Picture Book of Low Transverse Momentum Physics
Caltech ~1975
Geoffrey Fox

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- Inclusive

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A PICTURE BOOK OF
HIGH-ENERGY SCATTERING

by

G. C. Fox

MEMOIRS OF AN
ALSO INTERESTING
EXPERIMENTALIST
There is only one EXPERIMENT . . .

Beam ( $\pi^\pm$, $K^\pm$, $p$, $n$, $\delta$, Hyperons . . . )

--- $\rightarrow$ n final Particles

--- $\rightarrow$ mainly $\pi$ mesons

$<n> \approx 15$ at ISR

(Prob $\approx 1000$ GeV/c)

**MENU**

I) Categories of Scattering Processes

(Pictures 2 $\rightarrow$ 11)

2-Body

Quasi 2-Body

Multiparticle

Inclusive

Typical Life-History of a 2$\rightarrow$2 Reaction

II) Theoretical Description of 2 and Quasi 2-Body Reactions

(Pictures 12 $\rightarrow$ 25)

Regge Poles and Cuts

Geometrical Ideas

Diffraction

Symmetry Schemes

III) Regge Pole Description of Inclusive Reactions

(Pictures 26 $\rightarrow$ 29)
Picture 1: In the Beginning

According to last week’s lecture, there were $10^6$ particles. Now there is only one experiment in high energy physics – shown at the top of the first picture. Not all the $10^6$ particles are allowed as beams or target – there are only around 20 combinations ranging from $\pi^- p$ to $\gamma d$. However, any of the final particles may be chosen from our $10^6$ candidates and we can have from two (low energy) to a mean of 15 – 20 particles (ISR = CERN Intersecting Storage Rings) in the final state. Disregarding conservation laws and fact that 80% of those 15 particles were pions, we find some $10^{60}$ reactions. My task is thus $10^{54}$ times more difficult than last week’s speaker. I won’t bore you by taking $10^{54}$ times as long – rather it will be sufficient if my salary reflects greater complexity of my subject.

As ordained by the menu, I will first categorize scattering processes. Then I will describe the various "theoretical" ideas that have proved useful. This will mainly refer to $2 \rightarrow 2$ scattering; but in the last section, I will reveal how the Regge pole ideas have proved successful in inclusive reactions.

As befits my (official) status as an experimentalist, I will indicate how the various Caltech high energy scattering experiments will revolutionize our knowledge and more generally suggest which areas will see greatest progress in the weary years to come.
<table>
<thead>
<tr>
<th>Scattering Process</th>
<th>Observables</th>
<th>Approx. Number of Species Measured</th>
<th>% of Cross-Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^+ \pi^- \to \pi^+ \pi^- ) roughly constant with energy</td>
<td>Total ( d\sigma/dt ), Polarization R and A</td>
<td>40</td>
<td>ALL</td>
</tr>
<tr>
<td>( \pi^- \pi^0 ) falls like ( 1/\text{energy}^{0.5} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi^- \pi^0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| \( \pi^+ \pi^- \to n \) (Quasi 2 in final state) | \( d\sigma/dt \), Decay Density Matrix Elements | 400 | NONE | 75% | ZERO TO 20% |
| \( \pi^- \pi^0 \to n \) | | | | |
| \( p \) | | | | |

**Genuine Multiparticle Events** (\( \pi^+ \pi^- \to n \))

<table>
<thead>
<tr>
<th></th>
<th>Production Cross-Section</th>
<th>Differential Cross-Section Has 3n-4 Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^- \pi^0 \to n )</td>
<td>2000</td>
<td>NONE</td>
</tr>
<tr>
<td>( \pi^- \pi^0 \to n )</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>( p )</td>
<td></td>
<td>TO 80%</td>
</tr>
</tbody>
</table>

**Inclusive Reactions** (Roughly constant with energy)

\[ \frac{d^2\sigma}{dp_L dp_T} \]

| \( \pi^- \pi^0 \to n \) | 40 | ALL |
| \( p \) | | ALL |

| \( p \) | | | | |
Picture 2: Clerical Classification

The reader will remember that the observed particles fall into two classes: First, the "stable" particles ($\pi$, $K$, $p$, $\Lambda$, etc.) which if they decay at all do so weakly. Second, the unstable particles ($\rho$, $\Delta(1238)$...) which decay in the mere bat of an eyelid 1 fermi in length and fluttering at the speed of light. The latter are only seen as resonances (bumps in the mass distribution) of their strong interaction decay products.

The second picture indicates a convenient division of reactions based on the number and class of the involved particles. It also indicates the variety of data (number of different reactions measured) and makeup of the total cross-section at various energies. Let us consider the four classes of reactions in turn and so define our subject.
Picture 3: SPINLESS SCATTERING

\[ \pi^+ \pi^- \rightarrow \pi^+ \pi^- \]

is described by one complex amplitude \( A(S,t) \)

\( n^+ ' s \) are not available as a target - this is only a theorist's dream of a reaction!

\[ S = (p_1 + p_2)^2 \] corresponds to laboratory momentum of incident beam

\[ t = (p_1 - p_3)^2 \] corresponds to laboratory scattering angle of particle 1 to 3.

**OBSERVABLES**

Total Cross-Section (\( \sigma_{\text{total}} \)) \( \propto \text{Im} A(S,t=0) / S \) \( \quad \sum \text{over} \quad \pi^+, \text{Everything} \)

Differential Cross-Section \( d \sigma / d t \) \( \propto \frac{1}{S^2} \left\{ \text{Re} A(S,t)^2 + \text{Im} A(S,t)^2 \right\} \)

**TODAY'S PROBLEM**

WHAT (Experimental) and WHY (Theoretical) is dependence of \( \text{Re} A \) and \( \text{Im} A \) on \( S \) and \( t \).
The simplest reactions involve stable particles; we have some 40 of these. These processes are described by quantum mechanical amplitudes.

In the case of spinless particles, there is but one such amplitude which is a function of the invariants \( s \) and \( t \). The basic measurements are total cross-sections which are \( \text{Im} A(s,t=0) \) and differential cross-section \( \frac{d\sigma}{dt} \alpha |A(s,t)|^2 \). Here \( s \) is determined by lab momentum \( p_{\text{lab}} \) (at high energy, \( s = 2mp_{\text{lab}} \) where \( m \) is the target mass) and \( t \) by the scattering angle. As defined, \( t \) is \( \leq 0 \) in the physical region and \( t = 0 \) corresponds to forward scattering (\( \cos \theta_{\text{cms}} = 1 \)) and the most negative allowed value of \( t \) is \( t = -s \) at high energy, and this corresponds to backward scattering (\( \cos \theta_{\text{cms}} = -1 \)). The usual measurement is thus several \( t \)-values for fixed \( s \).

Although \( t \) is, as explained above, \( \leq 0 \) in the physical region the axioms of quantum field theory show \( A(s,t) \) to be an analytic function when continued in the complex \( s \) and \( t \) plane. Thus, one studies \( A(s,t) \) theoretically for all values of \( s \) and \( t \).

Today's lecture is, of course, the theoretical and experimental study of the form of \( A(s,t) \), i.e., why particles are produced whereas last week's lecture was on what particles were produced and what were their properties.

Theorists concentrate on stable particle reactions because the measurements \( |A(s,t)|^2 \) are simply related to the basic quantum mechanical amplitudes \( A(s,t) \), and there are no technical problems in either implementing theoretical notions for the process or applying them to the data. Experimentally the reactions have a very well defined signature; and as there are (at most) two particles in the final state, no great difficulties in counting the events. Correspondingly, the present data is at times very accurate; future data will be of the same type and fill in the picture at higher energies and different momentum transfers.
Picture 4: Spin \( \frac{1}{2} \), Spin 0 Elastic Scattering

\[ \text{e.g. } \pi^+ p \rightarrow \pi^+ p \]

Two Complex Amplitudes \( N(S,t) \) and \( F(S,t) \) describe process:

\( N(S,t) \) is Spin

- Nonflip Amplitude

[Diagram showing two spin states, \( \pi^+ \rightarrow \pi^+ \)]

where Spin state (\( \uparrow \)) has \( J_z = +\frac{1}{2} \) and (\( \downarrow \)) \( J_z = -\frac{1}{2} \).

\( F(S,t) \) is Spin

- Flip Amplitude

[Diagram showing two spin states, \( \pi^+ \rightarrow \pi^- \)]

Observables

\( \sigma_{\text{total}} \propto \text{Im} N(S,t=0)/S \) as simple as spinless case

\( \frac{d\sigma}{dt} \propto \left\{ \text{Re}N^2 + \text{Im}N^2 + \text{Re}F^2 + \text{Im}F^2 \right\}/S^2 \) which is twice as complicated as spinless example. There are extra observables which use polarized targets and/or observe the polarization (spin) of the final nucleon.

\( P \frac{d\sigma}{dt} \propto 2 \text{Im} (N F^*) \) : data good for elastic scattering; reasonable for charge exchange processes.

\( R \frac{d\sigma}{dt} \propto 2 \text{Re} (N F^*) \) : data poor and only exists for elastic scattering.

\( A \frac{d\sigma}{dt} \propto (\text{Im}N^2 - \text{Im}F^2) \) : requires measuring spins of initial and final particles.
Pack 4: Spin, the Essential Compilation

Spinless particle scattering is only a theorist's dream. The simplest real reaction involves at least two spin \( \frac{1}{2} \) particles. Then there are two quantum mechanical amplitudes: spin nonflip \( N(s,t) \); spinflip \( F(s,t) \). \( \frac{d\sigma}{dt} \) measures \( |N|^2 + |F|^2 \) and is less clearly related to \( N \) and \( F \) separately; the theorist has to work twice as hard to predict both amplitudes — (as we will see this is a nontrivial difficulty) — even then structure in one (e.g., a zero) may be obscured in the measurements \( \frac{d\sigma}{dt} \) by the other amplitude.

This technical problem may be solved by polarization measurements. Such experiments are the largest virtually untouched field where scattering experiments can be done at current (\(<\) NAL) accelerators. I will not dwell on the subject, I covered it in my colloquium last year.
Picture 5: Plague of Resonance Reactions

Even more horrible: four complex amplitudes corresponding to the four (2 Spin = 1) Spin States of Spin 3/2 \( \Delta^{++} \). However more observables:

\[
\frac{d\sigma}{dt} \propto \left[ |A_2|^2 + |A_3|^2 + |A_2|^2 + |A_2|^2 \right]
\]
\[
S_{2,3} \frac{d\sigma}{dt} \propto |A_2|^2 + |A_2|^2
\]
\[
S_{3,-1} \frac{d\sigma}{dt} \propto \text{Real} \left( A_2 A_2^* + A_2 A_2^* \right)
\]
\[
S_{3,1} \frac{d\sigma}{dt} \propto \text{Real} \left( A_3 A_3^* - A_2 A_2^* \right)
\]

where last 3 are obtained "free" (i.e. without special polarization effects) from decay distribution of p(\( \pi^+ \)) from \( \Delta^{++} \) decay.

i.e. \( W(\theta, \phi) \propto \frac{1}{6} + 2 S_{33}/3 + \cos \theta (1 - 4 S_{33})/2 \)

\[
-2/\sqrt{3} \quad S_{3,-1} \quad \sin^2 \theta \cos 2\phi
\]

\[
-2/\sqrt{3} \quad S_{3,1} \quad \sin 2\theta \cos \phi
\]

where \( \theta \) and \( \phi \) are polar angles of final proton in \( \Delta^{++} \) rest frame.
There are a lot (i.e., around 400) of the next class of reactions; namely, resonance production reactions. This might have been thought to be a good idea. However, the current data is of insufficiently poor quality that it has not been very helpful in understanding the basic dynamics of amplitudes (Regge poles and cuts, etc.) and has been most useful for symmetry schemes relating similar amplitudes. The low statistics follows as most of this data comes from untriggered bubble chamber experiments. Soon resonance reactions will come of age for data from spectrometers and triggered bubble chambers will be 100 to 1000 times better and should test and extend our knowledge of dynamics.

Note:

Disadvantage  
(i) Many amplitudes; even $\pi N \rightarrow \pi \Delta$ has 4, $\pi N \rightarrow \rho N$ has 6.

Advantage:  
(ii) Resonance decay gives information on the relative size of amplitudes; this is only obtained with polarized targets in stable particle reactions.

Disadvantage (?)  
(iii) Possible theoretical difficulty that can't see pure $\pi^+ p \rightarrow \pi^0 \Delta^{++}$ due to background from $\pi^+ p \rightarrow \pi^0 \pi^+ p$. Won't know if real trouble until we get data.
Picture 6: Most of the Cross-section

Genuine Multi Particle Reactions

 Too Many Variables
Can't Even Display Data!

Inclusive Reactions

Everything Else Summed Over

(i) Only 3 (3) continuous variables $s, p_L, (p_T^2)$ to specify momenta of $\pi^+p, \pi^0$. Theoretical description technically simple.

(ii) Scaling (Come and listen to Feynman)

(iii) Cross-sections are large as you use all events: thus early NAL and ISR results are dominantly in this field. At current accelerators, bubble chambers are doing well.

(iv) Recent Mueller formalism ties together theories of inclusive and exclusive reactions.

(Techniques require better data - see pictures 26-29).
Picture 6: Most of the Cross-section

The third category of data is the remaining 80% of the cross-section; the true multiparticle reaction. Some 2000 of these have been measured, mainly just production cross-sections. The full theory must now predict the amplitudes which are functions of at least 5 (in $2 \rightarrow 3$) continuous variables. This is only possible in restricted regions of phase space (e.g., the so-called multiregge region) and maybe experiments can be designed in the future to probe these special regions.

The study of these reactions was revolutionized by the concept of inclusive reactions where you find the cross-sections for producing 1 or more given particles and anything else. These will be discussed by chief revolutionary himself (RPF) and so I will just note:

(i) The theoretical description is technically simple (i.e., there are only three continuous variables $s$, $p_L$, and $p_{\perp}^2$. Of these $s$ drops out at high energy as inclusive cross-sections scale.

(ii) Cross-sections are large which allows ISR measurements (ISR can never measure non-diffractive two-body reactions) and accurate discussion at current energies with bubble chambers. Thus, experimentally the subject is booming.

(iii) Recent theoretical developments (Mueller) have related the theories of inclusive and exclusive reactions which is very pretty; to really test this requires accurate experiments - but they are perfectly possible and this will be a field of great advance in the future.

*An exclusive experiment is one of the types discussed in pictures 3-5 where all particles in a specific final state are measured.
Picture 7: Third Time Lucky
on Regge Poles

\[ \alpha(m^2) = \alpha_0 + \alpha' m^2 \]

\( \alpha_0 \) varies
\( \alpha' \) universal

One gets real live particles, for \( \alpha \) = integer (mesons) and half odd integer (baryons).

Forces are exchange degenerate for mesons and so there is a resonance for each spin: usually one gets only every other resonance i.e. as in A and nuclear trajectories, spacing in spin 2 units.
Gell-Mann and Firestone have explained to you how all observed resonances lie on Regge poles. First, they are straight lines, \( \alpha = \alpha_0 + \alpha' m^2 \); secondly, \( \alpha' \) is universal (same all trajectories) and around 0.85 to 1 \((\text{GeV/c})^{-2}\). This universality is great prediction of dual models. The picture shows three trajectories.

First mesons: the \( \rho - A_2 - g \) particles occur when \( \alpha \) = physical values 1, 2 or 3. They form a trajectory with \( \alpha_0 \approx 0.5 \). It happens that forces are EXD (exchange degenerate) for mesons and so you get a particle for every physical value of \( \alpha \).

Turning to baryons, \( N_1^* \) (1688) - \( N_2^* \) (2200) occur when \( j = \frac{1}{2}, \frac{5}{2}, \frac{9}{2} \) with \( \alpha_0 \approx -0.3 \) and are not EXD.

\( \Delta(1238) - \Delta(1950) - \Delta(2920) \) occur when \( j = \frac{3}{2}, \frac{7}{2}, \frac{11}{2} \) with \( \alpha_0 = 0 \) and are not EXD.

Both these last two trajectories have an EXD partner which fills in unoccupied \( J \) and lies a bit lower in \( \alpha \) v. \( m^2 \) plot. This splitting is expected when - as it happens for baryons - a (small) force perturbs a dominantly EXD (exchange degenerate) force.
Picture 8: 3 Views of $\pi^+ p \rightarrow \pi^+ p$

<table>
<thead>
<tr>
<th>INVARIANTS</th>
<th>$\pi^+ p \rightarrow \pi^+ p$ PHYSICAL REGION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = (p_1 + p_2)^2$</td>
<td>$S \approx (m + p)^2$</td>
</tr>
<tr>
<td>$t = (p_1 - p_2)^2$</td>
<td>$t &lt; 0$</td>
</tr>
<tr>
<td>$u = (p_1 - p_4)^2$</td>
<td>$u &lt; 0$</td>
</tr>
<tr>
<td>note: $S + t + u = 2m^2 + 2\mu^2$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PHYSICAL PROCESS</th>
<th>EFFECT ON $\pi^+ p \rightarrow \pi^+ p$</th>
</tr>
</thead>
</table>

A. Regge Trajectory

\[ S \rightarrow \Delta(1236) \rightarrow \pi^+ \rightarrow p \]

B. Regge Trajectory

\[ t \rightarrow \pi^- \rightarrow s \rightarrow g \rightarrow p \]

C. Nucleon Regge Trajectory

\[ u \rightarrow \pi^- \rightarrow N^*(1688) \rightarrow P \]

Low Energy Resonance Peaks

\[ s \rightarrow \Delta(1236) \rightarrow \Delta(1470) \rightarrow \Delta(2420) \]

\[ s = 1.234^2 \]

High Energy Forward Peak

\[ \frac{d\sigma}{dt} \propto S^2 |t|^{-2} \]  
\[ t > 0 \]

High Energy Backward Peak

\[ \frac{d\sigma}{du} \propto S^2 |u|^{-2} \]  
\[ u < 0 \]

Fixed $S$

Below Threshold (Bound State) True Resonance

Falls faster with $S$ than Forward Peak
Now consider $\pi^+ p$ elastic scattering; it has the direct (s) channel $\Delta$ trajectory shown in A.

But I can turn around a $\pi^+$ and a $p$ to get, * as shown in B, $p\bar{p} \rightarrow \pi^+ \pi^-$ which sees the $\rho$, $A_2$, $g$ trajectory at $t = m^2/\rho \ldots$). I can view these as exchange forces

\[
S \rightarrow \rho \rightarrow \pi^+ \rightarrow S
\]

which give $s^{\alpha}(t)$ behavior at large $s$ and fixed $t$ in physical region, (i.e., $t < 0$) for the basic process $\pi^+ p \rightarrow \pi^+ p$.

Again - as in C - I can turn around the two $\pi$ mesons to get $\pi^- p \rightarrow \pi^- p$ which sees the isospin ($I = 3/2$ and) $I = \frac{1}{2}$ resonances. For our basic reaction $\pi^+ p \rightarrow \pi^+ p$, this corresponds to backward scattering

\[
S \rightarrow p \rightarrow \pi^+ \rightarrow S
\]

behaving like $s^{\alpha}(u)$ for large $s$ and fixed $u$. Note we have introduced the variable $u$ which is linearly related to $s$ and $t$. The swap $t \leftrightarrow u$ just corresponds to $\cos \theta_{\text{c.m.s.}} + \cos \theta_{\text{c.m.s.}}$, i.e., forward + backward scattering. At high energies, $\cos \theta = -1$ is $u = 0$.

* i.e., replace an incoming particle by an outgoing antiparticle (or vice versa). This is called crossing and mathematically it corresponds to analytically continuing the scattering amplitude which is a function of two complex variables. This function represents the three different scattering processes $\pi^+ p \rightarrow \pi^+ p$, $p\bar{p} \rightarrow \pi^+ \pi^-$ and $\pi^- p \rightarrow \pi^- p$ in three different regions of the real $s$-$t$ plane. This is illustrated in the next figure.
Picture 9: Manna from Heaven

So if we look at a scattering process, i.e., $\pi^+ p \rightarrow \pi^+ p$, at low energy ($p_{\text{lab}} \leq 2$ GeV/c), we see direct channel resonances; after this energy, forward and backward peaks develop with $d\sigma/dt \propto s^{2\alpha-2}$ at fixed $t, u$. They are described by the Regge poles in twisted processes. Picture 9(a) - which is drawn to scale - shows this kinematically for $\pi^+ p \rightarrow \pi^+ p$. God said, "Let there be Regge poles - and there were and it was good." Fig. 9(b) shows that it is also crowded...

The figures also show the (beginning of) physical regions of the three processes mentioned in picture 8. The same analytic function, evaluated in these different regions gives the scattering amplitude for the different processes.
Picture 10
Life History of $\pi^+ p$ Scattering

$P_{lab}$ GeV/c

$\sigma_{total}$ ($\pi^+ p$)

$\Delta(1234)$

$\sigma_{total} = \text{constant}$

$\Delta(2420)$

$\Delta(1940)$

Serpukhov

$\sqrt{s}$ GeV/c

"THRESHOLD" $P_{lab} = 0.08$ GeV/c

$d\sigma/dt$

$mb/(GeV/c)^2$

0

-0.009

-0.018

"AT THE $\Delta(1234)$ RESONANCE"

$P_{lab} = 0.3$ GeV/c

1000

0

-0.1

-0.2

$t (GeV/c)^2$

"END OF THE RESONANCE REGION"

$P_{lab} = 2.07$ GeV/c

$d\sigma/dt$

$mb/(GeV/c)^2$

40

20

0

-1.58

-3.15

"HIGH ENERGY" $P_{lab} = 5$ GeV/c

Backward peak can't be seen on this scale
Picture 10: $\frac{dc}{dt}$ Grows Up

Picture 10 shows $N\bar{N}$ scattering in the resonance region and the development of the forward/backwards peaks as energy increases. Concurrently the obvious resonance peaks in the cross-section disappear.

Returning to Picture 9 note that the width observed in $t$ never gets any bigger even though the amount of space ($|t|$ range $\sim s$) grows as $s$ does. In Picture 9 top NAL/ISR energies are (before photographic reduction) 25/75 yards away, respectively. Nature has 25/75 yards of $t$-space to use but only cares to use $\sim 1$ cm of it at each end.
More or less constant cross-section of $\approx 5000 \text{ pb}$

(Diffractive Resonance Production 100 times Smaller)

```
100 pb  $\rightarrow$ 3 pb  $\rightarrow$ 0.003 pb
```

"Large" Non-Diffractive Processes $\approx 20$ times Smaller for Forward Scattering.
Picture 11: Accelerators

So to show true scope of current accelerators, we must invent logarithmic graph paper and the next slide shows current situation. For $s \leq 5$, we have resonance region where we can find direct channel Regge poles by the phase shift techniques discussed last week. Then there is a transition region where forward/backward peaks develop and the concept of duality was spawned. Here phase shift analysis is hard because if you have several resonances of varying spins at a given mass, they add up to give observed forward/backward peaks which get more and more pronounced as energy increases. Again, in this transition region, a high energy theory like Regge theory can only be applied qualitatively as even the necessary kinematic condition $s \gg m^2$ is not valid. Now the low energy dynamics ($\alpha' = 1$) suggest that $1 (\text{GeV/c})^2$ is a good scale for both masses (i.e., kinematics) and dynamics. So $s = 10$ ($p_{\text{lab}} = 5$) is current asymptotic domain where we try to quantitatively test high energy models. We will see that Regge poles do indeed emerge in this domain. However, there is also some evidence that there are other effects (cuts?) which only become apparent at higher energy. So $s = 100$ may be true asymptotia. If this is so, NAL is well placed to study it whereas Serpukhov and the lower energy machines cannot study this region.

Again note the size of cross-sections. Non-diffractive (e.g., $p \Rightarrow p$ exchange) processes are too small to be measured at ISR with its low luminosity. It (ISR) can, however, see elastic scattering very well. NAL will be able to do a reasonable job on non-diffractive processes up to $s = 400$.

* units are always GeV/c for mass. Remember the mass of the proton 0.938 GeV/c and that of the pion is 0.14 GeV/c.
Picture 12: Regge Poles meet Experimental Data

Take, for instance, \((\pi N CEK) \pi^- p \rightarrow \pi^0 n\)

\[ A(s,t) = s^\alpha(t) \left[ e^{-\pi \gamma(t)} - 1 \right] g(p \rightarrow \pi \pi) \cdot g(p \rightarrow NN) \]

Predictions:

(i) Shrinkage: \(ds/dt \propto s^\alpha(t) - 2\) : power of \(s\) decreases as \(-t\) increases

(ii) WSNZ (Wrong Signature Nonsense Zero)

\[ A(s,t) = 0 \text{ when } \alpha(t) = 0 \]

(iii) Factorization: Total Coupling constant is a product of one factor for top and one factor for bottom vertex.
Enough of Introduction. Picture 12 starts our comparison of Regge theory with experiment. Of the three basic predictions - shrinkage, WSNZ and factorization - the first, shrinkage, is most distinctive. Its discovery in $\pi^- p + \pi^0 n$ seven years ago probably convinced people that Regge poles existed (it was before I was invented, so I can't say).

Note that theory predicts that we should have Regge cuts (branch points in the j-plane) as well as Regge poles. Only if the cuts are weak, will we be able to see the poles. Liken unto the days in L. A. when the smog lifts, this does not appear to occur too often. However, as Picture 13 shows, the cuts are weak in $\pi^- p + \pi^0 n$ and we see shrinkage at the predicted rate.
**Picture 13(a): Shrinkage**

\[
\frac{d\sigma}{dt} = \frac{d\sigma}{dt}(t=0) \exp \left[ \frac{2a'(t)}{s} \log s \right]
\]

Only \( s \) dependence, slope of \( d\sigma/dt \) increases with \( s \approx 2p_{\text{lab}} \)

- \( p_{\text{lab}} \) (GeV/c)
  - 18.2
  - 4.25

\[
\log \left( \frac{5.9}{4.25} \right) \approx 0.34
\]

\[
\log \left( \frac{18.2}{4.25} \right) \approx 1.45
\]

**Picture 13(b):**

Fixed \( t = -0.175 \text{ (GeV/c)}^2 \)

\( a(t) = 0.37 \pm 0.01 \)

\( \chi^2 = 39 \text{ on 44 data points} \)

\[
d\sigma/dt = f(t) s^2 a(t)^{-2} \text{ implies } \log(s^2 d\sigma/dt) = \text{const} + 2a(t) \log s \text{ when plotted at fixed } t \text{ and varying } s.
\]

**Picture 13(c):**

\( \rho \) Regge Trajectory from Scattering

Data: \( a_E = 0.58 \pm 1 \)

\( \rho, A_2, g \) curve was \( 0.4 \pm 1 \) determined from real particles
Picture 13: Shrinkage

(a) This shows how shrinkage corresponds to an increase of the t slope of \( d\sigma/dt \) with increasing energy.

(b) Plots such as (a) are not quantitative. Best is to take \( d\sigma/dt \) at fixed \( t \) (interpolated, if necessary) and various lab momenta \( s \).

Then a Regge pole predicts that \( \log d\sigma/dt \) v. \( \log s \) should be a straight line whose slope gives the value of the Regge trajectory function at this \( t \)-value. Picture 13(b) shows a typical \( t \)-value where Regge theory is clearly a great success. We call the \( \alpha \) determined thus from experiment, \( \alpha_{\text{eff}} \).

(c) Similar success occurs in \( \pi^- p \rightarrow \pi^0 n \) at all \( t \) values and the results of many fits such as 13(b) are shown in 13(c). Note that the \( \rho \) Regge trajectory determined this way and that found from the \( \rho \rightarrow A_2 \rightarrow g \) particle masses are in splendid accord.
Picture 16(a) Total Cross-section Differences

\[ \Delta \sigma = \sigma_{\text{total}}(p\bar{p}) - \sigma_{\text{total}}(pp) \]

\[ \Delta \sigma \text{ mB} \]

\[ \Delta \sigma(p\bar{p}) = 0.37 \pm 0.04 \]

\[ \Delta \sigma(pp) = 0.68 \pm 0.04 \]

\[ \Delta \sigma(\pi^{-}p) = 0.67 \pm 0.06 \]

\[ \Delta \sigma(\pi^{+}p) = 0.50 \]

Picture 16(b) Slope of \( d\sigma / dt \)

\[ d\sigma / dt \text{ mB/(GeV/c)^2} \]

\[ d\sigma / dt = 0.58 \pm 0.02 \]

Best Fit

Corn Data

Serpunov Data

Picture 16(c) Shape of \( n^-p \rightarrow n^-p \)

\[ d\sigma / dt \text{ mB/(GeV/c)^2} \]

Best Fit

Corn Data

Serpunov Data

Picture 16(d) Rate of Growth of \( n^-p \rightarrow n^-p \)

\[ d\sigma / dt = \exp(kt) \]

\[ k = 0.22 \pm 0.08 \text{ (GeV/c)^2} \]

Rat7n predicts \[ t = 0.2 \pm 0.2 \text{ (GeV/c)^2} \]

Conclusion: Serpunov data not accurate enough to confirm or deny Regge pole theory. One must await Caltech NAL expt.
Picture 14: A Russian Plot

Clouds on the horizon at higher energy? Low statistic data from Serpukhov sees signs of deviations from this simple picture. Thus, one can also obtain $\alpha_\rho(0)$ from the total cross-section differences $\pi^- p$ minus $\pi^+ p$ (Picture 14(a)). The value gotten disagrees with that from $\pi^- p + \pi^0 n$ in the same energy range (Picture 14(b)) by two standard deviations. This - taken at face value - violates dispersion relations. The effect is not statistically large as the $\alpha_\rho(0) = .58$ line on the 14(a) graph shows.

Also there is no evidence for shrinkage at Serpukhov energies. But there is no evidence against it as the Regge pole curve shows in Picture 14(c).
Picture 15: $\gamma p \rightarrow \pi^+ n$

$(\pi^+ \text{ Photoproduction})$

$\alpha_{\text{eff}}$ has (t) slope $\approx 0$
i.e. no shrinkage.
Could be accident or asymptotia?
Study at NAL with vector dominance related reaction
$\pi^- p \rightarrow \rho^0 n$

$t$ (GeV/c)

Goodbye $\pi, B$

$\rho, A_2$

Regge Poles
Picture 15: Mr. Regge, All Dolled Up, Is Trampled On by Stanford Militants

We will see indirect evidence for deviations of $\pi N$ CEX from Regge pole theory (at current energies) later (Pictures 19, 20). However, some (most?) reactions show blatant violation. Picture 15 shows an $\alpha_{\text{eff}}$ determined as in 13(b) and (c) - but for a different reaction - namely, $\gamma p \rightarrow \pi^+ n$. Generally all photon induced processes show no shrinkage up to $t \approx -3 (\text{GeV/c})^2$. This may be because the Regge poles are small and we are seeing some new phenomena - perhaps we have reached asymptotia already? Alternatively, it may be an accident resulting from the combination of many Regge poles and assorted cuts. It is clearly necessary to study such processes at higher energy. This is not possible directly for photoproduction. However, similar features are observed for the vector dominance related reaction $\pi^- p \rightarrow \rho^0 n$ (replace $\rho^0$ by a photon) which at current energies exhibits the same lack of shrinkage seen in photon processes. This and related reactions will be studied in Caltech's NAL spectrometer experiment.
Picture 16: wrong signature nonsense zeros
Picture 16: On the Second Day, He Rose Again

We now come to the second Regge prediction which is the presence of a zero in the amplitude when the so-called signature factor \( e^{-i\pi\alpha_{\rho}(t)-1} \) in Picture 12) vanishes. This will lead to a dip in \( d\sigma/dt \) plotted v. \( t \) which may be partially filled in or even completely swamped by the presence of "cuts" in the \( j \)-plane. Picture 16 shows a sample of data; Wrong Signature Nonsense Zeroes (WSNZ) are seen in the top three diagrams for three different values of \( \alpha \) and three different exchanged particles. However, in each case, they are in the position predicted by simple Regge theory. Namely:

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Exchange</th>
<th>Signature Factor</th>
<th>WSNZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^- p \rightarrow \pi^- n )</td>
<td>( \rho )</td>
<td>( e^{-i\pi\alpha_{\rho}-1} )</td>
<td>( \alpha_{\rho} = 0 )</td>
</tr>
<tr>
<td>( \pi^+ p \rightarrow \pi^+ \Delta )</td>
<td>( A_2 )</td>
<td>( e^{-i\pi\alpha_{A_2}-1} )</td>
<td>( \alpha_{A_2} = -1 )</td>
</tr>
<tr>
<td>( \pi^+ p \rightarrow \pi^+ \pi^+ )</td>
<td>Nucleon</td>
<td>( e^{-i\pi\alpha_{N}-1} )</td>
<td>( \alpha_{N} = -\beta )</td>
</tr>
</tbody>
</table>

A splendid confirmation of theory? Yes but, like shrinkage, the situation is spotty.

Thus, \( \rho \) exchange \( \pi^- p \rightarrow \omega^0 n \) does not show any \( t = -0.6 \) dip and neither does \( N \) exchange \( \pi^+ p \rightarrow \rho^0 p \) have any \( u = -0.2 \) dip. We may continue our advertising campaign to note that the Caltech NAL \( \pi^- p \rightarrow \pi^- n \) experiment will, as a by-product, study \( \pi^- p \rightarrow \omega^0 n \) (\( \omega^0 \) decays into \( \pi^0 \gamma \)). They will have an example of a Regge success and a Regge failure.

There is some, but it is not perfect, correlation between the failures of the two Regge predictions of shrinkage and WSNZ. For instance, WSNZ are absent in all photoproduction reactions (except \( \gamma p \rightarrow \pi^0 p \)) and these - as exemplified in Picture 15 - show no shrinkage.
Picture 17: Factorization

\[ \frac{g_{AB}}{g_{CD}} = \frac{g_{A'B'}}{g_{C'D'}} \]

\[ \frac{g_{AB}}{g_{CD}} = \frac{g_{A'B'}}{g_{C'D'}} \]

(i) Data
\[
\frac{\pi^+ p \to \pi^+ p^*}{\pi^+ p \to \pi^+ p} = 0.04 \pm 0.01 = 0.05 \pm 0.01
\]

Tests Pomeranchuk Factorization but are \( p^* \) reactions Pomeranchuk Exchange?

(ii) \[
\frac{\pi^- p \to \pi^+ n}{\pi^+ p \to \pi^+ \Lambda} = \frac{\pi^- p \to \eta^0 n}{\pi^+ p \to \eta^0 \Delta^{++}} \text{ works well}
\]

and all reactions agree with Regge Pole theory but it tests exchange degeneracy (EXD) + Factorization

(iii) \[
\frac{\pi^- p \to \pi^+ n}{K^- n \to \pi^- \Lambda} = \frac{\pi^- p \to B^0 n}{K^- n \to B^- \Lambda} \text{ is a complicated example which works to } 20\% \text{ but it tests EXD + SU(3) + Factorization.}
\]

(iv) \( \pi^- \) exchange fails (could be cuts or "conspiracy")

(v) Test in Inclusive Reactions in Future
Picture 17: Factorization

We now come to the third simple Regge prediction listed in Picture 12. As illustrated in Picture 17, it claims that the amplitudes $T$ for four processes are related by:

$$\frac{T(AC \rightarrow BD)}{T(AC' \rightarrow BD')} = \frac{T(A'C \rightarrow B'D)}{T(A'C' \rightarrow B'D')}$$

Unfortunately, it is hard to test directly. The picture indicates that it is satisfied to 20% for the ratio of nucleon resonance $p^*(1688)$ production to elastic scattering for $\pi$ and proton beams. This is meant to test the factorization of the Pomeranchuk singularity. The data is of dubious quality and other recent data makes one doubt that $N^*$ production is, in fact, simple Pomeranchuk exchange.

Other tests can be found but they need additional assumptions. It is definitely known to fail for $\pi$ exchange processes in a rather pretty and well-defined way. This special nature of $\pi$ exchange has never been satisfactorily understood theoretically.

NAL data will test Pomeranchuk factorization from inclusive reactions. We must await this. Also Caltech's NAL spectrometer experiment will give $\pi\pi$ scattering parameters and allow the factorization test

$$\pi\pi \rightarrow \pi\pi = (\pi N + \pi N)^2 / NN + NN$$
Picture 18: ELASTIC SCATTERING

- $t$ (GeV/c)$^2$
- $f(l)$

**Graphical Elements:**
- Plot labeled "pp elastic"
- Data points for 7 GeV/c and 19.2 GeV/c
- Incident momentum $k$
- Target momentum $l$
- Orbital angular momentum $l = l' - k$
- Simple 2-body reactions are peripheral
- i.e. large $l$ dominates,
- As elastic scattering is central (small $l$ dominate)

**Equations and Observations:**
- $pp \rightarrow pp$
- $19.2$ GeV/c
- $l$ in Fermi
- $1/m_{NN}$
Picture 18: Geometrical Point of View

Some years ago, it was thought that so-called absorption corrections would correctly describe deviations from Regge pole theory. Now we know this doesn't work but this way of looking at things showed up some important systematics. Actually originally the basic formulae were sort of guessed intuitively. Now they have been justified by a new rigorous theory; it's a pity they were disproved phenomenologically so many years before theorists took an interest in the subject.

So far we have looked at data in terms of $t$ - now consider conjugate variable $t$ (angular momentum) or rather impact parameter $b = 1/k$; $k = \sqrt{s}/2$ c.m.s. momentum. Introduce Fourier-Bessel transform $f(b)$:

$$f(b) \propto \int_0^{\infty} A(s,t) J_n(b \sqrt{-t}) \, dt$$

$$A(s,t) \propto \int_0^{\infty} f(b) J_n(b \sqrt{-t}) \, b \, db$$

($n$ is the number of units of spinflip; in Picture 4, $n = 0$ for $N$, $n = 1$ for $F$.)

Now if I take pp elastic scattering around 20 GeV/c, its $f(b)$ is peaked near $b = 0$ and disappears for $b \gg 1$ fm. The large component at small $b$ is just reflection via unitarity of central collisions giving many inelastic events. According to the absorption model, the simple two-body processes are squeezed out of center and are dominated by peripheral collisions near $b = 1$ fm. This, on undoing Bessel transform, gives an amplitude $J_n(b \sqrt{-t})$ where $B = 1$ fm. This predicts* dips that depend on $n$, namely, $n = 0$, $t = -2$; $n = 1$, $t = -6$; $n = 2$, $t = -1.2$. The model predicts $t$-dependence and not $s$ dependence at all. Also $t$-dependence varies with helicity flip $n$. In simple Regge theory, zeroes are independent of $n$ (WSNZ depend only on $\alpha(t)$ which is amplitude independent).

*Using the known zeroes of $J_n(x)$. 

Picture 19: $\frac{d\sigma}{dt}(\Xi p \to \Xi p) - \frac{d\sigma}{dt}(\Xi p \to \Xi p) \propto \text{Im} N_{g,w}(s,t)$. Im Pomeranchuk

$= 0$ at "Cross-over Point" when
\[ \text{Im } N_{g,w} = 0. \]

Absorption predicts this zero at $t \approx -2 \ (\text{GeV}^2)\]
Regge poles predict zero at $t \approx -6 \ (\text{GeV}/c)^2$.

Cross-over $t = -0.16 \pm 0.02$

ELASTIC SCATTERING

5 GeV/c

$-t(\text{GeV}^2)$
Let's consider the reactions and dips of Fig. 16, in this new model. The absorption prediction agrees splendidly with $\pi N$ CEX which is $n = 1$ and has a dip at $t = -0.6$, $\pi^+ p$ backward which is $n = 0$ and vanishes at $u = -0.2$ (remember $n = 1$ gives $d\sigma/dt \propto -t$; $n = 0$ no $t$ factor. Thereby $d\sigma/dt$ shape gives $n$-value). Both these dips are also given correctly by WSNZ. However, Regge WSNZ correctly predicted that $\pi N \rightarrow n\Delta$ should have no dip at $-0.6$ whereas our absorption model can see no reason for a different dip structure here as compared with $\pi N$ CEX. Both are dominantly $n = 1$ reactions and should vanish at $t = -0.6$. One up for Mr. Regge.

However, the other two reactions in picture 19 are negative evidence for the absorption model. Thus, they are definite Regge failures; but according to the absorption models, they may be mixtures of many amplitudes and the predicted dips in one are filled in by other amplitudes.

More positively, absorption agrees with the "crossover" in picture 19 which reflect zeroes of $\text{Im}N(s,t)$ for $p$ and $\omega$ exchange. These crossover zeroes cannot be fitted in WSNZ which predict them at $-0.6$ not at the observed $-0.2$. 
Picture 20: EXPERIMENTAL γ EXCHANGE AMPLITUDES

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re N</td>
<td>Regge</td>
</tr>
<tr>
<td>Im N</td>
<td></td>
</tr>
<tr>
<td>Re F</td>
<td></td>
</tr>
<tr>
<td>Im f₀</td>
<td></td>
</tr>
</tbody>
</table>
Picture 20: My Kingdom for an Amplitude

The absorption model looks even worse if we examine the Re and Im parts of the dominant $\pi N$ CEX $n=1$ amplitude. The real part is very large near impact parameter $b=0$ - it had a double zero at $t=-.6$ faking an absorption single zero.

The situation with the $\pi N$ CEX $\rho$ exchange amplitudes is summarized in Picture 20. Absorption agrees with the imaginary parts of both $N$ and $F$ amplitudes. However, it disagrees with ReF which as discussed above has a large component at small impact parameter. Regge theory agrees with real and imaginary parts of $F$ but fails for $\Im N$; $\Re N$ is unclear at present. These amplitudes were extracted from looking at both $\partial \sigma/\partial t$, polarization plus $R$ and $A$ parameters, i.e., a complete set of observables. Such experiments are clearly vital in untangling the successes and failures of models.

Similarly, the $\pi N \to p\Delta$ failure of the absorption model also can be connected with the real part; in Regge pole theory (which is a splendid success here), the $\Im F_{p\Delta}$ has a $-.6$ zero but the real part is smooth - in agreement with the $\partial \sigma/\partial t$ data.

One generalization of these results is due to Harari. The absorption picture (always) works for imaginary, never for real parts. If Regge pole WSNZ predict similar $t$ dependence to absorption in $\Im A$ then Regge pole theory is good for whole amplitudes, including $\Re A$ which then disagrees with absorption. If Regge pole theory disagrees with absorption for $\Im A$, then it (Regge theory) is a poor approximation for $\Re A$ and in fact nobody knows how to calculate the real part.

This is not the only generalization from current data, and, in fact, there is some evidence against it. It may be tested by looking at other amplitudes ($n=2\rho$ exchange, $n=1\pi$ exchange) for other exchanges. This requires measurements of amplitudes not $\partial \sigma/\partial t$ and means polarization measurements of the type used in the $\rho$ exchange amplitude analysis above. It is important to note that amplitude structure does not vary greatly with energy and so these questions can be answered on the pre-NAL accelerators. In fact, they are the most important experiments (in high energy scattering) at these accelerators. Neat reactions are those like $\pi N \to K^*A$ because the $A$ measures its own polarization and so determines amplitudes (in a new situation!). This is Caltech spectrometer experiment at SLAC which is also good because at the same time, data can be taken on reactions like $\pi N \to QA$ which will solve all the open spectroscopy questions raised last week.
Now we come to an interesting class of reactions - the so-called diffractive reactions which in the language of Regge pole theory are meant to be described by the exchange of the so-called Pomeranchuk (or Pomeron) singularity. The constancy of pp total cross-sections in Picture 21(a) (needs to be confirmed by accurate NAL measured, as $K^+ p \sigma_c$ which was as constant as pp up to 20 GeV/c, rises at Serpukhov energies) implies $\sigma_{np}(0) = 1$. Up to 30 GeV/c, the $d\sigma/dt$ shrinks as nicely as $\pi N$ CEX. It is not known whether this is a property of the diffractive component or of lower lying $P', \omega$ trajectories. Indeed, it is not clear to me whether we'll ever know. Let us daydream for a line or two. Data on related reactions $\gamma p + \phi p$ and $K^+ p + K^p$ would be helpful. Also $K^0 p \rightarrow K^0 S p$ would tell us about $\omega$ exchange and $pp \rightarrow pp$ compared with $pp \rightarrow pp$ would show if the large t scattering had the positive charge conjugation expected (but not proven!) for diffractive processes. A better bet is some new theoretical insight.

Anyhow the new ISR data shows that diffraction is certainly not just the exchange of a Pomeron which is a moving Regge pole. $\log (d\sigma/dt)$ v. $\log (p_{lab})$ should be a straight line at fixed $t$ if this were so (cf. picture 13(b)) - it is consistent with this below 30 GeV/c (see picture 21(b)) but levels off somewhere between ISR and 30 GeV/c. Actually the ISR energy ratio (remember its energy ratios here 500/1500 not range 1000, that counts) is not that big and the current data is still consistent with many different asymptotic forms, e.g., fixed cut $(1-j)^n + behavior s/(logs)^{n+1}$ and we can have, say $1 \leq n \leq -1$, and fit the data without much trouble.

In any case, the picture 21(c) shows how striking the new ISR data is. At large $-t$ the cross-section has dropped by two orders of magnitude from the CERN data and a pronounced dip is present at $t \approx -1.2$ (GeV/c)$^2$. This new shape remains, within statistics, unchanged over the ISR energies.
Picture 22: Models for the pp Asymptotic Amplitude

pp elastic amplitude in impact parameter (b) space

Chou-Yang Model

Pomeron couplings proportional to photons $A(s,t) \propto g^2(t) \propto \frac{1}{(t - 0.71)^4}$ using dipole fit for $g(t)$. Note this (illegal?) quadruple pole in $A$ gives tail of high partial waves in agreement with data
There have been many explanations of the pp elastic data. However, most interest at present has been focused on the Chou-Yang model. Nobody understands why it works. Essentially we postulate that \( f(b) \), regarded as a matter distribution, should be identified with the charge distribution \( \rho(b) \) observed in ep → ep collisions. Actually more exactly ep only sees the charge distribution of one proton. In pp scattering, we say that \( f(b) \) is convolution of charge distributions for top and bottom vertices. Equivalently in t-space, we get the same answer if we identify the Pomeranchuk couplings with those of the photon. This prediction \( G^4(t) \) is much larger than experiment. More precisely we look again at the theory of constituent scattering and realize we should identify \( \rho(b) \) not directly with \( f(b) \) but write

\[
S(b) = \exp(-\rho(b))
= 1 - \rho(b) + \rho^2(b)/2
= 1 + i f(b)
\]

\[\uparrow\]

\text{correction}

\[\text{............... (P. T. O.)}\]
Picture 23: ISR pp $d\sigma/dt$ melt the Chou-Yang model

$E_{CM}=53$ GeV

- Barbiellini et al. (1972)
- This work

$G^2_E(t)$: Simple Pomeron is a photon model

Eikonal version of This with various Real Parts

$-t$ (GeV$^2$)
This is much nicer than before giving dip at $t = -1$ in agreement with data. We can postulate some additional mechanism to fill in dip a bit and get an exact fit. This is assigned to so-called "Real-part" in amplitude, but it is not clear where it comes from. (At low energies, it comes from $P', \omega, \lambda$ exchange, but these are zero at ISR energies. Otherwise, a fixed pole at $j = 1$ which is Chou-Yang model - can never have a real part.) The agreement is more precise than this figure indicates. Thus, the data has a slope change at $t = -0.15$ ($d\sigma/dt \propto \exp(At)$ goes from $A = 12.5$ to 10) and this is given by model!

One thing worries me about this. This model has a quadruple pole: at $t = m_p^2$ from dipole fit whereas true pp amplitude is not allowed any poles there... (ep dipole fit has two poles and real amplitude one pole). This violation may not matter at large $t$ but fit is best at small $t$ (especially slope change). Presumably it is quadruple pole which is giving high impact parameter part of $f(b)$; this controls small $t$. It is sad that a) large high impact parameter contribution is needed to fit data, b) this is given in model but model gets from a blatant violation of analyticity whereas it was analyticity that makes large high impact parameter implausible in first place.

It may be possible to test the photon type models in diffraction dissociation as the amplitude is just proportional to 

\[
\frac{P}{\sqrt{Q}} \times \frac{P}{\sqrt{Q}}
\]

both of which are known. This has been pursued by Ravndal. There are some difficulties; naively, this model predicts all diffraction dissociation vanishes at $t = 0$. Experimentally this is completely untrue although - as pointed out by Ravndal - the resonance components do appear to vanish at $t = 0$. However, the current experimental definitions of resonances are distinctly spurious (in view of recent $\pi^-p + (3\pi)^-p$ data from Serpukhov).

The situation is theoretically and experimentally confused. One can hope for some clarification from Caltech triggered bubble chamber experiment at SLAC on $\pi^+p + \pi^+p^*$. Also NAL spectrometer should be able to get useful data.
Picture 24

SERPICO

BNL, CERN

SLAC → NAL

ESS

NIMROD

BEVATRON

$\delta(1234)$

$\delta(1940)$ $\pi p \rightarrow n n$ shrinkage observed

$S \approx 2$ $P_{lab}$ (GeV/$c^2$)

$1$ $10$ $100$ $1000$ $10,000$

nucleon

$pp$ elastic "shrinkage"

"Shrinkage" in all processes

No shrinkage in

$\pi N \rightarrow p n$ and $NN \rightarrow NN$

$t = -1$ break in $pp$ elastic

"Shrinkage" in $pp$ elastic stops

$t = -1$ (Size $\approx 1$ fm) dip develops.

1 femt peripheral zeroes in

$Im A$ (non-diffractive).
Picture 24: Shrinkage and Size

It is appropriate now to summarize all this information on shrinkage. Note that the proton appears to be the same size at ISR energies as at 5 GeV/c. Remember in Picture 19 we found dips in imaginary parts of amplitudes corresponding to scattering off the edge of a proton of size 1 fermi. The same sort of size came out from the analysis of the pp elastic ISX data. If the Pomeranchuk had slope 1 then the slope of dσ/dt (a e^{-at} a = const + 2 α' log s) would increase and the typical size $b = \sqrt{2a/5}$ would also increase. Actually even this would be a small effect. Anyhow such data confirms the utility of amplitude analysis from 5 to 30 GeV/c as telling us information about the asymptotic amplitude.

Finally we note that the drastic change in the elastic data above 30 GeV/c suggests that 1 GeV/c is not the correct energy scale. More like 30 + 100 GeV/c seems the asymptotic scale. Perhaps NAL will find other indications of energy variation on this new scale?
(i) $\text{SU}(3)$ For Regge Vertices relates, for instance:

\[ \overrightarrow{g} \rightarrow \overrightarrow{K} \]

No known discrepancy.

(ii) EKB (Exchange Regeneracy) relates, for instance:

\[ K \rightarrow K \quad \text{to} \quad K \rightarrow A_2 \]

It is always qualitatively correct. There are some unexplained systematic deviations at low energies.

(iii) Quark model relates $K^+ \rightarrow K^+$ all processes to "quark-quark scattering" and extends $\text{SU}(3)$ to spin.

No failures when applied to cases where $\text{SU}(3)$ mass breaking can't mass things up. For instance Picture 25(b), shows $\Delta^{++}$ decay prediction: $g_{33} = \frac{3}{8}$, $g_{31} = \frac{15}{6}$, $g_{33} = 0$.

(iv) Vector-Meson Photon Analogy

\[ e.g. \quad \gamma^* \rightarrow K^+ \rightarrow K^0 \]

(a) Same good $\Delta^{++}$ prediction as in (iii)
(b) $B$ production vanishes at $t = 0$. This agrees with experiment and explains why spectroscopy hard
Picture 25: Symmetry Schemes

We will not discuss SU(3) and Exchange Degeneracy.

**Quark Model:**

This is very interesting as it probably has no real failures and yet it is a non-trivial and poorly understood symmetry. I have no space to discuss it. I will give the so-called Stodolsky-Sakurai test for $\Delta^{++}$ production.

$$\rho_{33} = \frac{3}{8}, \quad \rho_{3-1} = \sqrt{3}/8, \quad \rho_{31} = 0$$

which even, as shown in the Picture (25b), works down to very low energy. These symmetries are probably more general than Regge poles, i.e., they can be converted into Regge pole coupling symmetries but this is only reasonable for when poles dominate: they probably hold for total amplitude; poles plus cuts. For instance, they seem to give good results for $\pi N \to \rho \Delta$ and $\omega \Delta$ which are bad news for Regge poles.

**$\rho$-photon Analogy:**

Theorists have little imagination. Not only has it been proposed that Pomeran coupling are proportional to the photon but also that vector meson $\rho$ (and by EXD, tensor meson $A_2$) couplings are proportional to the photon's. This has better qualitative success. First, it gives the same Stodolsky-Sakurai distribution as quark model for $\Delta^{++}$ production. Perhaps it's unfair to hang two theories on the same data; anyhow the quark model predicted some non-trivial things for $\pi N \to \rho \Delta$ and $\pi N \to \omega \Delta$ as well. Meanwhile, the photon model also predicts inelastic resonance production by $\rho$, $A_2$, ..., exchange vanishes at $t = 0$. This is in nice agreement with experiment; in fact, it suppresses the cross-sections of these resonances and so explains some of the difficulties in meson spectroscopy for $A_1$ and B nonets.

Generally, the symmetry field has made lots of progress recently; as experiments have given us mediocre data on lots of reactions other than the very good data in 1 or 2 reactions necessary to probe Regge theory.
**Picture 26: Regge Theory for Inclusive Reactions**

**Grand-daddy:**

\[ 2 \rightarrow 2 \]

Total Cross-section

\[ \sum \text{over all final states} \]

\[ = \text{Im} \]

**Optical Theorem**

**Inclusive Reactions**

\[ a \rightarrow c \]

all other particles, except c, in final state are summed over.

\[ \text{Sum over all particles} \]

Analytic Continuation.

\[ \text{"Im"} \]

Generalized Optical Theorem.

**Fragmentation Limit (c fragment of a)**

\[ a \rightarrow c \]

\[ \text{Regge Poles} \]

Pomeron, \( s, t, \ldots \)

\[ \text{Regge Prediction} \]

(i) \( S \)-dependence:

\[ A(x, p_T^2) = B(x, p_T^2) S^{-\frac{1}{2}} \]

scaling

specific up

(ii) Factorization of Pomeron as \( S \rightarrow \infty \)

\[ \sigma(ab \rightarrow c : c \text{ fragment of } a) = \]

\[ \sigma_{\text{tot}}(ab) \]

Real live grand-daddy

\[ \sigma(ab' \rightarrow c : c \text{ fragment of } a) \]

Two body total cross-sections

\[ \sigma_{\text{tot}}(ab') \]
Regge poles have been usefully employed recently in inclusive reactions (peep back at Picture 6); the basis for this application was discovered by Mueller. A single particle inclusive reaction

(written ab(c)) can be bent into

and just as for real unbent 2 → 2 scattering we can use the optical theorem to convert this into

"Im"

an amplitude to which we can apply Regge theory.

Now we can identify three limits: First, we have the fragmentation limit; take, wolog, fragmentation of a where c comes off with a longitudinal momentum similar to a (fragmentation of b is obviously similar). This is illustrated in Picture 26 and at high energies this should be just like 2 → 2 total cross-sections and be described by exchange of Pomeranchuk and secondary Regge trajectories lying half a unit below in j-plane.

This gives

\[
\frac{d\sigma}{d^3p} = A + B s^{-\frac{1}{2}}
\]

where A and B are functions of x and \(p_\perp^2\). The new thing Regge theory tells us about A is factorization, i.e., in scaling limit:

\[
\frac{A}{\sigma_{tot}(ab)} \quad \text{and} \quad \frac{A}{\sigma_{tot}(ab')} \quad \text{.....}
\]

\(x = \) longitudinal momentum of c in c.m.s./maximum value
\(p_\perp^2 = \) transverse momentum squared of c
Picture 27(a): Factorization Test

\[(d^2 \sigma/dp_T) / \sigma_{\text{total}}\] integrated over all \(p_T\)

\[(d\sigma/dp_L dp_T) / \sigma_{\text{total}}\] for various \(p_T\) ranges

\[\text{Laboratory Longitudinal Momentum } p_L \text{ GeV/c}\]

- \(K^+ p \rightarrow \pi^- \) (11.8 GeV/c)
- \(p p \rightarrow \pi^+ \) (28.5 GeV/c)
- \(\gamma p \rightarrow \pi^- \) (10.4 GeV/c)

\(\pi^-\) is fragment of proton

Distributions are scaled by \(\sigma_{\text{total}}\) and should be equal for each \(p_T\) range - in scaling limit. \(\gamma p\) hasn't reached it yet. \(p p\) and \(K^+ p\) should be near. They agree to 40%.

Picture 27(b): Regge Energy Dependence Test

\(\gamma p \rightarrow \pi^- + \text{(ANYTHING)}\)

Regge Theory predicts behavior \(A + B s^{-1/2}\) i.e. straight lines on these graphs

\[s^{-1/2} (\text{GeV}^{-1})\]

\[0 < p_T < 0.1 \text{ GeV/c}\]

\[0.1 < p_T < 0.2 \text{ GeV/c}\]

\[0.2 < p_T < 0.3 \text{ GeV/c}\]
This fragmentation factorization is difficult to test at the moment because few processes have reached their scaling limit. New NAL data should revolutionize this. There will be so many tests that these will become our best tests of Pomeranchuk factorization. Meanwhile, we must use lower energy data.

There is much controversy about which reactions have (experimentally) and should (theoretically) reach their scaling limit (= Pomeranchuk dominance) at current accelerators. For $2 \rightarrow 2$ total cross-sections it wouldn't matter as Pomeron dominates at say 10 GeV/c. But generally $s^{-1/2}$ terms are 3x bigger in inclusive than in two-body reactions at same energy. It is believed that $K^+ p + \pi^- + \text{any}$ and $p p + \pi^- + \text{any}$, proton fragments should scale early whereas $\gamma p + \pi^- + \text{any}$ shouldn't. This is tested in Picture 27 which shows after scaling by $\sigma_{\text{total}}$, equality (to within 30%) $s^{-1/2}$ behavior of the non-scaling term needs to be tested better but it seems difficult to do conclusively at present. There are systematic errors when combining data from different experiments - the latter are also relatively low statistics as they come from bubble chambers. One of the nicest tests is shown in the other half of picture 27 with $\gamma p + \pi^- + \text{any}$ at three different energies from the same group (!) - agreement with theory is splendid.
Picture 28: Pionization

$\sigma_{[a b (c)]}$, when divided by $\sigma_{\text{total}} (ab)$, should have common scaling limit if Pomeron factorizes.
Picture 28: Pionization

In the second - so-called pionization limit - c has \( x \approx 0 \) in the c.m.s. and this is described in Regge theory by

\[
\begin{align*}
\text{scaling} & \quad \text{nonscaling} \\
X & + \quad Y s^{-1} \\
\frac{1}{s} & + \quad \frac{1}{s}
\end{align*}
\]

and now secondary term is no longer down by \( s^{-1} \) but \( s \)-dependence is

\[
X + Y s^{-1} k
\]

for each part of graph (above and below \( \gamma \)) shares energy and only has \( \gamma \) each. So second graph is down by \( (s^{-1})^1 - \alpha_p (0) \approx s^{-1} k \) not \( (1/s)^1 - \alpha_p (0) \)

\[ \approx s^{-1} k \]. We also get many factorization tests as the diagram factors at both vertices \( a \bar{a} \) and \( b \bar{b} \).

Picture 28 shows an interesting test of these ideas by Ferbel. The straight lines v. \( s^{-1} k \) and common value at \( s = 0 \) test both secondary Regge trajectories and Pomeranchuk factorization. Again better data at NAL will confirm this somewhat biased plot, i.e., many other extrapolations are possible.
Picture 29 triple Regge limit

$pp \rightarrow p + \text{anything}$

$s = 1995 \text{ (GeV/c)}^2$

$E \frac{d^3\sigma}{d^3p} \text{ mb/(GeV/c)}^2$

- $p_T = 0.7 \text{ GeV/c}$
- 0.8
- 0.9
- 1.0
- 1.1

- $\alpha = 0.45 + 0.95t$
- $\alpha = 0.45 + 0.75t$
The final limit is called the Triple Regge limit. This is achieved by taking one Regge limit and then summing

\[ \sum \]

Another Regge limit here gives us

\[ \alpha_1(t) \text{ can be Pomeron, p...} \]

see \( \alpha_1(t) \) for all \( t \)

\[ \alpha_2 = \text{Pomeron for scaling: see } \alpha_2(0) \text{ only} \]

For \( x \) near 1, \( \frac{d^3\sigma}{d^3p} \alpha_{(1-x)}^2 - 2\alpha_1(t) \) and one can hope to see "shrinkage" in \( t \)-dependence of \( (1-x)^n(t) \).

This is particularly interesting as it allows us to use Regge theory away from \( t = 0 \) and perhaps see not only shrinkage, but absorption dips (\(?\)), WSNZ, etc.!

Picture 29 shows some fine data from the ISR. As illustrated, it can be fitted by the sum of two triple Regge terms

(i) \( \alpha_2 = \text{Pomeranchuk, } \alpha_1 = f^0 \) where the data shows shrinkage of \( f^0 \);
(ii) \( \alpha_2 = f^0, \alpha_1 = \text{Pomeranchuk} \) to give the striking peak near \( x = 1 \) in proton distribution. (This can also be fitted by \( \alpha_1 = \alpha_2 = \text{Pomeranchuk;} \)
a so-called triple Pomeron coupling which terrifies theorists as it is meant to be zero.)

This particular application will advance quickly as NAL/ISR pour data in our eager hands.