Classwork

Meson Quantum Numbers in the Quark Model

Consider a two particle system, the $q\bar{q}$ system, which in the quark model form the mesons. Denote the quark $q$ as particle 1 and the antiquark $\bar{q}$ as particle 2. The quarks have spin $s_q = \frac{1}{2}$, mass $m_q$, and intrinsic parity $\eta_q = +1$. Let the four momentum and spin projection for the quark $q$ be $p_1$ and $\sigma_1$, and for the $\bar{q}$ $p_2$, $\sigma_2$. A two quark system may be written in their center-of-momentum frame

\[ |q(p_1, \sigma_1) \bar{q}(p_2, \sigma_2) \rangle \rightarrow |\hat{p}\sigma_1\sigma_2 \rangle \]  

(1)

where $\hat{p} = (\theta, \varphi)$ is the orientation of $q$ with respect to a defined coordinate system.

(1) In the $LS$ coupling scheme, write the two particle state in the total angular momentum basis $|JM\ell s\rangle$.

(2) Determine the allowed quantum numbers for $s$, $\ell$, and $J$.

(3) Using the action of the parity $P$ and c-parity $C$ operators on the state $|JM\ell s\rangle$, determine the allowed parity and c-parity quantum numbers $P$ and $C$. Determine the allowed $J^{PC}$ numbers for the $q\bar{q}$ system through $J = 3$. List the states through $J = 3$ that are not allowed, the so-called ‘exotic’ states.

\[ P |JM\ell s\rangle = (-1)^{\ell+1} |JM\ell s\rangle \]  

(2)

\[ C |JM\ell s\rangle = (-1)^{\ell+s} |JM\ell s\rangle \]  

(3)