This order can serve as a 'trans-world/time measuring rod'.

# Derived degrees (cont.)

$$\left\{\begin{array}{ll} (e_1, w_8, t_2), & (e_2, w_1, t_2) \\ (e_1, w_4, t_1), & (e_3, w_4, t_4) \end{array}\right\} > \ldots > \left\{\begin{array}{ll} (e_1, w_3, t_1), & (e_3, w_2, t_1) \\ (e_2, w_3, t_9), & (e_5, w_2, t_4) \end{array}\right\}$$

# Current analysis

**Definition 9: taller**

Let $e$ and $e'$ be entities, $w$ a world, and $t$ a time. Then,

$$e \textbf{ taller } e' \text{ at } w \text{ and } t \Leftrightarrow \delta((e, w, t)) > \delta((e', w, t))$$

(which can intuitively be understood as reading, '$e$ has more degrees of height than $e'$ at world $w$ and time $t$').

- Define **shorter** as the order dual ($<$) of **taller**.
- Define **as tall as** ($\geq$) as **not taller than**.

# Negation

**Claim**: (1) ⇔ (2) ⇔ (3)

1. John is not taller than Mary
2. Mary is as tall as John
3. John is as short as Mary

van Rooij (2008)

$$\neg(\textit{john}\ \textbf{taller}\ \textit{mary}) \Leftrightarrow \textit{mary}\ \textbf{as tall}\ \textit{john}$$
$$\Leftrightarrow \textit{john}\ \textbf{as short as}\ \textit{mary}$$

# Entity/sentential-level coordination

**Claim**: (1) ⇔ (2)

1. John is taller than Mary (is) and Sue (is)
2. John is taller than Mary or Sue (is)

von Stechow (1984)

$$(\textit{john}\ \textbf{taller}\ \textit{mary}) \wedge (\textit{john}\ \textbf{taller}\ \textit{sue}) \Leftrightarrow \textit{john}\ \textbf{taller}\ \max(\{\textit{mary}, \textit{sue}\})$$

$$\Leftrightarrow \textit{john}\ \textbf{taller}\ (\textit{mary} \sqcup \textit{sue})$$

# Quantification

1. John is taller than everyone (else at the current world and time)

$$\forall x \, (x \neq john \rightarrow (\delta((john, w, t)) > \delta((x, w, t))))$$

2. Every man is taller than some woman (at the current world and time)

$$\forall x \, (\mathbf{man} \, (x) \, (w) \, (t) \rightarrow \exists y \, (\mathbf{woman} \, (y) \, (w) \, (t) \wedge (\delta((x, w, t)) > \delta((y, w, t)))))$$

# Time and modality

1. John might be taller than Mary

$$\exists u\,(\delta((\mathit{john}, u, t))\ \textbf{taller}\ \delta((\mathit{mary}, u, t)))$$

2. John will be taller than Mary

$$\exists s\,(\delta((\mathit{john}, w, s))\ \textbf{taller}\ \delta((\mathit{mary}, w, s)) \wedge t \prec s)$$

# Inference patterns

Using

- properties and concepts of (linear) orders; and
- classical rules of inference, e.g., De Morgan's Law

we get a robust set of inferences involving comparatives.

# Future work

- Develop a natural language proof theory integrating ordinary comparatives that builds on the insights of the semantics of ordinary comparatives presented here (again see Moss (2011)).
  - What orders do other adjectives induce, e.g., **clever** (see Kamp (1975) among others)?
  - How are those orders determined?
- Integrate ‘comparative proof theory’ with a larger natural language proof theory that allows reasoning across construction types
  - Temporal expressions (**before**, **after**, etc.)
  - Quantifiers (**every**, **some**, etc.) via
- Monotonicity calculi (van Benthem (1986), Valencia (1991), Moss (2010) among others).

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