E370 HOMEWORK 5, SPRING 2010, PROF. W. E. BECKER

Confidence interval: Ex 8.3, 8.5, 8.6, 8.13, 8.17, 8.18, 8.22, 8.26, and 8.42.

8.3 (2'×4)

a. α = 0.01 or 1 percent, since we have a 99 percent confidence interval.
b. $Z_{0.005}$ = NORMSINV(0.005) = 2.576.
c. The point estimate of the population mean, i.e., the value of the sample mean should be the midpoint of the upper limit and the lower limit of the interval. Thus, it should be $94,245(=(65345+123145)/2).
d. The lower end of the interval: 65,345 = $\bar{x} - z_{\alpha/2}(\sigma/\sqrt{n})$ = 94,245 - 2.575 $(\sigma/10)$

The upper end of the interval: 123,145 = $\bar{x} + z_{\alpha/2}(\sigma/\sqrt{n})$ = 94,245 + 2.575 $(\sigma/10)$

Either one of the above two gives σ = 112,233.
(Note that the population standard deviation ($112,233) is extremely large relative to the estimated mean ($94,245). This is possible if the population of income of Cadillac owners is highly right skewed.)

8.5 When the sample size is increased fourfold, the margin of error will be one-half as large, because the standard error of the estimator will be one-half as large. Thus, the length of the confidence interval will be also one-half as large.

8.6 The width (or length) from the lower to the upper end is 18.60. Thus, $z_{\alpha/2}$ = (18.6/2)/(12/SQRT(16)) = 3.1. NORMSDIST(3.1) = 0.999. This implies that $\alpha/2 = 0.001$ and thus $\alpha = 0.002$. The corresponding level of confidence is 99.8 percent.

8.13 (2'×6)

a. 0.949931328 = 1-TDIST(1.72,21,1)
b. 0.005002535 = TDIST(3.707,6,1)
c. 0.974999839 = 1-TDIST(2.306,8,1)
d. -3.012282832 = -TINV(0.01,13)
e. 12.92442903 = TINV(0.001,3)
f. 2.807337296 = TINV(0.01,23)

8.17

a. n=100, x-bar=17.5, and s=6.75 are given. Because σ is unknown, we need to use t in the construction of a 95 percent confidence interval as follows. t = TINV(0.05,99) = 1.984. The confidence interval is thus $16.157 < \mu < 18.843$. Thus, the estimated population mean is \$16.16 to \$18.84. (If you use z, it would be $16.177 < \mu < 18.823$, where NORMINV(0.025,0,1) = 1.95996.)
b. The level of confidence changes only when there is a change in α. Changes in *n* and σ affect only the width of the interval and not the level of confidence; i.e.:
An increase in α decreases the level of confidence, and decreases the width (or length) of the interval.
An increase in *n* decreases the width (or length) of the interval.
An increase in the population standard deviation (σ) increases the width (or length) of the interval.

8.18 From the given sample data, x-bar =\$22,340.833 and s=\$2,081.131, respectively. Under the assumption of normality of the population of the car prices, the distribution of *x-bar* is normal with a mean of \$21,750 and a standard error of \$849.618 (=2081.13/SQRT(6)) . Thus, P(xbar>22,340.833) = P(t>0.6954) = 0.2589, where from EXCEL we have 0.2589 = TDIST(0.6954,5,1).

	A	B
1	19500	
2	25550	
3	21995	
4	22500	
5	23500	
6	21000	
7	22340.83	=AVERAGE(A1:A6)
8	2081.13	=STDEV(A1:A6)
9	0.6954	=(A7-21750)/(A8/SQRT(6))
10	0.2589	=TDIST(A9,6-1,1)

8.22 A 95 percent confidence interval for the population proportion of Indiana school bus drivers with blemished driving records will be approximately 0.2286<pi<0.4114, where z=1.96. Thus, the population proportion of all Indiana School bus drivers is 22.86% to 41.14%.

8.26 A 95 percent confidence interval for the population proportion with faulty brakes is 52% to 70% of all trucks.

8.42 The sample mean and the sample standard deviation are calculated to be \$1180.245 and \$1795.443, respectively. Thus, the 95 percent confidence interval for the population mean capitalization is \$339,961,000 to \$2,020,529,000, where TINV(0.05, 19) = 2.093025.

9.6 (3’×9)

a. H_0: $\mu = 87.4$
H_A: $\mu \neq 87.4$

b. H_0: $\mu = 0.05$
H_A: $\mu \neq 0.05$

c. H_0: $\mu \geq 3$
H_A: $\mu < 3$

d. H_0: $\pi \leq 0.10$
H_A: $\pi > 0.10$

e. H_0: $\mu \leq 1.8\%$
H_A: $\mu > 1.8\%$

f. H_0: $\mu = 378$
H_A: $\mu \neq 378$

g. H_0: $\mu \leq 1000$
H_A: $\mu > 1000$

h. H_0: $\pi \leq 0.07$
H_A: $\pi > 0.07$

i. H_0: $\pi \geq 0.06$
H_A: $\pi < 0.06$

9.7 (3’×2)

a. H_0: $\pi \leq 0.5$
H_A: $\pi > 0.5$

b. H_0: $\mu = 0.20$
H_A: $\mu \neq 0.20$

9.13 (9’×2)

H_o: $\mu \geq 29$; H_A: $\mu < 29$, $\sigma = 16$, $\alpha = 0.05$

a. n=64, $\bar{x} = 26$. Calculate the z statistic = (26-29)/(16/8) = -1.5 the p-value is 0.066807229 = NORMDIST(-1.5,0,1,1), which is greater than 0.05, so we do not reject the null hypothesis for the value $\alpha = 0.05$.

b. n=64, $\bar{x} = 23$. Calculate the z statistic = (23-29)/(16/8) = -3 the p-value is 0.001349967 = NORMDIST(-3,0,1,1), which is less than 0.05, so we reject the null hypothesis for $\alpha = 0.05$.

9.15 (2’+2’+9’)

a. Given the fact that we have a really small sample, it is important to know whether the population variance is known or not because if we know the population variance, we can use the z-test for hypothesis testing, whereas is we don't, we have to use the t-test, assuming the population is normally distributed.

b. Fisher probably did not know the standard deviation of the population, because it is a parameter that is difficult to obtain. We sample because we do not know parameter values.

c. (Population Variance = 1.5) Use EXCEL to find out the sample mean and standard deviation: Sample Mean = 1.58; Sample Standard Deviation = 1.229995483. If we assume we don't know the population standard deviation, we have to use the t-test = (1.58 - 0)/(1.23/3.16) = 4.06, $P(t \geq 4.06) = 0.001421$. If we assume we know the population variance to be 1.5, population standard deviation to be 1.225, we use the z- test = (1.58 - 0)/(1.225/3.16) = 4.08, $P(z \geq 4.08) = 0.00002$

The results are different because the sample size is small. Thus, the assumption of a known population variance may affect the conclusion drawn here. (NOTE: This demonstration of the difference between z and t can be done with any null hypothesize value of μ .

9.16 (3’+6’)

a. H_o: $\mu \geq 10{,}000$; H_A: $\mu < 10{,}000$, where μ is the mean mileage of a rental car in the company.

b. Using EXCEL, calculate the following: Sample Mean = 9296.555556; Sample Standard Deviation = 2862.787283. We must use the t-test and to do so we must assume the population to be normal. Calculate the t-test statistic = (9297-10000)/(2863/3) = -0.74. The associated p-value is 0.240225307 = TDIST(0.74,8,1). Thus, the company's claim cannot be accepted as true for any reasonable level of significance.

9.19 (9')

Set up: H_o: $\mu \geq 20$; H_A: $\mu < 20$, where μ is the average fill in a glass. $\alpha = 0.05$. Use EXCEL to calculate the following from the sample data: Mean = 19.93888889; Sample Standard Deviation = 0.149786886. Calculate the value of the t-statistic, assuming that the population is normally distributed, = (19.939 - 20)/(0.149/3) = -1.228. The associated p-value is TDIST(1.228,8,1) = 0.127. The hypothesis that this pub serves, on average, 20 ounces glasses or more cannot be rejected under $\alpha = 0.05$.

9.21 (9')

n = 1200. H_o: $\pi \geq 0.50$; H_A: $\pi < 0.50$. $\alpha = 0.05$. p = 0.45.

$z = (0.45 - 0.50) / \sqrt{0.50 \times (1 - 0.50) / 1200} = -3.47$.

p-value = NORMSDIST(-3.47) = 0.00026 < 0.05 = α. $\Rightarrow$ Reject the null and, at a significance level of 5%, we conclude that less than 50 percent of all hourly workers think their firm is a good place to work.

Could also be done with binomial distribution but no need to because n is big.

9.23 (9')

a. H_o: $\pi \geq 0.90$; H_A: $\pi < 0.90$.
b. n = 40. $\alpha = 0.05$. p = 0.85.

   $z = (0.85 - 0.90) / \sqrt{0.90 \times (1 - 0.90) / 40} = -0.05/0.04743 = -1.054092553$.

   p-value = NORMSDIST(-1.054) =0.146 > 0.05 = α. $\Rightarrow$ Fail to reject the null and, at a Type I error level of 5%, we don't have enough information to conclude that less than 90 percent of the nurses are passionate and caring, and so we don't have enough information to repute Ann's claims.

   Should be done with binomial distribution to obtain exact p-value, but because n > 30 normal approximation for the distribution of the sample proportion is acceptable. The exact p-value is 0.20:

0.206273 =BINOMDIST(0.85*40,40,0.9,1)