Unitarization procedures applied to a Strongly Interacting EWSBS

Rafael L. Delgado

A.Dobado, M.J.Herrero, Felipe J.Llanes-Estrada and J.J.Sanz-Cillero,

2015 Intern. Summer Workshop on Reaction Theory,
Indiana University, June 8-19, 2015

Electroweak symmetry breaking: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$

- Three would-be Goldstone bosons $\omega$.
- Equivalence theorem: for $s \gg 100$ GeV, identify them with the longitudinal components of $W$ and $Z$.
- A 125-126 GeV scalar “Higgs” resonance $\varphi$. 
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### Empirical situation

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- **IMPORTANT:** No new physics!! *If there is any...*
- Four scalar light modes, a strong gap.
- Natural: further spontaneous symmetry breaking at $f > v = 246$ GeV?
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H (125.9 GeV, PDG 2013)
W (80.4 GeV), Z (91.2 GeV)
Effective Field Theory + Unitarity: similarity with low–energy (i.e.: hadronic) physics

Chiral Perturbation Theory plus Dispersion Relations.
Simultaneous description of $\pi\pi \rightarrow \pi\pi$ and $\pi K\pi K \rightarrow \pi K\pi K$ up to 800-1000 MeV including resonances.

Lowest order ChPT (Weinberg Theorems) and even one-loop computations are only valid at very low energies.

A. Dobado, J.R. Peláez
We have no clue of what, how or if new physics...
Most general NLO Lagrangian for $\omega, h$ at low energy

\[
\mathcal{L} = \left[ 1 + 2a \frac{h}{v} + b \left( \frac{h}{v} \right)^2 \right] \frac{\partial_\mu \omega^a \partial_\mu \omega^b}{2} \left( \delta^{ab} + \frac{\omega^a \omega^b}{v^2} \right) \\
+ \frac{4a_4}{v^4} \partial_\mu \omega^a \partial_\nu \omega^a \partial_\mu \omega^b \partial_\nu \omega^b \\
+ \frac{4a_5}{v^4} \partial_\mu \omega^a \partial_\nu \omega^a \partial_\mu \omega^b \partial_\nu \omega^b \\
+ \frac{2d}{v^4} \partial_\mu h \partial_\mu h \partial_\mu \omega^a \partial_\nu \omega^a + \frac{2e}{v^4} \partial_\mu h \partial_\mu \omega^a \partial_\nu h \partial_\nu \omega^a \\
+ \frac{1}{2} \partial_\mu h \partial_\mu h + \frac{g}{v^4} (\partial_\mu h \partial_\mu h)^2
\]
We also consider the case of the $\gamma\gamma \to W_L^+ W_L^-$ and $\gamma\gamma \to Z_L^+ Z_L^-$ scattering (unitarization is work in progress).

Current efforts for measuring these channels (although only 2 events measured).


Wait for LHC Run–II and CMS–TOTEM.

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Particular cases of the theory

- $a^2 = b = 1$, SM
- $a^2 = b = 0$, Higgsless ECL$^2$
- $a^2 = 1 - \frac{v^2}{f^2}$, $b = 1 - \frac{2v^2}{f^2}$, $SO(5)/SO(4)$ MCHM$^3$
- $a^2 = b = \frac{v^2}{f^2}$, Dilaton$^4$

---


$^3$See, for example, K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719, 165 (2005)

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Experimental bounds on low-energy constants

As it would require measuring the coupling of two Higgses, there is no experimental bound over the value of $b$ parameter\(^5\). Over $a$, at a confidence level of $2\sigma$ (95%),

- CMS\(^6\) ............................................ $a \in (0.88, 1.15)$
- ATLAS\(^7\) ............................................ $a \in (0.96, 1.34)$

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\textsuperscript{5}Giardino, P.P., Aspects of LHC phonem., PhD Thesis (2013), Università di Pisa
\textsuperscript{6}Report No. CMS-PAS-HIG-14-009.
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The form of the partial wave is

\[
A_{IJ}(s) = \frac{1}{64\pi} \int_{-1}^{1} d(\cos \theta) P_J(\cos \theta) A_I(s, t, u)
\]

\[
= A_{IJ}^{(0)} + A_{IJ}^{(1)} + \ldots
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Which will be decomposed as

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A_{IJ}^{(0)} = Ks
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A_{IJ}^{(1)} = \left( B(\mu) + D \log \frac{s}{\mu^2} + E \log \frac{s}{\mu^2} \right) s^2
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As \( A_{IJ}(s) \) must be scale independent,

\[ B(\mu) = B(\mu_0) + (D + E) \log \frac{\mu^2}{\mu_0^2} \]
Unitarization procedures

\[ A^{IAM}(s) = \frac{[A^{(0)}(s)]^2}{A^{(0)}(s) - A^{(1)}(s)} \]

\[ A^{N/D}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)}} + \frac{1}{2} g(s) A_L(-s) \]

\[ A^{IK}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)}} + g(s) A_L(s) \]

\[ A^{0K}(s) = \frac{A_0(s)}{1 - iA_0(s)} \quad A_L(s) = \pi g(-s) D s^2 \]

\[ A_R(s) = \pi g(s) E s^2 \]

\[ g(s) = \frac{1}{\pi} \left( \frac{B(\mu)}{D + E} + \log \frac{-s}{\mu^2} \right) \]

PRD 91 (2015) 075017
Validity range of unitarization procedures

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<tr>
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- The IAM method cannot be used when $A^{(0)} = 0$, because it would give a vanishing value.
- The N/D and the IK methods cannot be used if $D + E = 0$, because in this case computing $A_L(s)$ and $A_R(s)$ is not possible.
- The naive K-matrix method,

$$A_0^K(s) = \frac{A_0(s)}{1 - iA_0(s)},$$

fails because it is not analytical in the first Riemann sheet and, consequently, it is not a proper partial wave compatible with microcausality.
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Scalar-isoscalar channels

From left to right and top to bottom, elastic $\omega\omega$, elastic $hh$, and cross channel $\omega\omega \to hh$, for $a = 0.88$, $b = 3$, $\mu = 3$ TeV and all NLO parameters set to 0. PRL 114 (2015) 221803, PRD 91 (2015) 075017.
Vector-isovector channels

We have taken $a = 0.88$ and $b = 1.5$, but while for the left plot all the NLO parameters vanish, for the right plot we have taken $a_4 = 0.003$, known to yield an IAM resonance according to the Barcelona group, PRD 90 (2014) 015035.

Scalar-isotensor channels ($IJ = 20$)

From left to right, $a = 0.88$, $a = 1.15$. We have taken $b = a^2$ and the NLO parameters set to zero. Both real and imaginary part shown. Real ones correspond to bottom lines at left and upper at low $E$ at right.

Isotensor-scalar channels ($IJ = 02$)

$a = 0.88$, $b = a^2$, $a_4 = -2a_5 = 3/(192\pi)$, all the other NLO param. set to zero. PRD 91 (2015) 075017.
Resonance from $W_L W_L \rightarrow hh$

$a = 1, \ b = 2, \ IAM,\ 
elastic\ channel\ W_L W_L \rightarrow W_L W_L$

Rafael L. Delgado,\ 
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Possible New Resonance from\ $W_L W_L$-hh Interchannel\ Coupling,
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Possible New Resonance from $W_L W_L$-$hh$ Interchannel Coupling,

PRL 114 (2015) 221803
Motion of the resonance mass and width

Dependence on $b$ with $a^2 = 1$ fixed (upper curve) and for $a = 1\xi$ and $b = 12\xi$ with $\xi = \nu/f$ as in the MCHM (lower blue curve).

PRL 114 (2015) 221803
Resonances in $W_L W_L \rightarrow W_L W_L$ due to $a_4$ and $a_5$ paramet.

Espriu, Yencho, Mescia
PRD88, 055002
PRD90, 015035
At right, exclusion regions include resonances with $M_{S,V} < 600$ GeV.
Resonances in $W_L W_L \rightarrow W_L W_L$ due to $a_4$ and $a_5$ paramet.

- $a = 0.90$, $b = a^2$
- PRD 91 (2015) 075017
- From left, clockwise, $IJ = 00, 11, 20$
- Excluding resonances $M_S < 700$ GeV, $M_V < 1.5$ TeV

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Unitarization procedures......
Resonances in $W_L W_L \rightarrow W_L W_L$ due to $a$ and $a_4$ parameters

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PRD 91 (2015) 075017

Rafael L. Delgado
Resonances in $W_L W_L \rightarrow W_L W_L$ due to $a$ and $b$ parameters

- **PRL & PRD** 91 (2015) 075017
- From left, clockwise, $IJ = 00, 11, 20$
- Excluding resonances $M_S < 700$ GeV, $M_V < 1.5$ TeV
- Constraint over $b$ even without data about $W_L W_L \rightarrow hh$ and $hh \rightarrow hh$ scattering processes.

Rafael L. Delgado
Resonances in $W_L W_L \rightarrow W_L W_L$ due to $b$, $g$, $d$ and $e$ parameters

Effective Theory, PRD 91 (2015) 075017, isoscalar channels ($I = J = 0$).
Two parameterizations have been considered (two effective Lagrangians obtained), giving the same results.

One loop computation for the process $\gamma\gamma \rightarrow \omega_L^a\omega_L^b$.

Simple result compared with the complexity of the computation.

$$\mathcal{M} = ie^2(\epsilon^\mu_1 \epsilon^\nu_2 T^{(1)}_{\mu\nu})A(s, t, u) + ie^2(\epsilon^\mu_1 \epsilon^\nu_2 T^{(2)}_{\mu\nu})B(s, t, u)$$

$$T^{(1)}_{\mu\nu} = \frac{s}{2}(\epsilon_1 \epsilon_2) - (\epsilon_1 k_2)(\epsilon_2 k_1)$$

$$T^{(2)}_{\mu\nu} = 2s(\epsilon_1 \Delta)(\epsilon_2 \Delta) - (t - u)^2(\epsilon_1 \epsilon_2) - 2(t - u)[(\epsilon_1 \Delta)(\epsilon_2 k_1) - (\epsilon_1 k_2)(\epsilon_2 \Delta)]$$

$$\Delta^\mu = p^\mu_1 - p^\mu_2$$
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\[
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\]

\[
T_{\mu\nu}^{(2)} = 2s(\epsilon_1 \Delta)(\epsilon_2 \Delta) - (t - u)^2(\epsilon_1 \epsilon_2)
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\[
\Delta^{\mu} = p_{1}^{\mu} - p_{2}^{\mu}
\]
\[ M(\gamma \gamma \rightarrow zz)_{\text{LO}} = 0 \]
\[ A(\gamma \gamma \rightarrow zz)_{\text{NLO}} = \frac{2ac_\gamma}{v^2} + \frac{(a^2 - 1)}{4\pi^2 v^2} \]
\[ B(\gamma \gamma \rightarrow zz)_{\text{NLO}} = 0 \]
\[ A(\gamma \gamma \rightarrow \omega^+ \omega^-)_{\text{LO}} = 2sB(\gamma \gamma \rightarrow \omega^+ \omega^-)_{\text{LO}} = -\frac{1}{t} - \frac{1}{\mu} \]
\[ A(\gamma \gamma \rightarrow \omega^+ \omega^-)_{\text{NLO}} = \frac{8(a_1^r - a_2^r + a_3^r)}{v^2} + \frac{2ac_\gamma}{v^2} + \frac{(a^2 - 1)}{8\pi^2 v^2} \]
\[ A(\gamma \gamma \rightarrow \omega^+ \omega^-)_{\text{NLO}} = 0 \]
Ref. JHEP1407 (2014) 149 (scattering $\gamma\gamma \rightarrow \omega_L^+\omega_L^-$) only contains the 1–loop computation.

The next steps will be...

- computing $\omega\omega \rightarrow hh$ matrix element,
- and performing the unitarization.

Both for $\gamma\gamma$ and $\omega_L\omega_L$ scattering, we should

- introduce fermion loops (work in progress),
- non–vanishing values for $M_H$, $M_W$, $M_Z$,
- and a full computation without using the equivalence theorem.

Besides, we are working on the $t\bar{t} \rightarrow \omega_L\omega_L$ channel.
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Work in progress

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New scalar particle + mass gap

- New physics would very likely imply strong interactions, in elastic $W_L W_L$ and inelastic $\rightarrow hh$ scattering.
- For $a^2 = b \neq 1$, strong elastic interactions are expected for $W_L W_L$, and a second, broad scalar analogous to the $\sigma$ in nuclear physics possibly appears. We identify a pole at 800 GeV or above in the second Riemann sheet very clearly, the question is whether it corresponds to a physical particle since it is so broad.
- Even if $a \approx 1$, with small $\lambda_i$ (higher powers of $h$), but we allow $b > a^2$, one can have strong dynamics resonating between the $W_L W_L$ and $hh$ channels, likewise possibly generating a new scalar pole of the scattering amplitude in the sub-TeV region.
- This fact allows to constrain $b$ even in the absence of data about $W_L W_L \rightarrow hh$ and $hh \rightarrow hh$, just looking at the $W_L W_L$ scattering.
- Finally, as an exception, for $a^2 = b = 1$, we recover the Minimal Standard Model with a light Higgs which is weakly interacting.
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- **SM → unitarity.**
- Higgsless model (now experimentally excluded) → unitarity violation in $WW$ scattering → new physics.
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- Not necessarily, with the present experimental bounds.
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Back Slides
1) IAM method

This method needs a NLO computation,

\[ \tilde{t}^\omega = \frac{t_0^\omega}{1 - \frac{t_0^\omega}{t_1^\omega}} , \]

where

\[ t_1^\omega = s^2 \left( \frac{D \log \left( \frac{s}{\mu^2} \right)}{s^2} + E \log \left( \frac{s}{\mu^2} \right) + (D + E) \log \left( \frac{\mu^2}{\mu_0^2} \right) \right) \]
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We have checked\(^8\), for the tree level case,

\[ \mathcal{L} = \frac{1}{2} g(\varphi / f) \partial_\mu \omega^a \partial^\mu \omega^b \left( \delta_{ab} + \frac{\omega^a \omega^b}{v^2 - \omega^2} \right) + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} M_\varphi^2 \varphi^2 - \lambda_3 \varphi^3 - \lambda_4 \varphi^4 + \ldots \]

\[ g(\varphi / f) = 1 + \sum_{n=1}^{\infty} g_n \left( \frac{\varphi}{f} \right)^n = 1 + 2\alpha \frac{\varphi}{f} + \beta \left( \frac{\varphi}{f} \right)^2 + \ldots \]

where \( a \equiv \alpha \nu / f, \ b = \beta \nu^2 / f^2 \), and so one, the concordance with the methods.

\(^8\text{See J.Phys. G41 (2014) 025002.}\)
Check at tree level

We have checked, for the tree level case,

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so that, for \( \tilde{t}_\omega, \)

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for \( \beta = \alpha^2 \) (elastic case),

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$N \to \infty$, with $\nu^2/N$ fixed. The amplitude $A_N$ to order $1/N$ is a Lippmann-Schwinger series,

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Note: actually, $N = 3$. For the (iso)scalar partial wave (chiral limit, $I = J = 0$),

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(elastic scattering at tree level only $\beta = \alpha^2$. See ref. J.Phys. G41 (2014) 025002). Ansatz

$$\tilde{t}^\omega(s) = \frac{N(s)}{D(s)},$$

where $N(s)$ has a left hand cut (and $\text{Im } N(s > 0) = 0$)

$D(s)$ has a right hand cut (and $\Re D(s < 0) = 0$);

$$D(s) = 1 - \frac{s}{\pi} \int_0^\infty ds' \frac{N(s')}{s'(s' - s - i\epsilon)}$$

$$N(s) = \frac{s}{\pi} \int_{-\infty}^0 ds' \frac{\text{Im } N(s')}{s'(s' - s - i\epsilon)}$$
$f = 2v$, $\beta = \alpha^2 = 1$, $\lambda_3 = \frac{M_\phi^2}{f}$, $\lambda_4 = \frac{M_\phi^2}{f^2}$. OX axis: s in TeV$^2$. 
Tree level, modulus of $\tilde{t}_\omega$, $K$ matrix

- All units in TeV.
- From top to bottom, $f = 1.2, 0.8, 0.4$ TeV
- $\Lambda = 3$ TeV
- $\mu = 100$ GeV
Im $t_\omega$ in the N/D method, $f = 1\,\text{TeV}, \beta = 1, m = 150\,\text{GeV}$
$\text{Re } t_\omega \text{ and } \text{Im } t_\omega$, large $N$, $f = 400 \text{ GeV}$
Re $t_\omega$ and Im $t_\omega$, large $N$, $f = 4$ TeV
Tree level, motion of the pole position of $t_\omega$
K–matrix, $M_\phi = 125$ GeV, $f \in (250$ GeV, $6$ TeV))