Quark Models
for baryons

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Basic idea of Constituent Quark Models (CQM)

Constituent Quarks

At variance with QCD quarks: CQ acquire mass & size
LQCD results: SU(6)× O(3) QM states up to ≈2.2 GeV
J. J. Dudek, R. G. Edwards

LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains
a long range spin-independent confinement
a short range spin dependent term

\[ \text{Spin-independence} \rightarrow \text{SU}(6) \text{ configurations} \]
SU(3)

3 flavours: \( q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \equiv (q_\alpha) \quad \alpha = 1, 2, 3 \)

* invariance under rotation in u, d, s space 
  
  \( \sim 15\% \) violation in mass spectrum

\( q \rightarrow q' = U q \)

\( U : 3 \times 3 \)

\( \text{unitary, unimodular} \)

\( SU(3) \)

\( i \Theta \sum_j n_j \lambda_j \)

\( e^{-\frac{i}{2} \sum_j n_j \lambda_j} \)

\( j = 1, \ldots, 8 \quad [ = 3^2 - 1 ] \)

\( \lambda_j : \text{Gell-Mann matrices} \)

\( [\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k \quad \text{LIE ALGEBRA} \)

\( f_{ijk} \quad \text{structure constants} \)

- \( \lambda_\kappa = \begin{pmatrix} \tau_\kappa & 0 \\ 0 & -\tau_\kappa \end{pmatrix} \quad \kappa = 1, 2, 3 \quad SU(2) \subset SU(3) \)

- only 2 commute: \( \lambda_3, \lambda_8 \)

\( T_3 = \frac{1}{2} \lambda_3 \quad Y = \frac{1}{\sqrt{3}} \lambda_8 \)

(3rd isospin comp. Hypercharge)
$(u, d)$ QUARKS and $SU(2)$

$\phi_q = \begin{pmatrix} u \\ d \end{pmatrix}$ in absence of e.m. interactions
$u$ and $d$ cannot be distinguished

→ INVARIANCE UNDER ROTATION IN THE CHARGE SPACE

$\phi_q \rightarrow \phi'_q = U \phi_q$

$U : 2 \times 2$

unitary
$\det(U) = 1$ [unimodular]

SU(2)

$\begin{pmatrix} p = u u d \text{ nucleon} \\ n = u d d \text{ isospin} \end{pmatrix}$

$U = e^{i \vec{T}_i \cdot \vec{n} \cdot \frac{1}{2}} \rightarrow$ generators of infinitesimal transformations

rotation axis of rotation
$Sp(\vec{n}) = 0$

$\tau_j : \cdot j = 1, 2, 3 \text{ (Pauli matrices)} \quad 3 = 2^2 - 1$

satisfy the Lie-algebra

$[\tau_i, \tau_j] = 2i \varepsilon_{ijk} \tau_k \quad ; i, j, k = 1, 2, 3$

describe the quark isospin

$\varepsilon_{ijk}$ (antisymmetric tensor) structure constants
Ex. Construct the baryon decuplet in the $Y$ ($Y = B + S$), $T_z$ plane
Young diagram technique for SU(N)
the fundamental N-dimensional representation is denoted by a box and the irr.rep.of three objects can be obtained as

\[
\begin{array}{c}
\square \otimes \square \otimes \square \otimes \square = \begin{array}{c}
\square \\
\otimes \\
\square \\
\square \\
\end{array}
\end{array}
\begin{array}{c}
A \\
M \\
M \\
S \\
\end{array}
\]

\[
N \otimes N \otimes N = \frac{1}{6} N(N-1)(N-2) + 2 \cdot \frac{1}{3} (N+1) N(N-1) + \frac{1}{6} (N+2)(N+1) N
\]

The advantage of this method is two-fold: first the pattern is general, being valid for any SU(N); furthermore each Young tableaux with n boxes defines an irreducible representation of the group Sn containing all the permutations of n objects and therefore it belongs to a definite symmetry type. The labels A,M,S refer to antisymmetry, mixed symmetry and symmetry for the exchange of the 3 quark coordinates. In the case of SU(2) (spin), the antisymmetric 3-quark state does not exist, because only two different states are available for three particles.
Young diagram rules:

1° rule  antisymmetry for box in columns, symmetry for box in rows

2° rule  the number of box in a column cannot exceed the number of states (N) accessible to each particle

3 rule   lower rows cannot have more box than the upper one

Not possible
DIMENSIONS for Young diagrams for SU(N). Ex. calculate also for SU(3), SU(6)

\[ d = \frac{F}{H} \]

\[ F = \text{product of all } N_i \]

\[ H = \text{product of "hooks" for each box} \]

\[
\begin{array}{cccc}
N & NH & NH2 & \\
N+1 & N & NH & \\
N-1 & N & \\
N-3 & \\
\end{array}
\]

\[
\begin{array}{c}
5 \\
3 \\
1 \\
1 \\
\end{array}
\]

\[
\begin{array}{c}
\frac{2 \cdot 1}{2 \cdot 1} = 1 \\
\frac{2 \cdot 3}{2 \cdot 1} = 3 \\
\frac{2 \cdot 3 \cdot 1}{3 \cdot 1 \cdot 1} = 2 \\
\frac{2 \cdot 3 \cdot 4}{3 \cdot 2 \cdot 1} = 4 \\
\end{array}
\]

Ex. calculate for SU(3) SU(6)
SPIN STATES SU(2)
If we adopt the standard angular momentum notation \( l((s_1,s_2)_{12},s_{-3})S > \), the explicit form of the 3q spin states is:

\[
\Phi_{MA} = |(\frac{1}{2},\frac{1}{2})_0, \frac{1}{2}\rangle \equiv |1\rangle_{\frac{3}{2}}^3
\]

\[
\Phi_{MS} = |(\frac{1}{2},\frac{1}{2})_1, \frac{1}{2}\rangle \equiv |1\rangle_{\frac{1}{2}}^2
\]

\[
\Phi_{S} = |(\frac{1}{2},\frac{1}{2})_1, \frac{3}{2}\rangle \equiv |1\rangle_{\frac{3}{2}}^2
\]

the suffixes indicate also the symmetry for the exchange of quarks in the pair with total spin \( S_{12} = 0 \) or 1.

The SU(3) irr.rep. are constructed following to same general scheme, the corresponding dimensions and symmetry types being

\[
SU(3): \quad 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 10
\]

\[
\begin{array}{cccc}
A & M & M & S \\
\end{array}
\]
In the case of non-strange baryons, the resulting states coincide with the standard isospin ones, detoned, similarly to the spin states by

$$\chi_{MA} \cdot \chi_{MS} \cdot \chi_{S}.$$ 

The strongest component of the quark-quark interaction is spin independent. In this case the flavour and spin states are combined into SU(6) multiplets with the dimensions

$$6 \otimes 6 \otimes 6 = 20 \oplus 70 \oplus 70 \oplus 56$$

$$\begin{array}{cccc}
A & M & M & S \\
\end{array}$$

Each SU(6) state can be analyzed with respect to its spin, SU(2), and flavour SU(3) content. Keeping in mind the symmetry properties of the various states involved, one can easily obtain the following decomposition:

$$20 = 4 \uparrow + 2 \downarrow$$

$$70 = 2 \uparrow + 2 \downarrow + 4 \uparrow + 2 \downarrow + 10$$

$$56 = 2 \downarrow + 4 \downarrow + 10$$

In the r.h. sides the suffixes denote of course the multiplicity 2S+1 of the 3q spin states, while the underlined numbers are the dimensions of the SU(3) flavour multiplets.
Each SU(6) state can be analyzed with respect to its spin, SU(2), and flavour SU(3) content. Keeping in mind the symmetry properties of the various states involved, one can easily obtain the following decomposition:

\[
\begin{align*}
20 &= 4_1 + 2_8 \\
70 &= 2_1 + 2_8 + 4_8 + 2_{10} \\
56 &= 2_8 + 4_{10}
\end{align*}
\]

Multiplication table:
What about colour?

- Y.D. above o.k.
- There is a $SU(3)$-singlet
- Baryons are colourless
- Is completely antisymmetric

\[ \frac{1}{\sqrt{6}} \epsilon^{abc} q^{a(1)} q^{b(2)} q^{c(3)} \]
Casimir Operators

- Defined in terms of the generators
- Commute with all generators

→ they have a definite value for each irreducible representation

→ they label the T.R.

\(SU(N)\): \(N-1\) Casimir op.

\(SU(2)\):

\[ t_i = \frac{1}{2} \tau_i \]

\[ C = \sum_{i=1}^{2} t_i^2 \rightarrow T(T+1) \]

\(SU(3)\):

\[ F_i = \frac{1}{2} \lambda_i \quad i = 1, \ldots, 8 \]

\[ C_1 = \sum_{i=1}^{8} F_i^4 \quad \text{unitary spin} \quad F^2 \]

\[ C_2 \rightarrow O(F^3) \]

dim. irr. repr. \(1 \ 3 \ \overline{3} \ 8 \ 6 \ 10 \)

\(F^2\) \(0 \ \frac{1}{3} \ \frac{5}{3} \ 3 \ 19\frac{2}{3} \ 6 \)
PROBLEMS

1. - Calculate the dimensions of \[ \begin{array}{c} \text{3} \\ \text{3} \end{array} \] in SU(3) and in SU(6)

2. - Show that \[ \begin{array}{c} \text{3} \\ \text{1} \end{array} \] is a SU(3) - singlet

3. - Show that in SU(3) one has \[ 3 \otimes 3 = 1 \oplus 8 \]

4. - Prove the SU(6) decomposition scheme
THREE-QUARK WAVE FUNCTION

$$\Psi_{3q} = \theta_{\text{colour}} \times \chi_{\text{spin}} \times \phi_{\text{iso}} \times \psi_{\text{space}}$$

\(\text{SU}(3)_c \quad \text{SU}(2) \quad \text{SU}(3)_f \quad \text{O}(3)\)

**SU(6) limit:** (spin-independent interaction)

\(\text{SU}(3)_c \quad \text{SU}(6) \quad \text{O}(3)\)

Permutation symmetry: \(\Psi_{3q}\) must be antisymmetric

\(\theta_{\text{colour}}\) is a colour singlet \(\Rightarrow A\)

the rest must be symmetric

\(\text{SU}(6) \& \text{O}(3)\) wf have the same symmetry (A, MS, MA, S)
SU(6) configurations for three quark states

\[
6 \times 6 \times 6 = 20 + 70 + 70 + 56
\]

\[\text{A M M S}\]

Notation

\[(d, L^{\pi})\]

d = \text{dim of SU(6) irrep}
L = \text{total orbital angular momentum}
\pi = \text{parity}
now New notation on PDG

\( |N^{2S+1}J_{\frac{1}{2}} \rangle \)

nucleon!

J total ang.
Momentum.
\[ \begin{align*}
I = \frac{3}{2}, & \quad J^p = \frac{3}{2}^+ \\
I = \frac{1}{2}, & \quad J^p = \frac{3}{2}^+ \\
I = \frac{1}{2}, & \quad J^p = \frac{3}{2}^- \\
I = \frac{1}{2}, & \quad J^p = \frac{1}{2}^+ 
\end{align*} \]

\[ \begin{align*}
N^X, \Delta & \quad J^P \\
\rightarrow & \quad N \\
\rightarrow & \quad \pi \\
\rightarrow & \quad X_{2I} 2J \\
X = S, P, D, \ldots & \quad \text{referred to the wave of the outgoing pion}
\end{align*} \]
HARMONIC OSCILLATOR STATES

\[ \vec{S} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2) \]

\[ \vec{\lambda} = \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) \]

\[ H = 3m + \frac{P^2_1 + P^2_2}{2m} + \frac{1}{2} \sum_{i<j} \kappa \frac{r^2_{ij}}{r_{ij}} + \frac{3}{2} \kappa (\vec{\rho}^2 + \vec{\lambda}^2) = \frac{3}{2} \kappa x^2 \]

\[ x = (\vec{\rho}^2 + \vec{\lambda}^2)^{1/4} \quad \text{hyperradius} \]

\[ E = (3 + N_{\text{h.o.}}) \hbar \omega \quad \omega = \frac{\alpha^2}{m} \]

\[ \Psi_{3q} \sim e^{\frac{-\alpha^2}{2}(\vec{\rho}^2 + \vec{\lambda}^2)} e^{\frac{\alpha^2}{2}(\vec{r}_1^2 + \vec{r}_2^2 + \vec{r}_3^2)} \]

\[ \Psi_{3q} \rightarrow \Psi_{\text{QSM}} \quad \text{without c.m. motion} \]

\[ N = 2(\nu + n_1) + l_3 + l_1 \]

\[ \alpha^2 = (3 \hbar m)^{1/2} \]
\[ E = (3 + N) \hbar \omega \quad \omega = \frac{\alpha^2}{\hbar} \]

\[ N = 2 (Y + \Omega) + \ell_2 + \ell_3 \]

\[ \alpha^2 = (Ze m)^{1/2} \]

General structure of the h.o. wave functions

\[ \Psi_{NLT} = \frac{N}{\text{norm}} P(g,\lambda) e^{\frac{-\alpha^2}{2} (g + \lambda^2)} \]

\[ \text{symm)} \uparrow \quad \text{factor polyn. degree } N \]

\[ \text{type} \]

\[ \text{combined to total ang. mom. } L \]
\[ \omega = 2 \quad 2^+_S \quad 2^+_M \quad 1^+_A \quad 0^+_M \quad 0^+_S \]

\[ \omega = 1 \quad 1^-_M \]

\[ \omega = 0 \quad 0^+_S \]
The general structure of the h.o. wave functions is

$$\psi_{2N+1} = \frac{\alpha}{2} \left( \frac{p^2 + \lambda^2}{\lambda} \right) Y_{\lambda}^{\Omega} \tilde{Y}_{\lambda}^{\Omega}(\Omega)$$

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$N$</th>
<th>$\nu$</th>
<th>$n$</th>
<th>$l_p$</th>
<th>$l_\lambda$</th>
<th>$L$</th>
<th>$\Pi$</th>
<th>$N/\Omega_2$</th>
<th>$P_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{00S}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\psi_{11M}^p$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-</td>
<td>$\alpha \sqrt{2/3}$</td>
<td>$p$</td>
</tr>
<tr>
<td>$\psi_{11M}^\lambda$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>$\alpha \sqrt{2/3}$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>$\psi_{20S}$</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>$1/\sqrt{3}$</td>
<td>$\alpha^2(p^2+\lambda^2) - 3$</td>
</tr>
<tr>
<td>$\psi_{20M}$</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>$\alpha^2/\sqrt{3}$</td>
<td>$p^2\lambda^2$</td>
</tr>
<tr>
<td>$\psi_{22S/22M}$</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>+</td>
<td>$2\alpha^2/\sqrt{15}$</td>
<td>$p^2$</td>
</tr>
<tr>
<td>$\psi_{22S/22M}$</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>+</td>
<td>$2\alpha^2/\sqrt{15}$</td>
<td>$\lambda^2$</td>
</tr>
<tr>
<td>$\psi_{20M}$</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+</td>
<td>$2\alpha^2/3$</td>
<td>$p\lambda$</td>
</tr>
<tr>
<td>$\psi_{21A}$</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>+</td>
<td>$2\alpha^2/3$</td>
<td>$p\lambda$</td>
</tr>
<tr>
<td>$\psi_{22M}$</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>+</td>
<td>$2\alpha^2/3$</td>
<td>$p\lambda$</td>
</tr>
</tbody>
</table>

TABLE 2 - The harmonic oscillator wave functions for the 3-quark system \([5,6]\) according to eq. (8). The presence of two items in the same line means that the correct symmetry property is obtained with a linear combination of the two wave functions. The quantity $J_\pi$ in the normalization factor is given by $4\alpha^3/\sqrt{\pi}$ and $(\pi = A, M, S)$ denotes the type of permutation symmetry. The parity $\Pi$ is $(\cdot)^N$. 


Table 6. Three-quark states with positive parity. For simplicity of notation, we have omitted the coupling to the total angular momentum $L$ of the second column.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>$L_{K_3}$</th>
<th>S</th>
<th>T</th>
<th>SU(6) configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}$</td>
<td>$0^+_1$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{00} Y_{000} O_S$</td>
</tr>
<tr>
<td>$P_{13}$</td>
<td>$2^+_1$</td>
<td>3</td>
<td>1</td>
<td>$\psi_{22} \frac{1}{\sqrt{2}} (Y_{200} - Y_{202}) \phi_M S + Y_{211} \phi_M A) S$</td>
</tr>
<tr>
<td>$P_{15}$</td>
<td>$2^+_1$</td>
<td>3</td>
<td>1</td>
<td>$\psi_{22} \frac{1}{\sqrt{2}} (Y_{200} + Y_{202}) O_S$</td>
</tr>
<tr>
<td>$P_{17}$</td>
<td>$2^+_1$</td>
<td>3</td>
<td>1</td>
<td>$\psi_{22} \frac{1}{\sqrt{2}} (Y_{200} - Y_{202}) \phi_M S + Y_{211} \phi_M A) S$</td>
</tr>
<tr>
<td>$P_{31}$</td>
<td>$0^+_1$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{00} Y_{000} \phi_S S$</td>
</tr>
<tr>
<td>$P_{33}$</td>
<td>$0^+_1$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{00} Y_{000} \phi_S S$</td>
</tr>
<tr>
<td>$F_{35}$</td>
<td>$2^+_1$</td>
<td>3</td>
<td>1</td>
<td>$\psi_{22} \frac{1}{\sqrt{2}} (Y_{200} - Y_{202}) \phi_M S + Y_{211} \phi_M A) S$</td>
</tr>
<tr>
<td>$F_{37}$</td>
<td>$2^+_1$</td>
<td>3</td>
<td>1</td>
<td>$\psi_{22} \frac{1}{\sqrt{2}} (Y_{200} + Y_{202}) \phi_S S$</td>
</tr>
</tbody>
</table>

$\Omega_S = \frac{1}{\sqrt{2}} [\chi_{MA} \phi_{MA} + \chi_{MS} \phi_{MS}]$,  

$\chi_{MS} = \langle \langle \frac{1}{2}, \frac{1}{2} \rangle^1 \frac{1}{2}, \frac{1}{2} \rangle$,  

$\chi_{MA} = \langle \langle \frac{1}{2}, \frac{1}{2} \rangle^0 \frac{1}{2}, \frac{1}{2} \rangle$,  

$\chi_{S} = \langle \langle \frac{1}{2}, \frac{1}{2} \rangle^1 \frac{3}{2}, \frac{1}{2} \rangle$.
Table 7. Three quark states with negative parity

<table>
<thead>
<tr>
<th>Resonances</th>
<th>$L^P_{S_T}$</th>
<th>S</th>
<th>T</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S1$</td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} \frac{1}{2} Y_{110}^{(1)} \Omega_M + Y_{101}^{(1)} \Omega_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} \frac{1}{2} Y_{110}^{(1)} \Omega_M + Y_{101}^{(1)} \Omega_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} Y_{110}^{(1)} \phi_M + Y_{101}^{(1)} \phi_S \chi_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} Y_{110}^{(1)} \phi_M + Y_{101}^{(1)} \phi_S \chi_S$</td>
</tr>
<tr>
<td>$D13$</td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} \frac{1}{2} Y_{110}^{(1)} \Omega_M + Y_{101}^{(1)} \Omega_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} \frac{1}{2} Y_{110}^{(1)} \Omega_M + Y_{101}^{(1)} \Omega_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} Y_{110}^{(1)} \phi_M + Y_{101}^{(1)} \phi_S \chi_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} Y_{110}^{(1)} \phi_M + Y_{101}^{(1)} \phi_S \chi_S$</td>
</tr>
<tr>
<td>$D15$</td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} \frac{1}{2} Y_{110}^{(1)} \Omega_M + Y_{101}^{(1)} \Omega_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} \frac{1}{2} Y_{110}^{(1)} \Omega_M + Y_{101}^{(1)} \Omega_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} Y_{110}^{(1)} \phi_M + Y_{101}^{(1)} \phi_S \chi_S$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} Y_{110}^{(1)} \phi_M + Y_{101}^{(1)} \phi_S \chi_S$</td>
</tr>
<tr>
<td>$S31$</td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} \frac{1}{2} Y_{110}^{(1)} \chi_M + Y_{101}^{(1)} \chi_{MS}$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} \frac{1}{2} Y_{110}^{(1)} \chi_M + Y_{101}^{(1)} \chi_{MS}$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} Y_{110}^{(1)} \chi_M + Y_{101}^{(1)} \chi_{MS}$</td>
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<td></td>
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<tr>
<td>$S33$</td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} \frac{1}{2} Y_{110}^{(1)} \chi_M + Y_{101}^{(1)} \chi_{MS}$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} \frac{1}{2} Y_{110}^{(1)} \chi_M + Y_{101}^{(1)} \chi_{MS}$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{11} Y_{110}^{(1)} \chi_M + Y_{101}^{(1)} \chi_{MS}$</td>
</tr>
<tr>
<td></td>
<td>$1^M_0$</td>
<td>1</td>
<td>1</td>
<td>$\psi_{21} Y_{110}^{(1)} \chi_M + Y_{101}^{(1)} \chi_{MS}$</td>
</tr>
</tbody>
</table>
Flavor wave functions

The flavor wave functions \( |(p, q), I, M_I, Y \rangle \)

(i) The octet baryons \((p, q) = (1, 1)\):

\[
|1, 1, 1/2, 1/2, 1 \rangle : \quad \phi_\rho(p) = \frac{[|udu\rangle - |duu\rangle]}{\sqrt{2}}, \\
\phi_\lambda(p) = \frac{[2|uud\rangle - |udu\rangle - |duu\rangle]}{\sqrt{6}}, \\
|1, 1, 1, 0 \rangle : \quad \phi_\rho(\Sigma^+) = \frac{[|suu\rangle - |usu\rangle]}{\sqrt{2}}, \\
\phi_\lambda(\Sigma^+) = \frac{[|suu\rangle + |usu\rangle - 2|uus\rangle]}{\sqrt{6}}, \\
|1, 1, 0, 0 \rangle : \quad \phi_\rho(\Lambda) = \frac{[2|uds\rangle - 2|dus\rangle - |dsu\rangle + |sdu\rangle - |sud\rangle + |usd\rangle]}{\sqrt{12}}, \\
\phi_\lambda(\Lambda) = \frac{[-|dsu\rangle - |sdu\rangle + |sud\rangle + |usd\rangle]}{2}, \\
|1, 1, 1/2, 1/2, -1 \rangle : \quad \phi_\rho(\Xi^0) = \frac{[|sus\rangle - |uss\rangle]}{\sqrt{2}}, \\
\phi_\lambda(\Xi^0) = \frac{[2|ssu\rangle - |sus\rangle - |uss\rangle]}{\sqrt{6}}.
\]

(ii) The decuplet baryons \((p, q) = (3, 0)\):

\[
|(3, 0), 3/2, 3/2, 1 \rangle : \quad \phi_S(\Delta^{++}) = |uuu\rangle, \\
|(3, 0), 1, 1, 0 \rangle : \quad \phi_S(\Sigma^+) = \frac{[|suu\rangle + |usu\rangle + |uus\rangle]}{\sqrt{3}}, \\
|(3, 0), 1/2, 1/2, -1 \rangle : \quad \phi_S(\Xi^0) = \frac{[|ssu\rangle + |sus\rangle + |uss\rangle]}{\sqrt{3}}, \\
|(3, 0), 0, 0, -2 \rangle : \quad \phi_S(\Omega^-) = |sss\rangle.
\]

(iii) The singlet baryons \((p, q) = (0, 0)\):

\[
|(0, 0), 0, 0, 0 \rangle : \quad \phi_A(\Lambda) = \frac{[|uds\rangle - |dus\rangle + |dsu\rangle - |sdu\rangle + |sud\rangle - |usd\rangle]}{\sqrt{6}}.
\]
The $S_3$ invariant space-spin-flavor ($\Psi = \psi \chi \phi$) baryon wave functions are given by

\begin{align*}
\Psi_{2\mathbf{56}, L^P} & : \frac{\psi_S(\chi_{\rho} \phi_{\rho} + \chi_{\lambda} \phi_{\lambda})}{\sqrt{2}}, \\
\Psi_{2\mathbf{70}, L^P} & : \frac{[\psi_{\rho}(\chi_{\rho} \phi_{\lambda} + \chi_{\lambda} \phi_{\rho}) + \psi_{\lambda}(\chi_{\rho} \phi_{\rho} - \chi_{\lambda} \phi_{\lambda})]}{2}, \\
\Psi_{4\mathbf{70}, L^P} & : \frac{(\psi_{\rho} \phi_{\rho} + \psi_{\lambda} \phi_{\lambda})}{\sqrt{2}}, \\
\Psi_{2\mathbf{20}, L^P} & : \frac{\psi_A(\chi_{\rho} \phi_{\rho} - \chi_{\lambda} \phi_{\lambda})}{\sqrt{2}}, \\
\Psi_{4\mathbf{10}, L^P} & : \frac{\psi_S \chi S \phi_S}{\sqrt{2}}, \\
\Psi_{2\mathbf{10}, L^P} & : \frac{(\psi_{\rho} \chi_{\rho} + \psi_{\lambda} \chi_{\lambda})}{\sqrt{2}}, \\
\Psi_{2\mathbf{1}, L^P} & : \frac{(\psi_{\rho} \chi_{\lambda} - \psi_{\lambda} \chi_{\rho})}{\sqrt{2}}, \\
\Psi_{4\mathbf{1}, L^P} & : \frac{\psi_A \chi S \phi_A}{\sqrt{2}}.
\end{align*}
Magnetic moments of Baryons

Single quark magnetic moment operator

\[ \vec{\mu}_j = \frac{e_j}{2m_j} \vec{\sigma}_j \]

\( \mu_B \) is given by the matrix element

\[ < B \frac{1}{2}, \frac{1}{2} ) | \sum_{j=1,2,3} \frac{e_j}{2m_j} \vec{\sigma}_j | B ( \frac{1}{2}, \frac{1}{2} > \]
the u pair is in a symmetric (triplet) spin state \( \chi(1, m) \)

(the antisimmetry is ensured by the color \( \mathrm{wf} \) )

\( \phi(1/2, s) \) spin state of the third quark

proton state

\[
\psi\left(\frac{1}{2}, \frac{1}{2}\right) = \sqrt{\frac{2}{3}} \chi(1, 1) \phi\left(\frac{1}{2}, -\frac{1}{2}\right) - \sqrt{\frac{1}{3}} \chi(1, 0) \phi\left(\frac{1}{2}, \frac{1}{2}\right)
\]

\[
\mu_p = \frac{2}{3} (2\mu_u - \mu_d) + \frac{1}{3} \mu_d = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d
\]

neutron (u and interchanged)

\[
\mu_n = \frac{4}{3} \mu_d - \frac{1}{3} \mu_u \quad \quad \frac{\mu_n}{\mu_p} = -\frac{2}{3} \quad (\exp - 0.685)
\]
Baryon magnetic moments in nuclear magnetons (n.m.), normalised to proton and lambda moments

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Magnetic moment in quark model</th>
<th>Predicted, n.m.</th>
<th>Observed, n.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$\frac{4}{3} \mu_u - \frac{1}{3} \mu_d$</td>
<td>+2.79</td>
<td>+2.793</td>
</tr>
<tr>
<td>$n$</td>
<td>$\frac{4}{3} \mu_d - \frac{1}{3} \mu_u$</td>
<td>−1.86</td>
<td>−1.913</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$\mu_s$</td>
<td>−0.61</td>
<td>−0.614 ± 0.005</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$\frac{4}{3} \mu_u - \frac{1}{3} \mu_s$</td>
<td>+2.68</td>
<td>+2.46 ± 0.01</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>$\frac{4}{3} \mu_d - \frac{1}{3} \mu_s$</td>
<td>−1.04</td>
<td>−1.16 ± 0.03</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$\frac{4}{3} \mu_s - \frac{1}{3} \mu_u$</td>
<td>−1.44</td>
<td>−1.25 ± 0.014</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>$\frac{4}{3} \mu_s - \frac{1}{3} \mu_d$</td>
<td>−0.51</td>
<td>−0.65 ± 0.01</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>$3 \mu_s$</td>
<td>−1.84</td>
<td>−2.02 ± 0.05</td>
</tr>
</tbody>
</table>
The baryon current

**Quarks are the fundamental carriers of the baryon charge**

\[
\mathbf{j}^{(B)}(x) = \sum_{i=1}^{3} \mathbf{j}(\mathbf{p}_i, \mathbf{q}_i, \mu) \delta^3(q_i - q)
\]

\[
\overline{q} \gamma_\mu q
\]

quark current

(pointlike quarks)

**Non-relativistic reduction:**

\[
\mathbf{j}(\mathbf{q}) = \sum_i e_i e^{i \mathbf{q} \cdot \mathbf{p}_i}
\]

\[
\mathbf{j}(\mathbf{q}) = \frac{i}{2m} \sum_i e_i \left[ \mathbf{p}_i + i \mathbf{q} (\mathbf{C}_i \times \mathbf{q}) \right] e^{i \mathbf{q} \cdot \mathbf{p}_i}
\]

\[
\mathbf{j}(\mathbf{q}) \rightarrow \text{quark spin}
\]

**Quark charge:**

\[
e_i = \frac{1}{2} \left[ \frac{1}{3} + \mathbf{e}_i \mathbf{C}_i \right]
\]

(U, D only)
\[ \langle r^2 \rangle_p = \frac{1}{\alpha^2} = 0.89 \text{ fm}^2, \quad \alpha = 1.23 \text{ fm}^{-1} \]
The Isgur and Karl model
(how to correct the defect of to the H.O. model)

\[
H = 3m + \frac{p^2 + p^2}{2m} + L(p, \lambda) + H_{hyp}(\rho, \lambda, \sigma_i) \quad \text{L} = \sum_{i<j} \left( \frac{1}{2} K r_{ij}^2 + U(r_{ij}) \right) \equiv V_{conf}
\]

The term \( L(p, \lambda) \) provides confinement again through a h.o. potential, to which however an anharmonic term \( U \) is added

\[
L = \sum_{i<j} \left( \frac{1}{2} K r_{ij}^2 + U(r_{ij}) \right) \equiv V_{conf}
\]

\[
H_{hyp} = \frac{2\alpha_s}{3m^2} \sum_{i<j} \left\{ \frac{8\pi}{3} \frac{\mathbf{S}_i \cdot \mathbf{S}_j \cdot \delta(r_{ij})}{r_{ij}^3} + \frac{1}{r_{ij}^3} \left[ \frac{3 (\mathbf{S}_i \cdot \mathbf{r}_{ij}) (\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right] \right\}
\]

\[
a_m = 3 (\alpha/\pi)^3 \int dp (\alpha p)^m U(\sqrt{2}p) \exp(-\alpha^2 p^2)
\]

through the quantities

\[
E_0 = \varepsilon_0 + a_0 ; \quad \Omega = \varepsilon_0 - a_0 / 2 + a_2 / 3 ; \quad \Delta = -5/4 a_0 + 5/3 a_2 - 1/3 a_4
\]

\[
E(56,0^+) = E_0 \quad E(70,1^-) = E_0 + \Omega
\]
\[
E(56',0^+) = E_0 + 2 \Omega - \Delta \quad E(70,0^+) = E_0 + 2 \Omega - \Delta / 2
\]
\[
E(56,2^+) = E_0 + 3 \Omega - \Delta / 5 \quad E(70,2^+) = E_0 + 3 \Omega - \Delta / 5
\]
The potential $U$ does not violate the SU(6) symmetry, but eliminates the degeneracy of the SU(6) multiplets in the various shells. The resulting energy pattern is richer than the h.o. one, with shifts depending only on the moments $a_m$ of the potential (see Table 5).

The last term in eq. (14) is called the hyperfine interaction, since it has the same form than the spin-dependent (Breit-Fermi) part of the electromagnetic interaction between charged particles:

$$H_{\text{hyp}} = \frac{1}{3m^2} \sum_i \sum_j \left( \frac{S_i \cdot S_j}{r_{ij}^3} \right)^2 \sum_{i' j'} J_{i i'} J_{j j'}$$

where $\%$ is the strong coupling constant and $S_i$ are the spins of the quarks. The spin-dependence violates SU(6) and the physical states are then superposition of SU(6) configurations. The effect of the various modification to the h.o. potential is shown pictorially in Fig. 4. The important consequence of $H_{\text{hyp}}$, besides giving rise to a mixing of the SU(6) h.o. states, is to produce a N-A mass difference: $2 \text{GeV}$.

Fig. 4. The h.o. spectrum (left), the arrangement of the SU(6) states after the introduction of the $U$ potential (middle) and some of the most relevant experimental levels (right). The lines show the mixing produced by the hyperfine part of the one-gluon-exchange interaction in order to build the physical states.
\[ \begin{aligned}
\mathcal{B} &= \langle B | g | B \rangle \\
\mathcal{J}_B &= \langle B | \mathcal{J} | B \rangle \\

&\text{for e.m. excitation}
\end{aligned} \]

**Example:** magnetic moments \( \{ N \} \)

\[ \mu = \langle N, J_z = +\frac{1}{2} | \sum_i \frac{e_i^2 \hbar}{2mc} \sigma_z^{(i)} | N, J_z = +\frac{1}{2} \rangle \]

\[ = 3 \cdot \frac{e_i \hbar}{2mc} \left( \frac{1}{3} \langle e_3 \sigma_z^{(3)} \rangle \right) \]

\[ \psi_{39^{\text{symm.}}} \]

\[ \psi_{\text{space norm.}} \]

nucleon isospin

\[ = \frac{e_i \hbar}{2mc} \cdot \frac{1 + s_0 c_0}{2} = 3 \mu_N \quad \text{proton} \]

\[ = -2 \mu_N \quad \text{neutron} \]

**Note:**

\[ \langle \sigma_9 \rangle = \frac{4}{3} \sigma_N \]

\[ \langle \sigma_9^0 c_0 \rangle = \frac{5}{9} \sigma_N c_0 \]

Ex. \( <r^2>_p = 1/\alpha^2 \)
Algebraic solution of the Coulomb problem

A) three dimensions \( \frac{1}{r} \)

\[ \begin{align*}
\therefore \text{angular momentum} & \rightarrow O(3) \text{ symmetry} \\
\therefore \text{Runge-Lenz vector} & \rightarrow O(4) \text{ symmetry}
\end{align*} \]

\[ H = -\frac{1}{2} \frac{1}{C_2(O(4)) + 1} \rightarrow E = -\frac{1}{2} n^2 \]

\[ C_2(O(4)) \text{ Casimir for } O(4) \]

since

\[ C_2(O(4)) = \omega (\omega + 2) \quad n = \omega + 1 \]

In general the eigenvalues of \( C_2(O(N)) = \omega (\omega + N - 2) \)
algebraic solution to the hyperCoulomb problem

B) six dimensions \( \frac{1}{x} \)

\[ \therefore \text{ hyperangular invariance} \rightarrow \text{O(6) symmetry} \]
\[ \therefore \text{"Runge-Lenz vector"} \rightarrow \text{O(7) symmetry} \]

\[ H = -\frac{1}{2 \left[ C_2(\text{O(7)}) + (5/2)^2 \right]} \rightarrow E = -\frac{1}{2 n^2} \]
\[ v + 1 \quad \left( (v-1)/2 \right)^2 \]

\[ C_2(\text{O(7)}) = \omega (\omega + 5) \quad n = \omega + 5/2 \]
\[ \omega (\omega + v - 1) \quad \omega + (v-1)/2 \]

(iachello, Giannini, Santopinto)
\[ H = \sum_{i=1}^{6} \frac{p_i^2}{2m} - \frac{x}{x} \]

\[ p_i = (\vec{p}_i, \vec{p}_i) \]

\[ q_i = (\vec{q}_i, \vec{q}_i) \]

\[ x = (\sum q_i^2)^{1/2} \]

\[ L_{ij} = q_i p_j - q_j p_i \]

\[ O(6) \]

15 generators

\[ [H, L_{ij}] = 0 \]

"Runge-Lenz" vector:

\[ M_i = \frac{1}{2m} (p_j L_{ij} + L_{ij} p_j) - \frac{2}{x} q_i \]

\[ [H, M_i] = 0 \]

generalization of classical R.L. vector
The Models
(CQM)

some other Constituent Quark Model
## different CQMs for bayons

<table>
<thead>
<tr>
<th></th>
<th>Kin. Energy</th>
<th>SU(6) inv</th>
<th>SU(6) viol</th>
<th>date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isgur-Karl</td>
<td>non rel</td>
<td>h.o. + shift</td>
<td>OGE</td>
<td>1978-9</td>
</tr>
<tr>
<td>Capstick-Isgur</td>
<td>rel</td>
<td>string + coul-like</td>
<td>OGE</td>
<td>1986</td>
</tr>
<tr>
<td>U(7) B.I.L.</td>
<td>rel M^2</td>
<td>vibr+L</td>
<td>Guersey-R</td>
<td>1994</td>
</tr>
<tr>
<td>Hyp. O(6)</td>
<td>non rel/rel</td>
<td>hyp.coul+linear</td>
<td>OGE</td>
<td>1995</td>
</tr>
<tr>
<td>Glozman Riska</td>
<td>non rel/relPlessas</td>
<td>h.o./linear</td>
<td>GBE</td>
<td>1996</td>
</tr>
<tr>
<td>Bonn</td>
<td>rel</td>
<td>linear 3-body</td>
<td>instanton</td>
<td>2001</td>
</tr>
</tbody>
</table>
Non strange spectrum

Capstick & Isgur’s Model


GB Model


V(x) = - \tau / x + \alpha x

Hypercentral Constituent Quark Model
hCQM

free parameters fixed from the spectrum

Comment
The description of the spectrum is the first task of a model builder

Predictions for:
photocouplings
transition form factors
elastic from factors

describe data (if possible)
understand what is missing
Hypercentral Constituent Quark Model

hCQM

free parameters fixed from the spectrum

Comment
The description of the spectrum is the first task of a model builder

Predictions for:
photocouplings
transition form factors
elastic from factors

describe data (if possible)
understand what is missing
LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains
  a long range \textit{spin-independent} confinement
  a short range spin dependent term

\textbf{Spin-independence} \rightarrow \textbf{SU(6) configurations}
SU(6) configurations for three quark states

\[
6 \times 6 \times 6 = 20 + 70 + 70 + 56
\]

Notation

\[(d, L^\pi)\]

d = \text{dim of SU(6) irrep}
L = \text{total orbital angular momentum}
\pi = \text{parity}
SU(6) configurations for three quark states

\[ 6 \times 6 \times 6 = 20 + 70 + 70 + 56 \]
\[ \text{A M M S} \]

Notation

\( (d, L^\pi) \)

d = \text{dim of SU(6) irrep}
L = \text{total orbital angular momentum}
\( \pi = \text{parity} \)
Jacobi coordinates

\begin{align*}
\rho & \quad 1 \\
\lambda & \quad 2 \\
3 &
\end{align*}

Hyperspherical Coordinates

\( (\rho, \Omega_\rho, \lambda, \Omega_\lambda) \Rightarrow (x, t, \Omega_\rho, \Omega_\lambda) \)

\begin{align*}
x &= \sqrt{\rho^2 + \lambda^2} \\
t &= \arctg \frac{\rho}{\lambda}
\end{align*}

\[ L^2(\Omega) Y[\gamma](\Omega) = -\gamma(\gamma + 4) Y[\gamma](\Omega) \quad \gamma = 2n + l_\rho + l_\lambda \]

\[ L^2(\Omega) \Leftrightarrow C_2(O(6)) \]

\[ Y[\gamma](\Omega) \]

\[ \gamma = 2n + l_\rho + l_\lambda \]

Hyperspherical harmonics

Hasenfratz et al. 1980:
\[ \sum_{i<j} V(r_{ij}) \approx V(x) + \ldots \]
\[ \sum V(r_{ij}) \text{ is approximately hypercentral} \]
Hypercentral Hypothesis

\[ V = V(x) \]

Factorization

\[ \psi(x, t, \Omega_\rho, \Omega_\lambda) = \psi_{\nu\gamma}(x) \quad Y_{[\gamma, l_\rho, l_\lambda]} \]

(“dynamics”) ("geometry")

Only one differential equation in x (hyperradial equation)

Hypercentral Model

\[ V(x) = -\frac{\tau}{x} + \alpha x \]

Hypercentral approximation of

\[ V = -\frac{b}{r} + cz \]

Genoa group, 1995
- QCD fundamental mechanism

- Flux tube model

3-body forces

Carlson et al, 1983
Capstick-Isgur 1986
hCQM 1995
Two analytical solutions

hyperCoulomb - τ/x

h. o. \[ \sum_{i<j} \frac{1}{2} k (r_i - r_j)^2 = \frac{3}{2} k x^2 \]
• H.O.
• W.fs  $e^{-\alpha^2 r^2}$
• F.F.  $e^{-\alpha^2 r^2/6}$
• Transition form factor:
• Polynomial $\times e^{-\alpha^2 r^2/6}$

• Hyp.
• W.fs  Polinomial $e^{-br}$
• F.F.  $:1/(1+Q^2/b^2)^{7/2}$
• Transition f.f.:
  Polynomial$\times$
  $1/(1+Q^2/b^2)^{(7+n)/2}$
\[ V(x) = -\frac{\tau}{x} + \alpha x \]
Quark-antiquark lattice potential  


\[ V = -\frac{b}{r} + cr \]
Introducing SU(6) violation

One Gluon Exchange

\[ V_{OGE} = -\frac{a}{r} + \text{Hyperfine interaction} \]
Hypercentral Model (1)

\[ H_{3q} = 3m + \sum_{i=1}^{3} \frac{p_i^2}{2m} + V(x) + H_{hyp} \]

• \( V(x) = -\frac{\tau}{x} + \alpha x \); \( H_{hyp} = A \left[ \sum_{i<j} V^S(r_i, r_j) \sigma_i \cdot \sigma_j + \text{tensor} \right] \)

• 3 parameters \( \tau, \alpha, A \) ← fixed to the spectrum, \( m = \frac{M}{3} \)

\[ \tau = 4.59 \]

\[ \alpha = 1.61 \text{ fm}^{-1} \]

\[ A \leftrightarrow (N - \Delta) \]

\[ x = \sqrt{\rho^2 + \lambda^2} \]

hyperradius
Results (predictions) with the Hypercentral Constituent Quark Model

for

- Helicity amplitudes
- Elastic nucleon form factors
The helicity amplitudes
HELICITY AMPLITUDES

Extracted from electroproduction of mesons
Definition

\[ A_{1/2} = < N^* J_z = 1/2 | H^{T}_{em} | N J_z = -1/2 > \]
\[ A_{3/2} = < N^* J_z = 3/2 | H^{T}_{em} | N J_z = 1/2 > \]
\[ S_{1/2} = < N^* J_z = 1/2 | H^{L}_{em} | N J_z = 1/2 > \]

\(N, N^*\) nucleon and resonance as 3q states
\(H^{T}_{em}, H^{L}_{em}\) model transition operator

§ results for the negative parity resonances:

Systematic predictions for transverse and longitudinal amplitudes
E. Santopinto, M.G., submitted to PR C
Definition

\[ A_{1/2} = \langle N* J_z = 1/2 | H_{\text{em}}^T | N J_z = -1/2 \rangle \]
\[ A_{3/2} = \langle N* J_z = 3/2 | H_{\text{em}}^T | N J_z = 1/2 \rangle \]
\[ S_{1/2} = \langle N* J_z = 1/2 | H_{\text{em}}^L | N J_z = 1/2 \rangle \]

N, N* nucleon and resonance as 3q states
\[ H_{\text{em}}^T H_{\text{em}}^L \] model transition operator

§ results for the negative parity resonances:

Systematic predictions for transverse and longitudinal amplitudes

Proton and neutron electro-excitation to 14 resonances
N(1520) $3/2^-$ transition amplitudes

N(1535) $1/2^-$
transition amplitudes

E. Santopinto, M.G.
Phys. Rev. C86,
065202 (2012)
Figure 15: (Color on line) The P11(1440) proton transverse (a) and longitudinal (b) helicity amplitudes predicted by the hCQM (full curves), in comparison with the data of refs. [134], [131] and the Maid2007 analysis [121] of the data by refs. [135], [132], [133] and [136]. The PDG point [63] is also shown. The figure is taken from ref. [46] (Copyright (2012) by the American Physical Society).

Neutron photocouplings

A\textsubscript{1/2} (10^{-3} \text{ GeV}^{1/2})

A\textsubscript{1/2} hCQM
A\textsubscript{1/2} Bonn

N(1440) N(1520) N(1525) N(1650) N(1675) N(1680) N(1710) N(1720)

Bonn: A.V. Anisovich et al., EPJ A49, 67 (2013)
• The hCQM seems to provide realistic three-quark wave functions
• The main reason is the presence of the hypercoulomb term

Solvable model

\[ V(x) = -\tau/x + \alpha x \]

linear term treated as a perturbation
wf mainly concentrated in the low \( x \) region

- energy levels expressed analytically
- unperturbed wf given by the \( 1/x \) term
- major contribution to the helicity amplitudes

Good results due to semplicity

$A_p^m \ N(1520)\ D13$

$r_p \ 0.5 \ fm$

$r_p \ 0.86 \ fm$

$A_m (10^{-3} \ GeV^2)$

$Q^2 (GeV^2)$

Green curves H.O.

Blue curves hCQM

$m = 3/2$

$m = 1/2$
The nucleon elastic form factors
- elastic scattering of polarized electrons on polarized protons

- measurement of polarizations asymmetry gives directly the ratio $G_{pE}^{p}/G_{pM}^{p}$

- discrepancy with Rosenbluth data (?)

- linear and strong decrease

- pointing towards a zero (!)

- new data (jan 2010) seem to confirm the behaviour
With a calculated radius of about 0.5 fm the e.m. form factors predicted by the hCQM are not good!

BUT

relativity is needed
Various levels

- relativistic kinetic energy
- Lorentz boosts
- Relativistic dynamics
- quark-antiquark pair effects (meson cloud)
- relativistic equations (BS, DS)
Relativistic corrections to form factors

- Breit frame
- Lorentz boosts applied to the initial and final state
- Expansion of current matrix elements up to first order in quark momentum

Results

\[ A_{\text{rel}}(Q^2) = F \cdot A_{n,\text{rel}}(Q_{\text{eff}}^2) \]

\[ F = \text{kin factor} \quad Q_{\text{eff}}^2 = Q^2 \left( M_N/E_N \right)^2 \]

De Sanctis et al. EPJ 1998
Full curves: hCQM with relativistic corrections
Dashed curves: hCQM in different frames
Elastic Form Factors in the hCQM


\[ \mu_p \frac{G_E^P(Q^2)}{G_M^P(Q^2)} \]

calculated

NO QUARK FORM FACTORS

\[ T_{NR}, \text{ BOOSTS, CURRENT EXPANSION} \]
Construction of a fully relativistic theory
Relativistic Dynamics

Three forms (Dirac):
Light (LF), Instant (IF), Point (PF)

**Point form:**
Composition of angular momentum states as in the non relativistic case

Moving three-quark states are obtained through (interaction free) Lorentz boosts (velocity states)
Construction of a fully relativistic theory
Relativistic Dynamics

Relativistic Hamiltonian Dynamics
for a fixed number of particles  (Dirac)

Construction of a representation of the Poincaré generators
\( P_\mu \) (tetramomentum), \( J_k \) (angular momenta), \( K_i \) (boosts)
obeying the Poincaré group commutation relations
in particular

\[
[P_k, K_i] = i \delta_{kj} H
\]

Moving three-quark states are obtained through
(interaction free) Lorentz boosts  (velocity states)

Three forms:
  Light (LF), Instant (IF), Point (PF)
Differ in the number and type of (interaction) free generators
**Point form:** 

\[ P_\mu \] interaction dependent

\[ J_k \] and \[ K_i \] free

Composition of angular momentum states as in the non relativistic case

Mass operator 

\[ M = M_0 + M_I \]

\[ M_0 = \sum_i \sqrt{p_i^2 + m^2} \]

\[ \sum_i \vec{p}_i = 0 \]

\[ \vec{P}_i \] undergo the same Wigner rotation -> \( M_0 \) is invariant

Similar reasoning for the hyperradius

The eigenstates of the relativistic hCQM are interpreted as eigenstates of the mass operator \( M \)

Moving three-quark states are obtained through (interaction free) Lorentz boosts (velocity states)
Calculated values!

- Boosts to initial and final states
- Expansion of current to any order
- Conserved current
Further support

Inelastic proton scattering as elastic scattering on CQ

(approximate) scaling function: $F(Q^2) = 1/(1 + 1/6 r_{CQ}^2 Q^2)$

$r_{CQ} \approx 0.2\,\text{fm}$

Ricco et al., PR D67, 094004 (2003)
$Q^2 F_2^p / F_1^p$

$Q^2$ (GeV/c)$^2$

$4 M_p^2/k_p$

Milbrath et al.
Gayou et al.
Pospischil et al.
Punjabi et al., Jones et al.
Puckett et al.
Relativistic hCQM
In Point Form

Y.B. Dong, M. Giannini, E. Santopinto, A. Vassallo,
Few-Body Syst. **55** (2014) **873-876**
• the medium $Q^2$ behaviour is fairly well reproduced
• there is lack of strength at low $Q^2$ (outer region) in the e.m. transitions

• emerging picture:
  quark core plus (meson or sea-quark) cloud
Conclusions First Part

- CQM provide a good systematic frame for baryon studies
- fair description of e.m. properties (specially N-N* transitions)
- possibility of understanding missing mechanisms
- quark antiquark pairs effects
  
  unquenching: important break through