II-1

CHAPTER II
THE LOGIC OF DIRECT TESTING

"So here's a question for you. How old did you say you were?"

Alice made a short calculation, and said
"Seven years and six months."

"Wrong!" Humpty Dumpty exclaimed triumphantly,
"You never said a word like it."

"I thought you meant 'How old are you?'" Alice explained.

"If I'd meant that, I'd have said it," said Humpty Dumpty.

--Lewis Carroll

Before a scientist can test a theory he or she obviously must know what it says. In this chapter we will concentrate on the logical aspects of understanding simple sentences. We will see that even the most trivial sounding claims have logical consequences which are at first psychologically surprising. For example, as we will prove later, the sentence "Only the good die young" directly implies that if you are bad you will have a rather long life.

I. Venn Diagrams and the Notion of Logical Content

The best way of revealing the logical content of simple sentences is through the method of Venn diagrams.¹

Let us represent the set of all the things which we will wish to talk about with a box (call it \(U\), for universe of discourse). Now consider

¹Named after John Venn, an English logician. A history of logic diagrams can be found in Venn's Symbolic Logic, rev. 2nd ed. (London, 1894).
some property $S$. Let the circle divide the universe into those objects which have the property $S$ (they lie inside the circle) and those which do not (they lie outside the circle):

Suppose we wish to claim that there are no objects in $U$ that have property $S$. (For example, let $U$ be the set of creatures alive today, and $S$ the set of dinosaurs.) Then we will diagram the content of this statement by showing that $S$ is not occupied:

Nothing (in $U$) has property $S$ (i.e., $S$ is empty).

Sometimes a sentence will claim that the not-$S$ set is empty. For example, if $U$ is the set of all people and $\overline{S}$ is the property of having original sin, some theologians would wish to claim the following:

No one (in $U$) is without original Sin (i.e. everyone has property $S$).
If we consider two properties, $S$ and $P$, they divide the universe up into four sets, (i) those objects which have both property $S$ and property $P$, (ii) those which are $S$, but not $P$, (iii) those which are not $S$, but $P$, (iv) those which have neither property $S$ nor property $P$:

We may wish to assert for some properties $S$ and $P$ that (i) is empty, i.e., that no $S$'s are $P$'s.¹ For example, if $U$ is the set of all material objects, $S$ is the set of all squirrels, and $P$ is the set of all professors, we would obtain the following diagram for the negative universal generalization:

No squirrels are professors.

We will diagram the corresponding affirmative existential claim as follows:

Some squirrels are professors.

¹We will let $S$ stand for both the property (e.g., the property of being square) and the set defined by that property (i.e., the set of all things with the property of being square).
(The $\checkmark$ shows that the sentence says that there is at least one object in the class of things which are both $S$ and $P$.)

Let us now systematically explore the logical content of simple sentences using Venn diagrams.

a. Sentences with "Some".

Consider the following sentence:

(1) Some dogs are black.

It is trivial to point out that sentence (1) is talking about dogs and asserting that at least some of them are black.

But if sentence (1) is true (and it is, of course), it must also be true that:

(2) Some black things are dogs.

One does not normally construe sentence (1) as talking about black objects and asserting that at least some of them are dogs, but this latter assertion is logically implicit in sentence (1) although we may not be psychologically aware of it.

To show that sentences (1) and (2) have the same logical content consider the following diagram:

![Venn Diagram]

(1) Some dogs are black (things).
(2) Some black things are dogs.

We will adopt the convention of putting a check ($\checkmark$) in any class which is occupied. Since the same Venn diagram pictures both (1) and (2), we say they are logically equivalent. Of course, there are important psychological and stylistic differences between them. (The student writing home for more
money would not be well advised to say, "Some of the rotten things in the world are my teeth" unless both parents are logicians).

To make sure you understand the above, fill in the diagram for the following sentence:

![Venn Diagram]

(3) Some dogs are not black.

Now construct another English sentence which is logically equivalent and write it below.

(4)

b. Sentences with "No" or "None".

We will adopt the convention of crossing out any classes which the sentence claims are unoccupied. Here is an example:

![Venn Diagram]

(5) No polar bears are black.

Sentence (5) is logically equivalent to the following:

(6) No black things are polar bears.

Sometimes English introduces a new word for the negation of a property. For example, doctrines which cannot be mistaken are said to be infallible.
It is customary to put the positive properties inside the circle. Here is an example:

![Venn Diagram](image)

No human utterances are infallible.

**EXERCISE A:**

1. Draw Venn diagrams for each of the sentences below. For simplicity assume that everything which is not good is bad and everything which is not pleasant is unpleasant. In each case begin with the same basic format:

![Venn Diagram](image)

(a) No good things are pleasant.
(b) No unpleasant things are good.
(c) None of the pleasant things are bad.
(d) None of the good things are unpleasant.
(e) No bad things are pleasant.

And now a harder one (marked !)

!(f) No bad things are unpleasant.

2. By looking at their Venn diagrams decide which of the above sentences (a-f) are logically equivalent.

**c.** Sentences with "All", "Every", "Each", or "Any".

It remains to discuss those sentences which express affirmative universal generalizations, such as:
All ravens are black.

Every metal expands when heated.

Each citizen has the right to life, liberty, and the pursuit of happiness.

Anyone who drinks also smokes.

We will diagram sentences such as the above as follows:

```
  Black Things  
   \    /      
    \  /       
     \|/        
    Ravens     

(7) All ravens are black.
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Sentence (7) asserts that there is no case of a non-black raven. Thus we cross out just that area on the diagram.

At first it may seem strange to you that an affirmative sentence should be diagrammed by denying the existence of a certain type of object. To motivate this way of looking at things, consider the following story.

After cooking a big dinner, Grandma retires to the couch while the relatives do the dishes. Cousin Susie puts all the tea cups on a tray and disappears into the kitchen. There is a huge crash and Grandma inquires anxiously, "Did the cups break?" There is a pause and then three sad pronouncements come from the kitchen:

Uncle Ned: "Not a single one survived."

Great-Aunt Nellie: "There ain't nary a cup but what's broken."

Little Carla: "All the cups are broken, Grandma."

These three sentences convey exactly the same information, but the first two are negatives while the third is affirmative. When you start out diagramming universal affirmatives you may wish to adopt Great-Aunt Nellie's form of expression. Thus (7) "All ravens are black" can be paraphrased as: There ain't no raven but what is black!
It can be shown that sentence (7) is logically equivalent to the following rather clumsy sentence (which is called its contrapositive):

(8) All non-black things are non-ravens.

To convince yourself of the equivalence, draw a Venn diagram of !(8).

Most people find it takes a while before they really believe that (7) and (8) are logically equivalent. For the moment, if you like, you can just memorize the following rule, called the Law of Contraposition:

"All A's are B's" is logically equivalent to "All non-B's are non-A's."

(In other words, if one negates both the subject and the predicate and switches them, the sentence says the same thing.)

EXERCISE B:

1. Draw Venn diagrams for each of the following sentences: (Use the same basic format as you did in Exercise A above.)

   (a) All good things are pleasant.
   (b) All pleasant things are good.
   (c) All bad things are pleasant.
   (d) All unpleasant things are good.
   (e) All good things are unpleasant.
   (f) All pleasant things are bad.
   !g) All unpleasant things are bad.
   !h) All bad things are unpleasant.

2. By looking at the Venn diagrams decide which, if any, of the above sentences (a-h) are logically equivalent.

3. Now use the Law of Contraposition to check your answers to (2).

4. Repeat exercises (1) and (2) above for the sentences formed by replacing "all" with "no," i.e. "No good things are pleasant."

5. Does the Law of Contraposition work on "no" sentences? Can you make up a rule which does generate a logically equivalent claim for "no" sentences?

6. Make up a rule which turns an "all" sentence into a "no" sentence which is its logical equivalent.
d. How to check for equivalence, compatibility and inconsistency.

We have already noted that certain pairs of sentences which look different actually have precisely the same logical content; that is, they are logically equivalent. Later on when we discuss testing, we will want to say that a certain experimental report contradicts or is inconsistent with the theory under test. A third possibility is that two sentences are consistent, but not logically equivalent. We shall say such sentences are compatible. Thus we can classify the relationships between sentences as follows:

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<table>
<thead>
<tr>
<th>Pairs of Sentences</th>
<th>Consistent</th>
<th>Inconsistent (or Incompatible)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent</td>
<td></td>
<td>Merely Compatible</td>
</tr>
</tbody>
</table>
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Let us look more carefully at each of these three possible relationships between sentences.

i) **Logical equivalence.** Two sentences are logically equivalent if and only if the conditions which would make them true (or false) are exactly the same.

There follow pairs of sentences which are logically equivalent:

- a) Some dogs are animals.
- a') Some animals are dogs.
- b) No corpses are conscious.
- b') All corpses are unconscious.
- c) There are no skyscrapers on the north side of Bloomington.
- c') If there are any skyscrapers in Bloomington, they are on the south side.
- d) All poor people are honest.
- d') All dishonest people are above the poverty level.

If two sentences can be represented by exactly the same Venn diagram, then they are logically equivalent.

ii) **Inconsistency.** Two sentences are inconsistent if and only if there is no possible state of affairs such that both would be true.
The following pairs are mutually inconsistent:

a) Some Brownies are kind.
   a') No Brownies are kind.

b) Some mistakes are not embarrassing.
   b') All mistakes are embarrassing.

If the Venn diagram of one sentence has a check (✓) in an area which
the other sentence says is empty (i.e., the area is crossed out), then
the two sentences are inconsistent.

iii) Compatibility. All pairs of sentences which are neither logically
equivalent nor inconsistent we will call compatible. Two sentences are
compatible if and only if there is some possible state of affairs in which
they could both be true.

Here are examples:

   a) Grass is green.
      a') Grass is delicious.
   b) Some swans are white.
      b') All swans are white (and there are many swans).
   c) All animals with hearts have lungs.
      c') All animals with lungs have hearts.

In the Venn diagrams of sentences which are compatible, there will always
be at least one area in which one sentence has marks (either checks or
crossing-out lines) while the other sentence leaves that area blank.

As we saw above, in diagramming "All A's are B's" we did not assume
that there are in fact any A's. Therefore, all we did was exclude the
area representing things which were A but not-B. We did not place any
checks in the area where A and B overlap.

As a result of this convention, it turns out that the sentences "All
A's are B's" and "All A's are not-B's" are compatible, not inconsistent
as you might have expected.

To make this result more intuitively acceptable, consider the following
fable. Suppose two census takers, Jones and Smith, divide up their duties
in Bloomington by using the railroad tracks. At the end of the day, they report as follows:

Jones: "There are no Ferrari's west of the tracks in Bloomington."

Smith: "There are no Ferrari's east of the tracks in Bloomington."

I think most people would readily agree that the two reports are compatible. There is no temptation whatsoever to think that if Jones is right, Smith must have made a mistake or vice versa.

Let us now transform each of their reports into a logically equivalent claim. (The material in parentheses is logically superfluous and is only put in for explanatory purposes.)

Jones: "All Ferrari's (if there are any in Bloomington) are east of the tracks" (since there aren't any west of the tracks).

Smith: "All Ferrari's are west of the tracks."

These new reports, being logically equivalent to the old ones, are still compatible. There is a possible world (namely one in which there are no Ferrari's whatsoever in Bloomington) in which both are true.

The reports of Smith and Jones only become inconsistent if we augment them as follows:

Jones': "There are no Ferrari's west of the tracks; however, there are some somewhere in Bloomington."

Smith'" "There are no Ferrari's east of the tracks; however, there are some somewhere in Bloomington."

In general, in this book we will use the sentence "All A's are B's" to mean "All A's (if there are any) are B's." We will not use it to mean "All A's (and there are some) are B's."

In most ordinary contexts, a sentence such as "All my aunts are rich" does imply that you in fact have some aunts. However, this is not always the case in scientific contexts. The claim that the volume of all ideal gases varies inversely with pressure does not imply that there are in fact any perfectly ideal gases. The law of the lever holds for all friction-free cases -- but of course there aren't any.
Much of science talks about what would happen under circumstances which in fact never obtain! This is one reason that we adopt the convention of not reading any existential import into "all" sentences.

Remember, the claim "All A's are B's" will not be interpreted as implying that there are any A's. The only sentences which we will take to assert existence are the ones involving "some".

[Notice how one's theory of scientific method influences the logical conventions one uses. Since an inductivist never lets a conjecture cross his or her mind unless there is strong inductive evidence for it, the inductivist would never have occasion to utter "All A's are B's" unless they had observed many cases where A is associated with B.

However, if one adopts the method of hypothesis testing one does need to be able to talk about conjectures of the form, "If there are any A's they are all B's." In this book we will choose the convention most convenient for the hypothesis tester.]

EXERCISE C:

1. For each of the following pairs, decide whether they are equivalent, compatible, or inconsistent. (Make no unwarranted assumptions about what does or does not exist.)

   a) All sane people are jolly.
   a') All insane people are jolly.
   ! b) All stannous nitrate samples are combustible.
   !b') All stannous nitrate samples are incombustible.
   c) All Puritans are intolerant
   c') No tolerant people are Puritans.
   d) All inadequate people are people who can't cope.
   d') No inadequate people can cope
   e) No inadequate people can cope.
   e') People who can cope are adequate.
   f) Some bellibones are furfuraceous.
   f') Nothing which is furfuraceous is a bellibone.
e. Arrow notation.

Since many scientific hypotheses are of the logical form "All A's are B's", it will be convenient to adopt a shorthand notation for them: $A \rightarrow B$, which can also be read as "If A, then B."

\[ A \rightarrow B \]

Anything which is not A will be symbolized as $\neg A$. Thus, "All good people are unpleasant" would be symbolized as: $G \rightarrow \neg P$. The expression which precedes the arrow is called the antecedent; that which follows is called the consequent. The entire sentence, $G \rightarrow \neg P$, is called a conditional statement.

**EXERCISES D:**

1. Express the following sentence in the suggested shorthand notation:
   
   (a) All Hoosiers are Basketball fans.
   
2. What kind of case would refute the above claim?

3. Consider this claim:
   
   (b) $\neg B \rightarrow \neg H$
   
   What kind of case would refute (b)?

   Are (a) and (b) logically equivalent?

4. Draw Venn diagrams for the following:
   
   (a) $A \rightarrow C$
   
   (b) $A \rightarrow \neg C$
   
   (c) $\neg A \rightarrow C$
   
   (d) $\neg A \rightarrow \neg C$
   
   (e) $C \rightarrow A$
   
   (f) $C \rightarrow \neg A$
   
   (g) $\neg C \rightarrow A$
   
   (h) $\neg C \rightarrow \neg A$

   Which, if any, are logically equivalent?

5. Consider the following:
   
   $A \rightarrow \neg B$
(a) Express the claim using "all."
(b) Express the claim using "no."
(c) Make up a "some" statement which is compatible with the claim.
(d) Make up a "some" statement which is inconsistent with the claim.

f. **Stylistic variants of affirmative universal generalizations.**

We have already seen that sentences which have the logical form \( A \rightarrow B \) can be expressed in English in a variety of ways, using words such as "all," "each," "every" and "any." There are lots of other stylistic variants of \( A \rightarrow B \), however, and some are not obvious at all. (One wonders how little kids ever learn natural language. Speaking logic would be much simpler for everyone!)

i) **Causes and other sufficient conditions.**

Many sentences which talk about causes and effects or necessary and sufficient conditions can be symbolized using the arrow notation. The most natural way to symbolize talk about causes and effects is to put the cause in the antecedent position and the effect in the consequent position:

\[
(1) \text{ Her Death was caused by a massive Stroke; } \\
\text{ or, } S \rightarrow D.
\]

(Note that "\( S \rightarrow D \)" means that massive strokes **invariably** result in death. Sometimes people who use the English sentence do not really mean to imply that there is a **universal** connection between the cause and the effect, as in "Smoking causes lung cancer.")

But causes do not always precede the arrow. We often reason from the absence of effects to the absence of causes:

\[
(2) \text{ Since there are no Tracks in the snow, my Dog couldn't have come this way; } \\
\text{ or, } \neg T \rightarrow \neg D.
\]

Not everything which appears in the antecedent place is a cause---for example, symptoms do not cause diseases, but they may nevertheless occur in the antecedent position if they provide sufficient grounds for a certain diagnosis.
(3) Whenever people break out in lavender Spots, it's a sure sign they've been strucken by the lambda Virus; or, \( S \rightarrow V \).

Regulations can also be symbolized using the arrow notation. Sufficient conditions occur in the antecedent position.

(4) At Mailorder U, 15 Dollars and 15 Credit hours are sufficient for Graduation.

\[(D \text{ and } C) \rightarrow G\]

(Note, however, that most rules express necessary conditions which will be discussed below.)

ii) Necessary conditions, prerequisites and other requirements.

Scientists often state laws of nature by citing physically necessary conditions.

(5) Calcium is necessary for Growth, or \( G \rightarrow C \); however, it is not sufficient, or \( \sim(C \rightarrow G) \). (Note the important difference between \( \sim(C \rightarrow G) \), which says that \( C \) doesn't always ensure \( G \), and \( C \rightarrow \sim G \), which says if \( C \) is the case, \( G \) never is present.)

Many legal statutes express necessary conditions.

(6) Anyone who wants to stay out of Court must pay their Taxes, or \( C \rightarrow T \).

(7) At least one year's Residency is required for Graduation, or \( G \rightarrow R \).

(If you prefer, you may wish to use the contrapositive form when symbolizing necessary conditions, as in "no year's residency, no graduation," or \( \sim R \rightarrow \sim G \).)

In non-technical language, necessary conditions are often marked by the words "only" or "unless." For example, how would you diagram and symbolize the following?

(8) Only Females get Pregnant.
You can solve this problem in many ways. (Some people prefer to just memorize a rule about "only.") Here is one way to think it out.

"Sentence (8) doesn't say that all females are pregnant. But it does say that if someone happens to be pregnant then that person is female—or in other words, it says that all pregnant people are females."

Thus (8) is represented as follows:

\[ P \rightarrow F \]

In other words, being female is a necessary (though not sufficient) condition for being pregnant.

It is even more difficult to symbolize "unless" sentences. Try this one:

(9) Bees certainly won't survive the winter unless you insulate the hive.

The sentence certainly doesn't say insulation is sufficient for survival. (After all, disease or predators may get in.) So we reject \( I \rightarrow S \). Even though the sentence speaks of "not surviving unless" the most straightforward way to symbolize it is using positive antecedents and consequents:

\[ S \rightarrow I \]

Isn't ordinary English rich (and tricky!) in its methods of expressing necessary conditions!

To summarize, \( A \rightarrow C \) simply means that if \( A \) is the case, \( C \) is also the case. The arrow shorthand in itself does not express the full nature of the relationship between antecedent and consequent. Sometimes the connection between \( A \) and \( C \) may be that of cause and effect (e.g., strokes cause death). In other cases the connection may be definitional (e.g.,
all vixens are foxes) or legal (e.g., all voters shall be at least 18 years old). Other times the relation between A and C may be more complex.

For example, consider the sentence "Whenever the barometer falls rapidly, there is a storm." Here we believe that there is no direct causal connection between the antecedent and consequent conditions. Rather both are effects of a common cause, the sudden influx of wet air.

EXERCISE E:

1. Symbolize each of the following conditionals using the underlined capital letters as abbreviations.
   (a) Only the Good die Young.
   (b) If an object is Dropped, it Accelerates toward the center of the earth.
   (c) All you need (for Happiness) is Love.
   (d) Love is necessary for Happiness.
   (e) Without Love, there is no (real) Happiness.
   (f) Love is not always enough (for Happiness).
   (g) In every Billiard ball collision, momentum is Conserved.
   (h) Momentum on a macroscopic scale is Conserved only if the collision is Elastic.
   (i) Monkeys raised without Cuddling are Neurotic.
   (j) Where there's Smoke, there's Fire.
   (k) One cannot legally Drive unless one is at least Fifteen.
   (l) Whenever an object is Heated, it Expands.
   (m) Only if she Tries will she Succeed (and even then she may not).
   (n) Too much Booze always results in a Hangover.
   (o) If anyone should happen to Break that bottle there would be a Mess.
   (p) All Strikes result in at least a temporary Loss of wages.
   (q) All Strikes are preceded by a Breakdown in negotiations.
   (r) Every Book in this room has a Call number starting with Q.
   (s) Anyone who is in College has a High school degree or the equivalent.
   (t) You can Vote for mayor only if you are a Resident of the city.
(u) The only way to Fail is if you don't Come to class.
(v) You will be Rich only if you Invest wisely.
(w) Only college Graduates can Stay in Crosby Hall.
(x) A necessary condition for Playing baseball is Quick reflexes.
(z) Unless one Never comes to class, one will Pass this course.

2. Symbolize the following sentences using the arrow shorthand. Above the arrow put a word or phrase which describes the nature of the relationship between antecedent and consequent, e.g., causal, definitional, legal, game rule, effects of a common cause, etc.

(a) No Rooks move Diagonally.
(b) Six months Residency is a necessary condition for In-state tuition.
(c) The Zone defense resulted in a Low shooting percentage.
(d) The player's Stepping out of bounds resulted in the ball's being Given to the other team.
(e) All Parallelograms necessarily have More than three sides.
(f) A Puppy is a young Dog.
(g) Any Tadpole which grows turns into a Frog.
(h) Whenever the red spots Disappear, the scarlet fever patient is nearly Well again.
(i) A conductor Moving through a magnetic field generates an Electric current.
(j) Infidelity is a sufficient condition for Divorce.
(k) His Cheating on the exam caused the teacher to Fail him.

g. More complicated claims.

In the ordinary use of language, sentences uttered in certain contexts have very definite implications which are not revealed by logical analysis. For example, suppose a day-care attendant announces cheerily, "Anyone who Finishes his or her milk, can have a Cookie." (F + C) Even a very young child would be upset if someone who didn't finish her milk got a cookie. (Only Humpty Dumpty would not object.)
Although what the attendant literally said was $F \Rightarrow C$, the child inferred him also to be asserting $\neg F \Rightarrow \neg C$. Although logic does not warrant this inference, the child was probably correct in believing that the attendant also intended to convey the second claim.

Also, as the examples in exercise E2 above indicate, Venn diagrams (or the arrow shorthand) are not rich enough to capture all of the meaning of simple sentences. In particular, they do not express the kind of connection which is claimed to hold between antecedent and consequent. (To do this one needs to use what is called modal logic.)

Neither do these very simple devices allow us to represent the logical content of more complicated sentences such as

"If the hypotenuse of triangle $A$ is longer than that of triangle $B$, then at least one side of triangle $A$ is longer than the corresponding side of triangle $B."

Here we will not attempt to develop notation for more complicated statements in any systematic way. (This problem is dealt with in symbolic logic classes and luckily for us, many scientific conjectures have a rather simple logical form.) However, we will make occasional use of intuitive shorthand like the following:

1. If something has Sugar in it, then it is both Tasty and Fattening.
   
   \[ S \Rightarrow (T \text{ and } F) \]

2. All Children are either Intelligent or Beautiful.

   \[ C \Rightarrow (I \text{ or } B) \]

3. If a car is either a Volvo or a Mercedes, then it has Disc brakes.

   \[ (V \text{ or } M) \Rightarrow D \]
4. Anyone who is both Just and Merciful is a Saint.

\((J \text{ and } M) \rightarrow S\)

We can produce Venn diagrams of these more complicated sentences by introducing a third circle. For example, sentence (1) above says the following:

1. \(S \rightarrow (T \text{ and } F)\)
   (i.e., everything which has property \(S\) is both \(T\) and \(F\) -- all other possibilities are ruled out.)

Now study the diagrams for the remaining three sentences above.

2. \(C \rightarrow (I \text{ or } B)\)
   (Of course, some children may be both intelligent and beautiful)

3. \((V \text{ or } M) \rightarrow D\)
EXERCISE F:

1. Put each of the following into the suggested shorthand notation and then draw a Venn diagram for it.

   (a) If someone is both Talented and Industrious, they will Succeed.
   
   (b) If one is either Talented or Industrious, one will Succeed.
   
   (c) All Talented people Succeed and all Industrious people Succeed. (Use only one arrow!)
   
   (d) Something which is both Dangerous and Pleasant is an Irresistible temptation.
   
   (e) Both Potassium and Sodium salts are Good conductors.
   
   (f) As pets, both Dogs and Cats are both Friendly and Amusing. (Excuse the bad English, but note the difference in how both "both's" are translated.)
   
   (g) Talent and Industry are both necessary for Success.

2. Suppose a mother says to a bunch of children,

   "Anyone who doesn't pick up his or her toys won't be going to the Movies."

Which of the following are logically equivalent to what she literally said? Which are probably part of what she meant (although she didn't explicitly say so)?
(a) If you pick up your toys, then you can go to the movies.
(b) If one of the kids shows up at the movies, then that child picked up his/her toys.
(c) You can go to the movies only if you pick up your toys.
(d) Unless you aren't going to the movies, pick up your toys.
(e) Only children who go to movies pick up their toys.

3. Many of the following sentences either explicitly state more than one conditional or implicitly express more than one conditional in normal contexts. Symbolize each sentence using as many conditionals as you need.

(a) All those who Believe and only those who believe will be Saved.
(b) Women should get equal Pay if and only if they do equal Work.
(c) The only time Janie Acts up is when Company comes and then she always does.
(d) If you Register, you can Vote. (What else is probably implied?)
(e) I'm going to the Lake unless company Comes.
(f) Jill's receiving a Scholarship is a necessary and sufficient condition for her going to Graduate school.
(g) That naughty dog Plays with the squeaky toy just at those times when the puppy Wants it.
(h) If You go skinny dipping I'll go too. Otherwise, forget it.
(i) Whenever the Switch is on the plug is Hot.
(j) Whenever there's a Rainbow, there are Water droplets in the air.
(k) The only thing which makes me Sneeze is Asparagus.

II. The Logic of Testing for Simple Sentences

We have used Venn diagrams as a means of analyzing the content of certain simple sentences. They are also useful in describing the different ways of testing such sentences.

a. Testing sentences with "Some".

Suppose you want to find out whether the sentence below is true or false:
(1) Some women are bald.

How would you proceed?

Perhaps the first method of testing which occurs to you is roughly as follows:

Test #1: Take a large sample of women (preferably from an old folks home). Now check to see whether any of them are bald.

We will describe the above test as follows: The domain of the test is a sample of women -- one only investigates women. The property-in-question in the test is baldness -- one only looks to see whether the members of the sample are bald or not.

There is another test which could be used to evaluate the truth of sentence (1):

Test #2: Take a large sample of bald people (perhaps the customers of a toupee shop) and check to see whether any of them are women.

In this case the test domain is a sample of bald people and the property-in-question is that of being female.

Test #1 and test #2 have different domains but they provide equally appropriate ways of testing sentence (1). Either test may lead to the discovery of at least one bald woman. Should this occur we will say that the test result verifies the conjecture. But what if our researches fail to turn up a single bald woman? Can we then say that the conjecture "Some women are bald" is false? Unfortunately, we cannot. Since it is impossible in a finite number of tests to exhaust either test domain, we can never conclusively refute the conjecture.

EXERCISES G:

1. Draw a Venn diagram for "Some women are bald". Locate the domains for test #1 and test #2 as described above. Can you design a test for the conjecture which uses a sample of men as a test domain? Why not?
2. For each of the sentences below, draw a Venn diagram, describe two different ways of testing it (specify both the test domain and the property-in-question) and say which test would probably be the easiest one to carry out.

(a) Some fleas live on dogs.
(b) Some chickens have two hearts.
(c) Some sodium compounds are not soluble.

b. Testing sentences with "No".

Let us now explore the ways of testing a negative universal generalization such as the following:

(2) No one year-old child can walk.

As above, there are two possible tests. We can either sample the population of one year-olds or choose the set of ambulatory people as our test domain.

If our investigations yield even a single one year-old who can walk, we will definitely have refuted the conjecture. But what if we fail to find any counterexamples to the claim? Even so, the conjecture will not be verified. No finite search can ever exhaust the test domains. Therefore, we can never be sure the claim is true.

EXERCISE H:

1. Draw a Venn diagram for each of the following sentences. Describe what would count as a refuting instance of each.
(a) No Fish has Feathers.

(b) No animal which has Lungs is without a Heart.

(c) No Large group is Free from racial prejudice.

(d) No Complex society is Insensitive to principles of fairness.

c. Testing sentences with "All" and other conditionals.

The most common type of scientific hypothesis is the affirmative
universal generalization. Testing sentences of this type works in exactly
the same way as for "No" sentences, but let us review the procedure once
more using an example where the conjecture is expressed as a conditional.

Considering the following conditional uttered on a snowy morning:

"If a Dog were to walk across the backyard, he would leave
Tracks."

\[ D \rightarrow T \]

As you read this sentence you might first picture a dog ambling across the
yard and then picture a set of tracks in the snow.

Can you imagine a series of events which would refute the claim that
\[ D \rightarrow T \]?

It might be as follows:

First scene: A dog walks across the snowy yard (e.g., an
instance of D).

Second scene: The yard is completely free of tracks (a case of
\( \neg T \)) because the dog has a little snow blower
attached to his rear end.

Moral: A case of D and \( \neg T \) refutes D \( \rightarrow T \).

Now consider the following use of the same conditional, D \( \rightarrow T \):
Standing on my back porch on a snowy morning, anxiously looking for my dog, I
reason this way:

"If a Dog were to walk across the backyard, he would leave Tracks
(D \( \rightarrow T \)). This means that if there are no Tracks, then my Dog
couldn't have walked across here. (\( \neg T \rightarrow \neg D \))."
\(~T \rightarrow \neg D\) is said to be the contrapositive of \(D \rightarrow T\). It is formed by inverting the antecedent and consequent and adding negation signs. \(\neg T \rightarrow \neg D\) is logically equivalent to \(D \rightarrow T\). But for most people contrapositives are not psychologically equivalent.

To help convince you that \(\neg T \rightarrow \neg D\) makes the same claim as \(D \rightarrow T\) let's ask what sort of case would refute \(\neg T \rightarrow \neg D\). Clearly what is needed is a case of \(\neg T\) and \(D\), e.g., a case where there are no tracks yet a dog has walked that way. The example of a dog with a snow blower refutes \(\neg T \rightarrow \neg D\) just like it refutes \(D \rightarrow T\).

Moral: A case of \(\neg T\) and \(D\) (or alternatively we could write \(D\) and \(\neg T\)) refutes \(\neg T \rightarrow \neg D\) (or alternatively we could write \(D \rightarrow T\)).

EXERCISES I:

1. Draw a Venn diagram for each of the following, describe two different ways of testing it, and specify what would count as a refutation.
   
   (a) Every time I travel by Train, I go to Boston.
   (b) Every time I go to Boston, I travel by Train.
   (c) All Arsenic compounds are Poisonous.
   (d) Each and every portion of the Universe contains Hydrogen.

2. For each of the following conditionals do the following series of tasks:
   
   i. Symbolize the conditional in the most natural way to you.
   ii. Symbolize the conditional using the contrapositive version.
   iii. Write down the kind of case which would refute the conditional.
   iv. Describe in practical terms what sort of samples could be used to test the conditional.

   a. (Example) If a mother Smokes, the new born child has Nicotine in its blood.
      
      i. \(S \rightarrow N\)
      ii. \(\neg N \rightarrow \neg S\)
      iii. \(S\) and \(\neg N\)
iv. (i) Take sample of pregnant women who smoke and see if their children have nicotine in their blood.

(ii) Take sample of children who are nicotine-free and see if their mothers smoked during pregnancy.

b. If you eat a normal American diet but don't drink milk, then you will have a calcium deficiency.

c. Only people with a schizophrenic parent become schizophrenic.

d. You can't teach an old dog new tricks.

e. If you don't smoke, you won't get lung cancer.

f. A nitro group can be added to a benzene ring just in those cases where the ring is electrophilic.

3. For each of the following sentences, list one, or if possible more, opposing claims about Jones which are inconsistent with the original sentence:

(a) Jones is fat and beautiful.

(b) Jones is both just and merciful.

(c) Jones is either at Bear's or at Nick's.

(d) Jones is a left-handed British tennis player.

(e) Jones is neither funny nor smart.

(f) Jones has a red car.

(g) Jones is either late or angry or both.

4. For each of the following claims, describe a case (or if possible several cases) which is inconsistent with it:

(a) Fat people are just and merciful.

(b) Fat, beautiful people are just.

(c) Beautiful people are unjust and lack mercy.

(d) Just people are either fat or beautiful (and possibly both.)

(e) Fat, beautiful people are unjust, but merciful.
To summarize what you have learned so far, complete the following table. (Some items have already been filled in.)

<table>
<thead>
<tr>
<th>SENTENCE TYPES</th>
<th>Some A's are B's</th>
<th>No A's are B's</th>
<th>All A's are B's</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain of Test #1</td>
<td>Sample of A's</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property-in-question</td>
<td>B or not B?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domain of Test #2</td>
<td>Sample of not B's</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property-in-question</td>
<td>A or not A?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If the sentence were true, would it be logically possible ever to succeed in verifying it after finite testing?</td>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If so, what would a verifying instance look like?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>If the sentence were false, would it be logically possible ever to succeed in refuting it after finite testing?</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If so, what would a refuting instance look like?</td>
<td>Something which is both A and B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
d. A reminder that so far we are only describing the logic of testing.

It is important to remember that this chapter deals only with the logic of testing. As far as logic is concerned the sentences "All dreams are significant" and "All nitrates are soluble" are completely on a par. Both are affirmative universal generalizations. Both can be refuted by a single counter example. Neither can be verified by a finite number of instances.

However, for purposes of actual scientific testing, they are very different indeed. It is rather easy to specify what counts as a soluble salt, although our definition may be somewhat arbitrary. (Perhaps we shall stipulate that anything whose solubility is over 5 g./100 ml. at 25°C is to be regarded as soluble.) But what counts as a significant dream? In this case it is very difficult to get clear on what is meant. Here the problem in testing is conceptual, because the concepts used in the conjecture are not very precise.

There are also immense practical difficulties in testing the first conjecture. Standard laboratory procedures exist for finding out whether a salt is a nitrate or not, and nitrates are powders which can be stored in jars and examined repeatedly. But dreams are much more difficult to work with. Here we can only investigate our subject matter indirectly through memories or perhaps hypnosis.

To summarize, the logic of testing is the same for the two conjectures. Their overall testability, however, is quite different. We will come back to the conceptual, empirical, and pragmatic aspects of testing later.

III. Degrees of Logical Testability and Comparisons of Logical Content

Other things being equal, scientists prefer scientific conjectures which are of high content. It turns out that the stronger a theory is (i.e., the more claims it makes about the world), the more testable it is (i.e., the more logical possibilities there are of refuting it.)

Let me illustrate the relationship between degree of content and degree of testability with a rather silly example. Compare the following
three sentences:

(1) All poodles have parasites.
(2) All dogs have parasites.
(3) All dogs have fleas.

Sentence 2 says more than sentence 1. It asserts that terriers as well as poodles have parasites. There are correspondingly more ways to test 2 than 1. To test 1, we would probably choose as our test domain the class of poodles. To test 2 we could sample poodles but we could also sample sheepdogs and collies.

When we compare sentences 2 and 3 we see that 3 says more than 2--it asserts not only that dogs have some sort of parasite, but also that fleas are included. If we were to find a dog that had only mites and no fleas, we would have refuted 3, but not 2. Thus, there are more possibilities for falsifying 3 than 2.

The general rule is this:

The content of a universal generalization, be it affirmative (expressed with "all") or negative (expressed with "no"), is directly related to the number of logically possible states of affairs which it rules out.

EXERCISE J:

1. Which statement in each of the following pairs has more content (and therefore a higher degree of testability)? For each pair, give an example of a report which would refute one conjecture, but not the other.

(a) Nitrates are water-soluble.
(b) The nitrates of alkali metals are water-soluble.
(c) Planets travel in elliptical paths.
(d) Planets travel in ellipses with the sun at one focus.
(e) All swans are white.
(f) All swans are black or white.
(g) All swans are white. (Assume you suspect that there are more white things than swans in the universe.)

(h) Only swans are white.

(i) All organic compounds have a strong smell.

(j) The class of organic compounds called esters have a strong fruity smell.

A special case of the general rule concerning content given above is:

The broader the antecedent of a conditional sentence and the more precise the consequent, the more content the conditional sentence has and the more ways there are to test it.

The intuitive idea is that a high content sentence makes precise assertions about a wide domain of objects or events.

There is another way of talking about the content of conditionals. One may think of them as inference licenses. Suppose that it is true that all prunes are wrinkled \((P \rightarrow W)\). This conditional "licenses" you to argue in the following fashion:

All prunes are wrinkled.
There is a prune in my fruit cup.

(therefore) There is a wrinkled object in my fruit cup.

This form of argument is called modus ponens and it can be symbolized as follows (we let \(a\) stand for a particular prune, namely the one in your fruit cup):

\[ P \rightarrow W \]
\[ Pa \quad (i.e., \; a \; is \; P) \]

\[ Wa \quad (i.e., \; a \; is \; W) \]

There is a second form of inference which can be made on the basis of \(P \rightarrow W\):
All prunes are wrinkled.
This strange object in my fruit cup is not wrinkled.
(therefore) It isn't a prune.

This second form of argument is called modus tollens and can be symbolized as follows (b stands for the puzzling object in your fruit cup):

\[ P \rightarrow W \]
\[ \sim wb \]
\[ \sim pb \]

On the basis of \( P \rightarrow W \), given any prune one can infer something about it using modus ponens. Using modus tollens one can infer something about any non-wrinkled object.

So the inference license, \( P \rightarrow W \), is useful in two domains, \( P \) and \( \sim W \).

We can now state the rule for content in a different way:

Each conditional has at least two inference domains.
The wider the inference domains, the greater the content of the conditional.

The inference domains are the same as the sample domains used for testing purposes. Again we note that the more content a conjecture has (i.e., the more inferences it licenses us to make), the more testable it is (i.e., the more tests we can perform on it).

EXERCISES K:
1. Which member of each of the following pairs of conjectures has more content? What inferences does each license? What are the test domains of each? (Before trying to answer, you should express both conjectures using the standard shorthand notation).

   (a) All prunes are wrinkled.
   (b) Both prunes and raisins are wrinkled.
   (c) All prunes are wrinkled.
   (d) All objects which are both prunes and purple are wrinkled.
   (e) All prunes are wrinkled.
   (f) All prunes are both sour and wrinkled.
(g) All prunes are sour and wrinkled.
(h) All prunes are sour or wrinkled.

(i) Only compounds which contain hydrogen are acidic.
(j) Only compounds which contain ionizable hydrogen are acidic.

(k) Unless a baby monkey is cuddled, it will not be a good parent when it grows up.
(l) Unless a baby monkey is cuddled while it is being fed, it will not be a good parent when it grows up.

(m) The combination of oxygen, fuel, and a spark is sufficient for combustion.
(n) The combination of oxygen, fuel, and a spark is both necessary and sufficient for combustion.

2. Draw Venn diagrams for the paired sentences (a-j) above using three circles for each sentence. (For example, in la, put in a circle for raisins too.) What generalization can you make about the pictures of the sentences with higher content?

3. Which statement in each of the following pairs has a higher content, or are they not comparable? For each pair which is comparable, describe an instance which would refute one but not the other.

(a) All of Bach's pieces for the well-tempered clavichord have a contrapuntal form.
(b) All of Bach's music has a contrapuntal form.

(c) Only elephants have tusks.
(d) Only elephants above 5 years of age have tusks.

(e) All crystals of sodium chloride are right-angled solids (the angle between any two adjoining faces is 90°).
(f) All alkali metal chlorides (including sodium) are cubic solids.

(g) Every twenty years there is a war.
(h) Every ten years there is a war.
4. Two italicized rules for comparing the content of conditionals have been given in this section. Are they equivalent? Explain your answer.

*** *** *** ***

We have discussed the logic of testing simple sentences and shown a few of the simple inferences which are licensed by conditionals. In order to understand the testing of more complicated scientific systems, we will need to know a little bit more about deductive logic. So we now turn to a brief discussion of valid inferences.

REVIEW EXERCISES FOR CHAPTER II

1. Venn Diagrams

Let $U =$ set of all acts
$C =$ crimes
$S =$ sins
$\neg C =$ acts which are not crimes (i.e., legally permitted)
$\neg S =$ acts which are not sins (i.e., morally permitted)

Diagram each of the following in the box provided.

(a) All crimes are sins.

(b) All sins are crimes.
(c) No crimes are morally permissible.

(d) No morally permissible acts are legally permissible.

(e) All morally permissible acts are legally permissible.

(f) Some acts which are not crimes are morally permissible.

2. **Logical Equivalence and Inconsistency**

(a) Which (if any) of the above sentences are logically equivalent?

(b) Which pairs (if any) of the above sentences are inconsistent?
3. Logical Equivalence, Etc., continued

Consider the claim:

\[ A \to \sim B \] (i.e., \( A \rightarrow \sim B \)).

For each of the following sentences say whether it is logically equivalent to, compatible with, or inconsistent with the claim in the box. (If it's impossible to say, put a question mark.)

(a) All A's are B's.

(b) Some A's are B's.

(c) All A's are either \( \sim B \) or C.

(d) Only things which are both A and C are not-B's.

(e) It's not the case that some A's are B's.

(f) If something is B, then it is both C and not-A.

4. Translation

Symbolize each of the following conditionals using an arrow and the underlined letters.

(a) The baby \underline{laughs} if you play \underline{Peek} with it.

(b) One hundred and twenty \underline{Hours} is a requirement for \underline{Graduation}.

(c) If only I could \underline{remember} the answer, then I'd \underline{write} it down.

(d) Only \underline{Adults} are permitted to \underline{buy} alcohol. (The speaker knows that there are also other requirements, such as the adult's not being in prison, its not being Sunday, etc.)
(e) No Squirrels are Meat-eaters.

(f) Eat your Spinach for only then will you be Healthy.

5. Testing

One midsummer's eve after studying for an exam you have the following nightmare:

The Grand Inquisitor escorts you into a room which has a spotlight shining down on a table. The G.I. says, "Draw a picture of what you see."

You note that there are 10 cards on the table. Some are face-up and some are face-down. Some of the cards have diagonal stripes on their backs; others are checked. The cards whose faces are showing all have letters on them.

You draw the following picture, numbering the cards for future reference.

```
(1) (2) (3) (4) (5)
(6) (7) (8) (9) (10)
```

"Fine," says the G.I. "Now listen carefully. I want you to test the following conjecture by picking up cards and turning them over. The conjecture is:

All the cards on this table with Striped backs have Vowels on their faces."
"What's a vowel?" you ask. "A, E, I, O or U," says the G.I.

"O.K.," you say and start to pick up a card.

"Wait," booms the G.I. "If you even touch a single card which is not relevant to the testing of the conjecture, you will be shot. Likewise, if you fail to check a relevant card."

(a) Which cards should you turn over? (You may refer to them by number.)

(b) When you finish turning these cards over, will you be able to say for certain whether the conjecture is true or false? Explain.

6. Testing of a Complicated Sentence

Your bitter friend mutters, "If someone is both Rich and Famous, then you can be darn sure that person is either Lucky or Dishonest, or both."

(a) Symbolize your friend's claim.

(b) What kind (or kinds) of person could you point to in order to refute your friend's claim? (Give all possibilities if there are more than one.)

7. More on Testing

Preliminary investigations suggest that a certain chemical known as the Q-factor may act as a cancer preventative.

Scientists are entertaining two conjectures:

Conjecture A: All people without cancer have the Q-factor in their blood.

Conjecture B: All people with cancer lack the Q-factor in their blood.

Let $C =$ people with cancer

$Q =$ people with the Q-factor in their blood
(a) Draw a Venn diagram of each conjecture.

A later study turns up ten people who lack the Q-factor but do not have cancer.

(b) In the light of this study what can you say about Conjecture A?

Would this study count as a test of Conjecture A?

Does the study refute Conjecture A?

Or verify it?

Or neither?

(c) What relevance does this study have for Conjecture B?

8. Regulations

A local ordinance reads as follows: "An apartment dwelling family may keep a dog only if someone in the family is blind, unless the dog has been with the family previous to its first occupancy of the apartment for a minimum of three years."

Draw a Venn diagram showing who is excluded from keeping a dog. (Hint: Make the universe the class of apartment dwelling families and use only three circles. Label them clearly.)