A braid diagram consists of four interlocking exact sequences.

1 Braid diagrams and Mayer-Vietoris exact sequences

In the braid diagram below

\[ C_{n+1} \rightarrow F_{n+1} \rightarrow B_n \rightarrow E_n \rightarrow \]
\[ \downarrow D_{n+1} \quad \downarrow A_n \quad \downarrow D_n \quad \downarrow A_{n-1} \]
\[ \downarrow B_{n+1} \quad \downarrow E_{n+1} \quad \downarrow C_n \quad \downarrow F_n \]

the four exact sequences are ABE, ACF, BDF, and CDE. There is a Mayer-Vietoris exact sequence

\[ \cdots \rightarrow A_n \rightarrow B_n \oplus C_n \rightarrow D_n \rightarrow \cdots \]

This comment is useful for two reasons. First it produces a Mayer-Vietories exact sequence. Secondly, it gives a guide for placement of groups when writing down a braid diagram.
2 Where do braid diagrams come from?

They come from a commutative diagram

\[
\begin{array}{cccccccc}
A_n & 
\rightarrow & B_n & 
\rightarrow & E_n & 
\rightarrow & \\
\downarrow & & \downarrow & & \downarrow & & \cdots & \\
C_n & 
\rightarrow & D_n & 
\rightarrow & E'_n & 
\rightarrow & \\
\downarrow & & \downarrow & & \downarrow & & \\
F_n & 
\rightarrow & & & & & F'_n
\end{array}
\]

where the dotted lines are isomorphisms and all vertical and horizontal sequences are exact.

3 Two examples

Example 1. Let \( X = U \cup V \) be an excisive decomposition (\( H_*(U, U \cap V) \rightarrow H_*(X, V) \) is an isomorphism). Then there is a braid diagram with

\[
\begin{align*}
A_n &= H_n(U \cap V) \\
B_n &= H_n U \\
C_n &= H_n V \\
D_n &= H_n X \\
E_n &= H_n(U, U \cap V) \\
F_n &= H_n(V, U \cap V)
\end{align*}
\]

More generally, the homology of a homotopy pushout diagram (a pushout diagram where two maps are cofibrations) gives a braid diagram.
Example 2. Let \( X = U \cup V \) be an excisive decomposition. Then there is a braid diagram with

\[
\begin{align*}
A_n &= H_n(X, U \cap V) \\
B_n &= H_n(X, U) \\
C_n &= H_n(X, V) \\
D_n &= H_n(U \cup V) \\
E_n &= H_{n-1}(U, U \cap V) \\
F_n &= H_{n-1}(V, U \cap V)
\end{align*}
\]