SPIN-PARITY ANALYSIS OF 3π DECAYS IN THE A₂ MESON REGION


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A Dalitz plot with 600 events in the A₂ mass region has been obtained at 7 GeV/c in the reaction π⁻p → A₂⁻p, where A₂⁻ → π⁺π⁻π⁻, using a system of wire spark chambers in connection with the CERN-Missing-Mass Spectrometer. A spin parity analysis shows that for the overall A₂ region (1260 < M < 1360 MeV) \( J^P = 2^+ \) is favoured \( P(x^2) = 400/0 \) over \( 1^- \). When the A region is divided into \( A_{2\text{high}} \) and \( A_{2\text{low}} \) as suggested by the missing mass spectrum, the distributions still favour \( 2^+ \) over \( 1^- \), particularly for \( A_{2\text{low}} \).

Since 1967, when the first Missing-Mass Spectrometer data on the \( A₂ \) region [1] presented evidence that in the mass region between 1260 and 1360 MeV there exist in fact two peaks separated by a narrow hole, supporting evidence in different reactions and centre of mass energies have been presented [2-4].

A possible explanation of the effect is the presence of two resonances very close in mass with the same spin parity or a double pole [1]. An alternative would be to have two resonances with different quantum numbers [5] as perhaps suggested by the KK spectrum of ref. 3. In order to obtain direct experimental evidence on the spin parity, we have analysed the Dalitz plot of \( A_{2\text{low}} \) and \( A_{2\text{high}} \) obtained in the reaction \( \pi^-p \rightarrow A^-p \) at incident momentum of 7 GeV/c and four-momentum transfers to the proton in the range 0.18 - 0.32 (GeV/c)².

675 events have been obtained in the \( A₂ \) region. A comparison between the two peaks shows no clear difference of the density distribution of the Dalitz plot, and we find \( 2^+ \) favoured for both the \( A_{2\text{high}} \) and \( A_{2\text{low}} \) regions, but other hypotheses cannot be definitely ruled out. The spin-parity analysis on the total \( A₂ \) fits well \( J^P = 2^+ \) whereas other assignments are rejected \( (P(x^2) < 0.5\%) \).

The laboratory angles of the three charged decay pions of the \( A₂ \) were measured with a system of two-wide-gap wire spark chambers with magnetostrictive read-out [6]. They were placed as seen in fig. 1 in the forward direction of the CERN missing-mass spectrometer [7], which determined the laboratory angle and momentum of the recoil proton and therefore the missing mass. A missing-mass spectrum of the \( A₂ \) meson with high statistics, obtained in the same run, has been published [1]. The mass-bite accepted with full efficiency was 850 < M < 1650 MeV.

In 65% of the cases, all three pions of an \( A₂ \) decay went into the fiducial area \((150 \times 150 \text{ cm}^2)\) of the wire chambers corresponding to an angular acceptance ± 21°. Data acquisition, sampling, and control of the entire system was done by an online computer.

Identification of 3π events. We have selected the events in the wire chambers which have three tracks coming from the target and giving a good geometrical reconstruction to the proton vertex. Applying three-momentum conservation to these events, with the hypothesis that in the reaction

\[
\pi^- + p \rightarrow p + X^- \\
(1) \\
(2) \\
(3) \\
(4)
\]

only the decay \( X^- \rightarrow \pi^+\pi^-\pi^- \) is present, gives the momenta \( p_i \) \((i = 1, 2, 3)\) of the decay pions

\[
\sum_{i=1}^{3} p_i = p_1 - p_3
\]

where \( p_1 \) and \( p_3 \) are known from the missing-mass spectrometer. Therefore the invariant mass of \( X^- \)
Fig. 1. CERN Missing-Mass Spectrometer (schematic layout) with wire chambers in the forward direction for decay analysis.

\[ M_X^2 = \sum_{i=1}^{3} (p_i^2 + m_i^2)^{1/2} - \sum_{i=1}^{3} p_i \]  

(3)

can be obtained without using a magnet. This method fails for events which are coplanar in the laboratory system (see below under biases).

A constraint to eliminate the events where \( X^- \rightarrow \pi^+\pi^-\pi^- + n\pi^0\)'s is obtained by requiring energy conservation. We define the difference \( \Delta E \) between the out-going and the incident energy in reaction (1):

\[ \Delta E = \sum_{i=1}^{3} (p_i^2 + m_i^2)^{1/2} + (p_3^2 + M_2^2)^{1/2} - (p_1^2 + M_1^2)^{1/2} - M_2 \]  

(4)

where \( P_1, M_1 \) refer to the incident pion, \( M_2 \) to the target proton, \( P_3 \) and \( M_3 \) refer to the recoil proton. \( \Delta E \) is shown for all our data in fig. 2a. The sharp peak at \( \Delta E = 0 \) contains the events with \( X^- \rightarrow \pi^+\pi^-\pi^- \), whereas the events with other decay modes are in the long tails.

In order to obtain a pure sample of three charged pion decays, we apply a cut \( \Delta E < 15 \text{ MeV} \), given by the experimental resolution. This cut reduces the total mass spectrum (fig. 2b) to the three charged pion spectrum shown in fig. 2c. The difference between figs. 2b and 2c, i.e. \( |\Delta E| > 15 \text{ MeV} \), is shown in fig. 2d, where the remaining small \( A_2 \) peak is due to uncertainty in the \( \Delta E \) cut and other decay modes of the \( A_2 \).

A Dalitz-plot (fig. 3a) has been constructed for the \( A_2 \) region (1260 < \( M < 1360 \text{ GeV} \)) containing, after all cuts, 675 events. For each event we have plotted six points, since we cannot distinguish between different charges. These data have been obtained in 58 hours of running.

**Biases.** Two kinds of biases have been examined in detail by a Monte-Carlo calculation in which possible polarization effects of the \( A_2 \) were not taken into account.

i) Geometrical biases due to limits in the finite solid angle of the wire chambers (including the "beam killers", see fig. 1). The loss of events due to solid angle cuts amounts to 35%. The lost events are uniformly distributed over the whole Dalitz plot.

ii) Coplanarity bias. Since our method fails for coplanar events, they fall into the tails of the \( \Delta E \) distribution (fig. 2a) and they are removed by the \( |\Delta E| < 15 \text{ MeV} \) cut. This is not a uniform loss, but it deviates from uniformity at maximum 25% at the border of the Dalitz plot (whereas the most interesting hypotheses 1' and 2' differ mainly in the center; their distinction being therefore not affected).

**Spin-parity analysis.** The density distribution of the Dalitz plot depends on the spin-parity of the resonance. In the analysis, an incoherent
Fig. 2. Selection of events with $X^- \rightarrow 3$ charged pions. 
a) Difference $\Delta E$ between the outgoing and the incident energy; 
b) Total missing mass spectrum; c) Events for $|\Delta E| < 15$ MeV ($A_2$ enhanced); d) Events for $|\Delta E| > 15$ MeV.
Table 1. Confidence levels for different spin-parities in the $A_2$ region.

<table>
<thead>
<tr>
<th>Mass interval</th>
<th>Hypothesis</th>
<th>$%$ resonance</th>
<th>$% p\pi$ background</th>
<th>$% 3\pi$ background</th>
<th>$P(X^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_2^{\text{low}}$</td>
<td>$2^+$</td>
<td>40</td>
<td>10</td>
<td>50</td>
<td>38%</td>
</tr>
<tr>
<td>$1254 &lt; M &lt; 1307$ MeV</td>
<td>$1^-$</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>0.3%</td>
</tr>
<tr>
<td>$A_2^{\text{high}}$</td>
<td>$2^+$</td>
<td>40</td>
<td>0</td>
<td>60</td>
<td>54%</td>
</tr>
<tr>
<td>$1307 &lt; M &lt; 1360$ MeV</td>
<td>$1^-$</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>10%</td>
</tr>
<tr>
<td>Total $A_2$</td>
<td>$2^+$</td>
<td>40</td>
<td>20</td>
<td>40</td>
<td>38%</td>
</tr>
<tr>
<td>$1260 &lt; M &lt; 1360$ MeV</td>
<td>$1^-$</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>0.1%</td>
</tr>
<tr>
<td>$2^-(p)$</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>$1^+(d)$</td>
<td>40</td>
<td>30</td>
<td>30</td>
<td>0.1%</td>
<td></td>
</tr>
</tbody>
</table>

mixture of resonance and two background terms have been assumed. For the resonant term we have used the theoretical distributions of Zemach [7] for the $p\pi$ decay of a $3\pi$ resonance. For the background we have assumed two terms, a $p\pi$ background with a distribution uniform in the $p$ bands and a $3\pi$ background uniform over the whole Dalitz plot.

Since our events must enter six times in the Dalitz plot, we have studied only the density distribution of one sextant where each event is counted only once. For this purpose we have used as variables the three-pion mass $M$ and the polar coordinates $r$ and $\theta$ within the sextant of the Dalitz plot. From $r$ we define a new variable $S = (r/R)^2$ where $R$ is the distance from the centre to the edge of the Dalitz plot for the particular $M$ value.

If one integrates over $M$ (taking into account the Breit-Wigner shape of the resonance) and $\theta$, the distribution of the events as a function of $S$ has a characteristic shape for each particular spin-parity value of the resonance. A uniform $3\pi$ background adds a contributions which is constant as a function of $S$. The $p\pi$ background has a very similar shape to a $2^-$ or $1^+$ state.

Results. The main aim of the analysis is to obtain further information on the possible spin parities of the two regions $A_2^{\text{low}}$ and $A_2^{\text{high}}$. The existence of the $\pi\gamma$ decay mode apparently over the full width [3] of the $A_2$ suggests that the assignments $1^-$, $2^+$ are the most interesting. In particular $1^-$ has been suggested for $A_2^{\text{low}}$ [2] to account for the absence of $KK$ in the $A_2^{\text{low}}$ region in the BNL experiment [3].

* Equal spin-parity for $A_2^{\text{high}}$ and $A_2^{\text{low}}$ would most easily explain the interference between the two peaks in the total MMS + CBS $A_2$ mass-spectrum [2].

![Fig. 3. Spin parity analysis of the $A_2$ Dalitz plot.](image-url)
a) Dalitz plot for $1260 < M < 1360$ MeV;  

b) Radial density distribution of one sextant of the Dalitz plot, fitted with the hypotheses $J^P = 2^+$ (solid line) and $J^P = 1^-$ (dashed line);  

c) Radial density distributions for $A_2^{\text{low}}$ (1254 – 1307 MeV) and $A_2^{\text{high}}$ (1307 – 1360 MeV).
Since the statistics and mass resolution for our decay analyzed 3 pion events selected above were insufficient to detect the hole in the centre of the A₂ we used the information from the overall split A₂ [2] to devise the A₂ region into two halves and examine these independently. We assume that in the A₂ there exist two independent resonances with masses of 1284 and 1330 MeV and widths of 30 MeV. Fig. 3c shows the radial Dalitz plot distribution for the two mass regions 1254 < M₃π < 1307 and 1307 < M₃π < 1360. No significant difference can be seen in the two distributions. When we fit the two regions A₂ₗow and A₂ₕighb with 2⁺ and 1⁻ hypotheses we find that 2⁺ is favored in both regions, although the 1⁻ cannot be ruled out for A₂ₕighb.

The results are shown in table 1.

We have also examined the spin parity for the overall A₂ mass region 1260 < M₃π < 1360. For the overall sample the 2⁺ is strongly preferred over the assignments 1⁻, 1⁺, 2⁻, as shown for example for 1⁻ in fig. 3b. In table 1 we have listed the best χ² obtained for each hypothesis. We have made the condition that the resonance be at least 30%, which is deduced from the amplitude of the A₂ peak in the missing-mass spectrum. The percentage of uniform background is also at least 30%, from the comparison of the pp mass spectrum relative to the total 3π spectrum.

We conclude that the 3π Dalitz plots for the mass regions of A₂ₗow and A₂ₕighb taken separately favor spin parity 2⁺. Assuming a single spin parity for the overall region we obtain a clear preference for 2⁺ over other simple spin parity hypotheses [8].

References

5. For possible theoretical interpretations we refer to the H. Harari Rapporteur's talk, at 14th Int. Conf. on High-Energy Physics, Vienna (1968) p. 195.