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**\*p. 3, Definitions 1.2 and 1.3.** The exposition of the definition of a CW-complex is flawed in two ways. First, the text uses non-standard notation by letting  $e_i^n$  denote a closed subset of  $X$ , when traditionally  $e_i^n$  denotes a subspace homeomorphism to the interior of the  $n$ -disk, with the homeomorphism given by the characteristic map. Second, it is unclear exactly what a CW-structure on a topological space is, and it is thus unclear when two CW-complexes with the same underlying space are the same. The definition of cells and CW-complex is clarified as follows.

If  $X$  is a topological space and  $A$  is a subspace, then  $X$  is obtained from  $A$  by attaching  $n$ -cells if there exist maps  $\phi_i^n : (D^n, S^{n-1}) \rightarrow (X, A)$ ,  $i \in I$  so that the set theoretic function  $A \amalg_{i \in I} (\text{int } D_i^n) \rightarrow X$  given by the inclusion of  $A$  and the restriction of the  $\phi_i$ 's is a bijection and so that the continuous map  $A \amalg_{i \in I} D_i^n \rightarrow X$  is a quotient map.

The cells of  $X$  rel  $A$  are the images  $e_i^n$  of the interiors of the disks  $D_i^n$  via  $\phi_i^n$ . In particular,  $X$  is the disjoint union (set-theoretically) of  $A$  and the cells of  $X$  rel  $A$ .

A relative CW complex (relative to  $A$ ) is a pair of spaces  $A \subset X$  and a filtration

$$A = X^{-1} \subset X^0 \subset X^1 \subset \cdots \subset X^n \subset \cdots \subset X.$$

The filtration satisfies the conditions that  $X = \cup_n X^n$ ,  $X^0$  is the disjoint union of  $A$  with a discrete space,  $X^n$  is obtained from  $X^{n-1}$  by attaching  $n$ -cells, and the topology on  $X$  satisfies the requirement that a subset  $B \subset X$  is closed if and only if  $B \cap X^n$  is closed for all  $n$ . The definition of a CW-complex is obtained by letting  $A$  be the empty set.

**\*p. 4, line -13.** Change “The largest  $n$ ” to “The smallest  $n$ ”

**\*p. 7, line 11.**  $\partial \left( \sum_{i=1}^{\ell} r_i \langle \sigma_i \rangle \right) = \sum_{i=1}^{\ell} r_i \partial \langle \sigma_i \rangle$ .

**\*p. 9, line 11.** Change “ $\pi(a_i \otimes b_i)$ ” to “ $\pi(a_i, b_i)$ ”

**\*p. 11, line 11.** Change “finitely generated groups” to “finitely generated abelian groups”

**\*p. 17, line -2.** Change “ $\gamma : C \rightarrow A$ ” to “ $\gamma : C \rightarrow B$ .”

**\*p. 24.** In the right top corner of the first commutative diagram replace “ $\mathbf{Z}$ ” by “ $\mathbf{Z}/2$ .” Thus the sequence should read:

$$0 \rightarrow \text{Hom}(\mathbf{Z}/2, \mathbf{Z}) \rightarrow \text{Hom}(\mathbf{Z}/2, \mathbf{Z}) \rightarrow \text{Hom}(\mathbf{Z}/2, \mathbf{Z}/2)$$

**\*p. 26.** In the first displayed exact sequence of the proof of Proposition 2.4, replace “ $R/A$ ” by “ $R/a$ .” Thus the sequence should read:

$$0 \rightarrow R \xrightarrow{\times a} R \rightarrow R/a \rightarrow 0$$

**\*p. 32, line 11.** Replace  $\mathbf{Z}$  by  $\mathbf{Z} - \{0\}$ .

**p. 36 line -7.** Note that in Definition 2.21, an acyclic chain complex  $C_*$  need not have  $H_0(C_*) = 0$ , and in particular  $C_*$  is not exact at  $C_0$ . This contrasts with the definition of acyclic complex used in Chapter 11, (c.f. p. 334) where in that context one assumes  $H_0(C_*) = 0$ .

**\*p. 38 line 6.** Change the  $C_0$  to  $C_n$ .

**\*p. 38 line -4.** Change the  $C_0$  to  $M'$ . The sentence should read “Since  $\epsilon' \circ (f_0 - g_0) = (\varphi - \varphi) \circ \epsilon = 0 : P_0 \rightarrow M', \dots$ ”

**\*p. 39.** In the first commutative diagram, the vertical arrow is mislabelled. It should be labelled  $f_n - g_n - s_{n-1}\partial_n$ . Also, the  $C_1$  at the lower left should be replaced by  $C_{n+1}$ .

**\*p. 40 line 3.** Change to “Since  $R_n$  is projective...”

**\*p. 40 line 10.** Change the “-” sign to a “+” sign; i.e.  $i\epsilon(p) + \Phi(r)$ .

**\*p. 44 line 18.** Replace “ $A_{q+2}$ ” by “ $B_{q+2}$ ”.

**\*p. 46.** Add a paragraph at the end of Part 2 of Exercise 30: The splitting of this map is obtained by splitting the inclusion  $i : Z_* \rightarrow C_*$ , passing to a chain map

$$(C_*, \partial) \rightarrow (H_*(C_*), 0),$$

applying  $\text{Hom}_R(-, M)$ , and taking cohomology.

**\*p. 48.** Replace the second to last sentence by “Since  $\mathbf{Q}$  and  $\mathbf{R}$  are both flat and injective as  $\mathbf{Z}$ -modules,  $\text{Tor}(-, \mathbf{Q})$ ,  $\text{Tor}(-, \mathbf{R})$ ,  $\text{Ext}(-, \mathbf{Q})$ , and  $\text{Ext}(-, \mathbf{R})$  all vanish.”

**\*p. 49.** The definition of a free functor  $F : \mathcal{A} \rightarrow \mathcal{C}$  is not complete. The precise definition is as follows. For each  $q \in \mathbf{Z}$  one is given an indexed set  $\{b_j \in F_q(M_j)\}_{j \in J}$  where  $M_j \in \mathcal{M}$  such that for every  $X \in \text{Ob } \mathcal{A}$ ,  $F_q(X)$  is a free  $R$  module with basis  $\{F_q(u)(b_j) \mid u \in \text{Hom}_{\mathcal{A}}(M_j, X)\}$ .

**\*p. 53.** If  $C_*$  and  $D_*$  are free chain complexes, the splitting in the Künneth exact sequence is obtained just like the splitting in the Universal Coefficient Theorem. If  $D_*$  is not free, then reasoning is more complicated and involves finding a chain homotopy equivalence  $D'_* \rightarrow D_*$  where  $D'_*$  is a free chain complex. For details, see the discussion in *A Course in Homological Algebra*, by Hilton and Stammback.

**\*p. 54.** In the statement of the Eilenberg-Zilber Theorem, replace everything after “naturally equivalent;” and before “for any” with “more precisely, there exist natural transformations  $A : F \rightarrow F'$  and  $B : F' \rightarrow F$  so that  $A(\sigma) = \text{pr}_X \sigma \otimes \text{pr}_Y \sigma$  and  $B(\tau \otimes \rho) = \tau \times \rho$  for any singular 0-simplices  $\sigma, \tau$ , and  $\rho$  in  $X \times Y$ ,  $X$ , and  $Y$  respectively. Furthermore,”

**\*p. 57.** In the exact sequence of Definition 3.8 change “ $H^*(X \times Y)$ ” to “ $H^{p+q}(X \times Y)$ .”

**\*p. 60.** The proof that  $1 \cup \alpha = \alpha$  is wrong. Instead, first extend  $1 \in S^0(X)$  to act on all chains by declaring  $1(z) = 0$  if  $z \in S_p(X)$  with  $p > 0$ . Then define a natural

transformation  $C : S_*(X) \rightarrow S_*(X)$  as the composite of a diagonal approximation  $\tau : S_*(X) \rightarrow S_*(X) \otimes S_*(X)$  and the map  $E : S_*(X) \rightarrow S_*(X)$  given by  $E(z \otimes w) = 1(z)w$ . The map  $E$  is easily checked to be a natural chain map, and hence  $C$  is a natural chain map. By the uniqueness part of the acyclic models theorem,  $C$  is chain homotopic to the identity. If  $\alpha \in S^*(X)$ ,  $E^*(\alpha)(z \otimes w) = \alpha(E(z \otimes w)) = \alpha(1(z)w) = 1(z)\alpha(w) = (1 \times^{alg} \alpha)(z \otimes w)$ . Therefore,  $C^*(\alpha) = \tau^*(E^*(\alpha)) = \tau^*(1 \times^{alg} \alpha) = 1 \cup \alpha$ . Passing to cohomology and using the fact that  $C$  is chain homotopic to the identity gives  $1 \cup \alpha = \alpha$ .

**\*p. 60.** Replace the first sentence of the proof of part 2. of Theorem 3.13 by the following.

The compositions of Eilenberg-Zilber maps

$$S_*(X \times Y \times Z) \rightarrow S_*(X \times Y) \otimes S_*Z \rightarrow S_*X \otimes S_*Y \otimes S_*Z$$

$$S_*(X \times Y \times Z) \rightarrow S_*(X) \otimes S_*(Y \times Z) \rightarrow S_*X \otimes S_*Y \otimes S_*Z$$

are natural transformations of functors on  $TOP^3$  (triples of spaces). The functor  $(X, Y, Z) \mapsto S_*(X \times Y \times Z)$  is free and acyclic on the models  $(\Delta^p, \Delta^p, \Delta^p)$ . The functor  $(X, Y, Z) \mapsto S_*(X) \otimes S_*(Y) \otimes S_*(Z)$  is free and acyclic on the models  $(\Delta^p, \Delta^q, \Delta^r)$ .

**\*p. 62.** In Definition 3.14 change “ $S_q(X)$ ” to “ $S_p(X)$ .”

**\*p. 71, line 8.** Change to “If  $M$  is closed...”

**\*p. 71, Theorem 3.26.** Add: “The integers  $\mathbb{Z}$  can be replaced by  $\mathbb{Z}/2$  in Theorem 3.26, and all assertions continue to hold. Moreover, with  $\mathbb{Z}/2$  coefficients the assertions hold for non-orientable manifolds as well.”

**\*p. 72, line 8.** Solving Exercise 48 requires knowing that the homology groups of a compact manifold are finitely generated abelian groups. This can be shown by proving that any manifold embeds in  $\mathbb{R}^N$  for some  $N$  in such a way that it is a retract of a finite subcomplex of  $\mathbb{R}^N$ . Morse theory gives an easy proof that a smooth compact manifold is homotopy equivalent to a CW-complex with finitely many cells.

**\*p. 73, line 5.** Change to “ $(4k - 2)$ -dimensional.”

**\*p. 73, Theorem 3.27.** As stated, the last sentence of Theorem 3.27 is only true for forms with non-negative signature. A more precise and complete statement is the following. If an even indefinite form  $Q$  has signature  $\sigma$  and rank  $r$ , let  $m = \frac{1}{8}|\sigma|$ ,  $\epsilon$  be the sign of  $\sigma$ , i.e.  $\epsilon = \frac{\sigma}{|\sigma|}$  if  $\sigma \neq 0$  and  $\epsilon = 0$  if  $\sigma = 0$ , and let  $\ell = \frac{1}{2}(r - |\sigma|)$ , so that  $\ell > 0$ . Then  $Q$  is equivalent to

$$\oplus_{\ell} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \oplus_m \epsilon E_8.$$

**\*p. 74, lines 8–11.** The list should read:

There are

1	even, positive definite	rank 8	forms
2	”	rank 16	”
24	”	rank 24	”
$\geq 10^7$	”	rank 32	”
$\geq 10^{51}$	”	rank 40	”

This data is taken from the book *Symmetric Bilinear Forms* by Milnor and Husemoller.

**\*p, 75, line -6.** Insert “oriented”, i.e. “Now suppose that  $M$  is a closed and oriented manifold of dimension  $2k - 1$ .”

**p, 75, Exercise 55.** Add the hypothesis that  $X$  is a finite complex. This guarantees that the abelian groups  $H^n(X; \mathbf{Q}/\mathbf{Z})$  are torsion.

**\*p, 77, line -1.** Change “ $g(x)$ ” to  $g \cdot x$ .”

**\*p, 80, line 1.** That a locally trivial bundle is the same thing as a fiber bundle with structure group  $\text{Homeo}(F)$  depends on what topology one uses on  $\text{Homeo}(F)$ , since with our definition we require the transition functions  $U \rightarrow G$  to be continuous. A solution would be to topologize  $\text{Homeo}(F)$  so that a map  $X \rightarrow \text{Homeo}(F)$  is continuous if and only if the adjoint map  $X \times F \rightarrow F$  is continuous. Often, but not always, this condition is satisfied by the compact open topology (see Theorem 6.5).

**\*p, 85, line 3 and 4.** Switch  $\varphi$  and  $\varphi'$ .

**\*p, 85, line 7.** Change “ $E/G$ ” to “ $P/G$ .”

**\*p, 88, line -4.** Change the sentence starting “This is clearly a homomorphism...” to “This is an anti-homomorphism: if  $a \in A$ ,  $\tilde{\gamma}_1$  is a lift of  $\gamma_1$  starting at  $a$ , and  $\tilde{\gamma}_2$  is a lift of  $\gamma_2$  starting at  $\tilde{\gamma}_1(1)$ , then  $\tilde{\gamma}_1\tilde{\gamma}_2$  is a lift of  $\gamma_1\gamma_2$  starting at  $a$ . Thus the function  $\pi_1(B, *) \rightarrow \text{Aut}(A)$  is an anti-homomorphism, which can be turned into a homomorphism by composing with the map  $\text{Aut}(A) \rightarrow \text{Aut}(A)$  given by  $f \mapsto f^{-1}$ .”

**\*p, 89, line 16.** In Definition 4.9, note that  $r$  is a  $G$ -homeomorphism.

**\*p, 90, Definition 4.11.** Change to “...from  $p : E \rightarrow B$  to  $p' : E' \rightarrow B'$  is a pair of...” and label the vertical arrows in the diagram.

**\*p, 90, line -7.** Change “ $p^{-1}(U')$ ” to “ $(p')^{-1}(U')$ .”

**\*p, 91, line -10.** Change “ $q : f^*(E) \rightarrow B$ ” to “ $q : f^*(E) \rightarrow B'$ ” and “ $E \times B$ ” to “ $B' \times E$ .”

**\*p, 99, line 4.** Change to “...on the right,  $A$  is... ”.

**\*p, 99, Second displayed formula and Exercise 74.** Change to “ $\text{Hom}_{\mathbb{Z}\pi}(S_*\tilde{X}, A_\rho)$ ”.

**\*p, 99, line -4.** Delete “with the trivial left  $\pi$  action”.

**\*p, 99, line -2.** Replace the “ $M$ ” with “ $m$ ”.

**\*p. 100, lines 11 and 12.** Replace “ $\text{Hom}_{\mathbf{Z}}(S_*X, \mathbf{Z})$ ” with “ $\text{Hom}_{\mathbf{Z}}(S_*\tilde{X}, \mathbf{Z})$ ” and “compact,” with “a CW-complex of finite type (i.e. a finite number of cells in each dimension), then”.

**\*p. 100, line 20.** Change to “For each cell  $e$  of  $X$ , choose a cell  $\tilde{e}$  above  $e$  in  $\tilde{X}$ .”

**\*p. 101, line 1.** Change to “Given  $n > 1$ , let  $\rho : \pi_1(\mathbb{R}P^n) \dots$ ”

**\*p. 101, line -15.** Change to “let  $V$  be an open set in  $M$ .”

**\*p 104, line 8.** Change to  $\gamma_{\sigma}^{-1}(a)$  in the displayed formula, so it should read

$$a\sigma \mapsto \gamma_{\sigma}^{-1}(a)(\sigma \circ f_0^k) + \sum_{m=1}^k (-1)^m a(\sigma \circ f_m^k).$$

**\*p. 108, line 12.** Replace “ $C_*(\tilde{B} \otimes_{\mathbf{Z}\pi} V)$ ” with “ $C_*(\tilde{B}) \otimes_{\mathbf{Z}\pi} V$ ”

**\*p. 109, line 16.** Change to “...can be taken to be  $X = \dots$ ”.

**\*p. 109, line 17.** Change to “...is open in  $X_i$  for all  $i$ .”

**\*p. 112, line 13.** Delete “with finitely many cells in each dimension” and add a line:  
 “4. The product of two CW-complexes, one of which has a finite number of cells in each dimension.”

**\*p. 113, line 20.** Add a line: “3. If  $X$  and  $Y$  are CW-complexes, so is  $k(X \times Y)$ .”

**\*p. 116, line 11.** Change  $f^*(E) \rightarrow B$  to  $f^*(E) \rightarrow X$ .

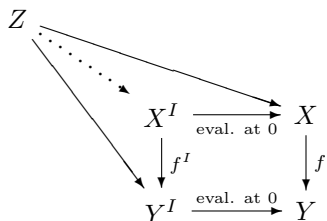
**\*p. 118, figure at bottom of page.** The “missing” edge on the box on the left should be the right edge, not the top edge.

**\*p. 122, line 5.** Change “(” to “(”.

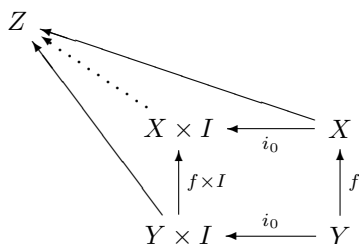
**\*p. 124, line -3.** After the displayed equation change to “where  $\alpha_s$  is the path  $t \mapsto \alpha(st)$ .”

**\*p. 126.** In the left commutative triangle near the bottom of the page, label the southwest arrow “ $p$ .”

**\*p. 128.** The first two diagrams on p. 128 are wrong. The first should be:



and the second:



**\*p. 128** Delete the sentence “In particular the neighborhood  $U = \{x \in X | u(x) < 1\}$  of  $A$  deformation retracts to  $A$ .”

**\*p. 130, line 5.** The sketch of the Proof of Theorem 6.23 is not on target. It works if  $A$  is the  $k$ -skeleton of a  $(k+1)$ -dimensional complex  $X$ , but for a general subcomplex, additional constructions are needed.

**\*p. 133, line -2.** Replace “ $H(-, 1)$ ” by “ $F(-, 1)$ .”

**\*p. 135, line 6.** Change to “Thus  $f$  is homotopic to a map with image in the fiber  $F$ , so ...”

**\*p. 136, line above Definition 6.32.** Change to “compactly generated topology obtained from the compact-open topology”

**\*p. 136.** The displayed equation in Definition 6.32 has a typo: one of the “ $\cup$ ” should be a “ $\times$ .” It should read

$$X \wedge Y = \frac{X \times Y}{X \vee Y} = \frac{X \times Y}{X \times \{y_0\} \cup \{x_0\} \times Y}$$

**\*p. 137, line 7.** Replace “ $\Sigma X / X \times \{0\}$ ” with “ $(X \times I) / (X \times \{0\})$ ”

**\*p. 127, 138 and the proof of Theorem 6.39 part 1, p. 138.** The following result is needed to justify the the notion of “the” fiber of a map.

**Proposition.** *Let  $p : E \rightarrow B$  and  $p' : E' \rightarrow B$  be fibrations and suppose there exists a homotopy equivalence  $h : E \rightarrow E'$  so that  $p'h$  is homotopic to  $p$ . Then  $p$  and  $p'$  are fiber homotopy equivalent. In particular the fibers of  $p$  and  $p'$  are homotopy equivalent.*

A simple consequence of this fact is that if one turns a map into a fibration in two different ways, then the resulting fibrations are fiber homotopy equivalent. This is needed to justify

the last step of the proof of Theorem 6.39 part 1. The commutative triangle on p. 140 shows that the inclusion  $F \rightarrow E$  can be replaced by the fibration  $(P_f)_0 \rightarrow E$  with homotopy fiber  $\Omega_{b_0}B$ . But this is not the same fibration as  $P_i \rightarrow E$  in Theorem 6.18 starting with the inclusion map  $i : F \subset E$ . The proposition shows that if the fiber of *some* fibration replacing  $i : F \subset E$  is  $\Omega_{b_0}B$ , then the fiber of every fibration replacing  $i : F \subset B$  has the homotopy type of  $\Omega_{b_0}B$ .

**Proof.** (due to Dold) First, we may assume that  $p'h = p$  by using the homotopy lifting property (HLP) for  $p'$ : just lift a homotopy from  $p'h$  to  $p$  starting at  $h$ . Its other endpoint is a map  $\hat{h}$  homotopic to  $h$  satisfying  $p'\hat{h} = p$ . Since  $h$  and  $\hat{h}$  are homotopic,  $\hat{h}$  is also a homotopy equivalence.

So assume  $p'h = p$  and let  $h' : E' \rightarrow E$  be a homotopy inverse for  $h$ . Let  $F : E' \times [0, 1] \rightarrow E'$  be a homotopy between  $hh'$  and  $\text{Id}_{E'}$ . Let  $\bar{G} = p'F$ , so  $\bar{G} : E' \times [0, 1] \rightarrow B$ . Since  $\bar{G}(e', 0) = p'(F(e', 0)) = p'(h(h'(e')))) = p(h'(e'))$ , the HLP for  $p$  implies that there is a lift  $G : E' \times [0, 1] \rightarrow E$  of  $\bar{G}$  with  $G(e', 0) = h'(e')$ . Then  $p(G(e', 1)) = \bar{G}(e', 1) = p'(F(e', 1)) = p'(e')$ . In other words, if we define  $h'' : E' \rightarrow E$  to be  $G(-, 1)$ , i.e.  $h''(e') = G(e', 1)$ , then  $h''$  is a homotopy inverse for  $h$  which preserves fibers, i.e.  $ph'' = p'$ . We will show that  $h''$  is a *fiber homotopy* inverse of  $h$ .

Given a homotopies  $R, S : X \times [0, 1] \rightarrow Y$  let  $R^{-1}$  denote the *reverse* homotopy, i.e.  $R^{-1}(x, t) = R(x, 1 - t)$  and let  $R * S$  denote the *composite* homotopy (assuming  $R(x, 1) = S(x, 0)$ )

$$R * S(x, t) = \begin{cases} R(x, 2t) & \text{if } t \leq 1/2, \\ S(x, 2t - 1) & \text{if } t \geq 1/2. \end{cases}$$

Let  $H : E' \times [0, 1] \rightarrow E'$  be the composite  $H = (hG)^{-1} * F$ , which is defined since  $hG(e', 0) = hh'(e') = F(0)$ . Thus  $H$  is a homotopy from  $hh''$  to  $\text{Id}_{E'}$ . Since  $p'F = \bar{G} = pG = p'hG$ ,  $p'H(e', t) = p'H(e', 1 - t)$ . In other words, viewing  $p'H$  as a loop  $[0, 1] \rightarrow \text{Map}(E', B)$ , this loop is obtained by traveling along a path and then returning along the same path. There is an obvious nullhomotopy obtained by traveling less and less along the path and returning. Precisely, define  $\bar{K} : E' \times [0, 1] \times [0, 1]$  by

$$\bar{K}(e', t, s) = \begin{cases} p'H(e', (1 - s)t) & \text{if } t \leq 1/2, \\ p'H(e', (1 - s)(1 - t)) & \text{if } t \geq 1/2. \end{cases}$$

Then  $\bar{K}(e', t, 0) = p'H(e', t)$ ,  $\bar{K}(e', t, 1) = p'(e')$ ,  $\bar{K}(e', 0, s) = p'(e')$ , and  $\bar{K}(e', 1, s) = p'(e')$ .

We will use the HLP to lift  $\bar{K}$  to a fiber preserving homotopy using an argument similar to the argument on the bottom of page 118. Let  $U \subset I \times I$  be the union of the three sides

$$U = \{(t, s) \mid s = 0\} \cup \{(t, s) \mid t = 0\} \cup \{(t, s) \mid t = 1\}.$$

Let  $K : E' \times U \rightarrow E'$  be the map

$$K(e', t, s) = \begin{cases} H(e', t) & \text{if } s = 0, \\ h(h''(e')) & \text{if } t = 0, \\ e' & \text{if } t = 1. \end{cases}$$

Since there is a homeomorphism  $I \times I \cong I \times I$  taking  $U$  to  $I \times \{0\} = \{(t, s) \mid s = 0\}$ , the HLP implies that  $K$  extends to a map  $K : E' \times I \times I \rightarrow E'$  satisfying  $p'K = \bar{K}$ . Let  $D : E' \times I \rightarrow E'$  be the endpoint of this map, i.e.  $D(e', t) = K(e', t, 1)$ . Then  $D(e', 0) = h(h''(e'))$ ,  $D(e', 1) = e'$ , and  $p'(D(e', t)) = \bar{K}(e', t, 1) = p'(e')$ . In other words,  $D$  is a fiber preserving homotopy between  $hh''$  and  $\text{Id}_{E'}$ .

Now repeat the entire argument to  $h''$  to find a map  $h''' : E \rightarrow E'$  and fiber preserving homotopy between  $h''h'''$  and  $\text{Id}_E$ . Use the notation " $\sim_F$ " for fiber preserving homotopic.

Then

$$h''h \sim_F h''hh''h''' \sim_F h''h''' \sim_F \text{Id}_E.$$

In other words  $h : E \rightarrow E'$  and  $h'' : E' \rightarrow E$  are fiber homotopy inverses.

**\*p. 141, line -1.** Change “ $\nu(f \vee g)$ ” to “ $(f \vee g) \circ \nu$ .”

**\*p. 142, line -7.** Change to “... an “inversion” map  $\varphi : Z \rightarrow Z$  which ...”

**\*p. 144, line 16.** Change to “..a map  $f : X \rightarrow Y$  of CW complexes is a homotopy equivalence ...”

**\*p. 147, line 2.** Replace “Chapter 3” by “Chapter 5.”

**\*p. 156, in figure.** Replace “ $f_0 \cong_v f_1$ ” by “ $f_0 \cong_v f_2$ ”

**\*p. 157, line 1.** Replace “action” by “right action.”

**\*p. 157, line 3 and line 15.** Replace “ $[u][f]$ ” by “ $[f][u]$ .”

**\*p. 157, line 16.** Replace “ $[u][f_0]$ ” by “ $[f_0][u]$ .”

**\*p. 159, line -5.** Replace “ $u \cdot [f]$ ” by “ $[f][u]$ .”

**\*p. 159, line -1.** Delete “for all  $n$ ”

**\*p. 166, paragraph following item 4.** delete “relative” twice.

**\*p. 167, line -3.** Change “homology” to “cohomology.”

**\*p. 167-187.** The  $n$ -skeleton of a CW complex  $X$  is denoted by  $X_n$  in these pages, and by  $X^n$  in the rest of the book.

**\*p. 167.** Exercise 117 is wrong.

**\*p. 168, last line of Section 7.1.** Change to “corresponding generalized homology and cohomology theories”

**\*p. 170, line 8.** Change to “..., so  $\pi_1(X_{n+1}, X_n) = 0$  for  $n \geq 1$ ”.

**\*p. 171, line 11.** Change “..., which equals  $[f_i] \in [S^n, Y] = \dots$ ” to “..., which equals  $[g \circ f_i] \in [S^n, Y] = \dots$ ”

**\*p. 173, line -3.** Replace “ $g'$ ” by  $f_1$ .

**\*p. 174, line 12.** Replace “ $S^{n-1}$ ” by  $D^n$ .

**\*p. 175, line 12.** Replace “ $C^n$ ” by  $C^{n+1}$ .



**\*p. 175, line 3.** Change to "...but maybe even on the  $(n - 1)$ -cells..."

**\*p.176, line 19.** Change " $f$  can be extended to  $X_{n-1}$ " to " $f$  can be extended to  $X_n$ ."

**\*p.178, line -7.** Change " $H^n(X, \pi)$ " to " $H^n(X; \pi)$ "

**pg 186, line -8.** Change "...does not factor through a face map." to "...does not factor through a degeneracy map, i.e. a linear projection onto one of its  $n - 1$  dimensional faces."

**\*p.189, line -6.** Change "n-simple" to " $n$ -simple."

**\*p.201, Exercise 132.** Replace "with negatives given by ..." by "with negatives given by composing with a map  $f : S^k \rightarrow S^k$  of degree  $-1$ ,

$$-[V_0] = [f(V_0)]."$$

**\*p.212, line -15.** Change "leads" to "lead"

**\*p.217, line -6.** Change "spectral" to "spectra"

**\*p.219, line -9.** Change "...the homotopy fiber of  $f : B \rightarrow BG$  is in fact a homotopy equivalence." to "...the homotopy fiber of  $f : B \rightarrow BG$  is in fact homotopy equivalent to  $E$ ."

**\*p. 220, line 19.** Change "...continuous groups.." to "...topological groups..."

**\*p. 225, line 3.** Replace " $\alpha^{-1}(X \times U) \rightarrow D^\ell/S^\ell$ " with " $\alpha^{-1}(X_+ \wedge p^{-1}(U)) \rightarrow D^\ell/S^{\ell-1}$  where  $p : EG_\ell \times_{G_\ell} \text{int } D^n \rightarrow BG_\ell$ ".

**\*p. 230, line 1.** Change "with" to "which is the union of"

**\*p. 232, line 6.** Replace "... axioms A1, A2, A3, and A5." by "... axioms A1, A2, A3, and A4."

**\*p. 234, line 9.** Replace "... $\nu(Q \hookrightarrow n)$ " by "...  $\nu(Q \hookrightarrow N)$ ."

**\*p. 237, line -1.** Replace "1.2..." by "1, 2, ..."

**\*p. 240, line 1.** To say the filtration preserves the grading means that  $F_p = \bigoplus_n (F_p \cap A_n) = \bigoplus_n F_{p, n-p}$  where we think of  $A = \bigoplus_n A_n$ .

**\*p. 241, Definition 9.21, part 2.** The second condition should read:

2. there is a convergent filtration of  $A_*$  so that for each  $n$  the colimit  $E_{p, n-p}^\infty = \text{colim}_{r \rightarrow \infty} E_{p, n-p}^r$  is isomorphic to the associated graded module  $\text{Gr}(A_n)_p$ .

**\*p. 244, line -3.** Delete " $(k, q) \neq (0, 0)$ "

**\*p. 245, line 1.** Replace “ $H_0(\Omega S^k) = 0$ ” by “ $H_0(\Omega S^k) = \mathbf{Z}$ ”

**\*p. 246, line 13.** Replace “Since  $F_{-1,n-1} = 0$ ” by “Since  $F_{-1,n+1} = 0$ ”

**\*p. 248, line -3.** Replace “ $E_{k,0}^k$ ” by “ $E_{k,0}^r$ ”

**\*p. 250, Equation (9.9) and the exact sequence on line 19.** Replace “ $E_{1,1}^\infty$ ” by “ $F_{1,1}$ .” Thus the exact sequence should read

$$F_{1,1} \rightarrow H_2(E) \rightarrow H_2(B) \rightarrow H_0(B; H_1(F)) \rightarrow H_1(E) \rightarrow H_1(B) \rightarrow 0.$$

Also, further in that paragraph (line -11) Change “ $H_2(E)$ ” to “ $H_1(E)$ .”

**\*p. 251, line -13.** Change ”cohomology” to ”homology.”

**\*p. 255, line 10.** Similar comment as p 240.

**\*p. 255, Definition 9.21.** The second condition should read:

2. there is a convergent filtration of  $A^*$  so that for each  $n$  the limit  $E_\infty^{p,n-p} = \bigcap_{r \geq r_0} E_r^{p,n-p}$  is isomorphic to the associated graded module  $\text{Gr}(A^n)^p$ .

**\*p. 258, line -9.** Replace “... for path space...” by “... for the path space...”

**\*p. 261, line 1 and 2.** Replace both occurrences of  $d^3$  by  $d_3$ .

**\*p. 264, line 21.** Change “ $\tilde{X} \rightarrow \tilde{X} \times_G EG \rightarrow X$ ” to “ $\tilde{X} \rightarrow \tilde{X} \times_G EG \rightarrow BG$ .”

**\*p 265, lines -11 and line-7** Change  $d_k$  to  $d_{k+1}$ .

**p. 273, line -7.** Change “ $H_{n-1}(X)$ ” to “ $H_n(X)$ ”

**\*p. 274 line 6 and 7** change two ” $< n$ ” to “ $\leq n$ ” i.e. “... is a  $\mathcal{C}$ -isomorphism for  $0 < i \leq n$ .”

**\*p. 274 line - 7.** Omit “with  $H_i(X, A) \in \mathcal{C}$  for  $i < n$ .”

**\*p. 275 line - 7.** Change “ $\pi_{k-1}(L) = \pi_k(X, A) = 0$ .” to “ $\pi_{k-1}(L) = \pi_k(X, A)$ .”

**\*p. 276, line -3.** Change “ $H_n(T)$  is finite...” to “ $H_k(T)$  is finite...”

**\*p. 278, line -6.** Change “ $H^5(F; \mathbf{Z}/2)$ ” to “ $H^5(Y; \mathbf{Z}/2)$ ”

**p. \*279, line -14.** Change “ $[X, \Omega SX]_0$ ” to “ $[X, \Omega SY]_0$ ”

**\*p. 286.** Theorem 10.21 is an immediate consequence of the Serre exact sequence for cohomology.

**\*p. 289, line 20.** Change “te” to “the”.

**\*p. 291, line -2.** Change “ $\tau(\iota_2^2)$ ” to “ $\tau(\iota_1^2)$ ”.

**\*p. 293, line 16.** Change “defining  $x_r = 0$  for  $r < 0$ ” to “defining  $y_r = 0$  for  $r < 0$ ”.

**\*p. 295, first paragraph.** All occurrences of the digit “8” should be changed to a “9” in this paragraph. Thus the paragraph should read as follows.

Let  $y \in H^5(SX)$  denote the non-zero element. Suppose to the contrary that  $Sh$  is nullhomotopic. Then  $SX$  is homotopy equivalent to the wedge  $S^5 \vee S^9$ . In particular the map  $Sq^4 : H^5(SX) \rightarrow H^9(SX)$  is trivial, since if  $y$  is the non-zero element of  $H^5(S^5 \vee S^9)$ , then  $y$  is pulled back from  $H^5(S^5)$  via the projection  $S^5 \vee S^9 \rightarrow S^5$ , but  $H^9(S^5) = 0$  and so by naturality  $Sq^4(y) = 0$ .

**p. \*295, lines 19 and 22.** Change “ $H^{k+n}(S^n B)$ ” to “ $\tilde{H}^{k+n}(S^n(B_+))$ ”.

**p. \*297, line 9.** Change “ $u$  to  $\tilde{u}$ ” to “ $\tilde{u}$  to  $u$ ”.

**p. \*306, line 15.** Change “no-zero” to “non-zero”.

**\*p. 309, statement of Theorem 10.39.** Change “Stiefel-Whitney numbers” to “Stiefel-Whitney classes”.

**\*p. 310, line -11.** Replace “ $H^*(BO(n-1))$ ” by “ $H^*(BO(n-1))$ ”

**\*p. 311.** The first exact sequence should read:

$$\cdots \rightarrow H^{k-1}(BO(n)) \rightarrow H^{k-1}(BO(n-1)) \rightarrow H^{k-n}(BO(n)) \otimes H^{n-1}(S^{n-1}) \xrightarrow{d_n} H^k(BO(n)) \rightarrow H^k(BO(n-1)) \rightarrow \cdots$$

and the sequence (10.18) should read:

$$0 \rightarrow H^{k-n}(BO(n)) \otimes H^{n-1}(S^{n-1}) \xrightarrow{d_n} H^k(BO(n)) \rightarrow H^k(BO(n-1)) \rightarrow 0$$

Also, on line 15, change “ $d_n([S^{n-1}]^* \cup \alpha) = w_n \cup \alpha$ ” to “ $d_n(\alpha \otimes [S^{n-1}]^*) = \alpha \cup w_n$ ”.

**\*p. 313, last line.** The third condition should read: “3.  $H_i(L_{(P)}(X); \mathbf{Z}_{(P)}) = H_i(L_{(P)}(X); \mathbf{Z})$  for  $i > 0$ .”

**\*p. 315-316.** In the discussion from the middle of page 315 to the middle of page 316 all cohomology should be with rational coefficients.

**\*p. 318.** Replace the statement of Theorem 10.46 by

*The map taking a manifold to its Stiefel-Whitney numbers induces an isomorphism*

$$\bigoplus_{\alpha \in P_n} w_\alpha : \Omega_n^O \rightarrow \bigoplus_{\alpha \in P_n} \mathbf{Z}/2.$$

*In other words, two closed manifolds are bordant if and only if they have the same Stiefel-Whitney numbers.*

*Moreover,  $\Omega_n^O$  is a polynomial ring over  $\mathbf{Z}/2$  on generators  $x_k \in \Omega_k^O$ , one for each non-negative integer  $k$  not of the form  $2^m - 1$ . Thus  $\Omega_n^O$  is a  $\mathbf{Z}/2$  vector space of rank the number of partitions in  $P_n$  of the form  $(i_1, \dots, i_n)$  satisfying  $i_k = 0$  when  $k = 2^j - 1$ .*

- \*p. 320, line 7.** Change “ $\pi_k(X)$ ” to “ $\pi_{k-1}(X)$ ”
- \*p. 321, line 2.** Change “ $S^n$ ” to “ $S^{n+1}$ ”
- \*p. 327, line -8.** This line should read  
*We assume all rings have the property that  $R^m \cong R^n$  implies  $m = n$ .*
- \*p. 333, line -7.** The summation should be over  $j$ , not  $i$ .
- \*p. 334, line 5.** Change “ $|n| \leq N$ ” to “ $|n| \geq N$ ” and on the next line change “ $n \geq N$ ” to “ $n < N$ ”.
- \*p. 334, line 20.** Change “a acyclic” to “an acyclic”
- \*p. 335, line 11.** Change “an chain” to “a chain”
- \*p. 337, line 9.** Change “ $s\partial\partial(y)$ ” to “ $ss\partial(y)$ ”
- \*p. 339, line -8.** Replace “ $C'_n \rightarrow C_n$ ” by “ $C_n \rightarrow C'_n$ ”
- \*p. 339, line -4.** Delete “acyclic”
- \*p. 340, line -15.** Replace the proof of Lemma 11.24 by the following :
- Proof.** (taken from [7, p. 48]). Equivalently we will find a chain map  $t : C'' \rightarrow C$  which splits  $p$ . Let  $\delta''$  be a chain contraction for  $C''$ . Let  $\sigma : C'' \rightarrow C$  be a sequence of homomorphisms  $\sigma_k : C''_k \rightarrow C_k$  which split  $C_k \rightarrow C''_k$ . Finally let  $t = \partial\sigma\delta'' + \sigma\delta''\partial''$ . Note that  $\partial t = \partial\sigma\delta''\partial'' = t\partial''$ , so  $t$  is a chain map. Note  $pt = p\partial\sigma\delta'' + \delta''\partial'' = \partial''p\sigma\delta'' + \delta''\partial'' = \partial''\delta'' + \delta''\partial'' = \text{Id}_{C''}$ , so  $p$  splits  $t$ .
- \*p. 340, line -3.** Change “Lemma 11.23” to “Theorem 11.23.”
- \*p. 341, line -11.** Change “ $\text{Cone}(C)$ ” to “ $\text{SCone}(C)$ .”
- \*p. 342, line 13.** Change “ $C(g \circ f)$ ” to “ $C(g \circ f)_n$ .”
- \*p. 345, line 1.** Change “ $(k+1)$ -cell” to “ $(k+2)$ -cell” and “ $i \neq k, k+1$ ” to “ $i \neq k, k+2$ .”
- \*p. 345, line 7 and line 15.** Change “ $C(\tilde{L}, \tilde{K})$ ” to “ $C_*(\tilde{L}, \tilde{K})$ .”
- \*p. 346, line 5.** Change “over  $S$ ” to “over  $R$ .”
- p. \*346, line -4.** Change “ $\Delta_R(C_*(X)) \in R^\times$ ” to “ $\Delta_R(C_*(\tilde{X})) \in R^\times / \pm 1$ .”
- \*p. 347.** The displayed equation in Proposition 11.34 should read:
- $$\det(\rho(\tau(f))) = \Delta_R(Y) / \Delta_R(X) \in R^\times / \pm G.$$
- \*p. 348, Exercise 203.** Change “ $\tilde{s}\partial - \partial\tilde{s}$ ” to “ $\tilde{s}\partial + \partial\tilde{s}$ .”

**\*p. 350, Exercise 204.** Change “ $\mathbf{R}^3 - \{\infty\}$ ” to “ $\mathbf{R}^3 \cup \{\infty\}$ .”

**\*p. 350, Exercise 205.** Change “the real projective plane” to “real projective 3-space.”

**\*p. 354, line 3.** Change “covers” to “cover.”

**\*p. 355, line -14.** Change “ $2 \cos(b\pi/p)$ ” to “ $2 \sin(b\pi/p)$ .”

**\*p. 356, line 3.** Change to “In particular,  $q \equiv (q')^{\pm 1} \pmod{p}$ .”

**\*p. 356, line 9.** Change

$$1 = (\zeta^a - 1)(\zeta^{-a} - 1)(\zeta^{ar} - 1)(\zeta^{-ar} - 1)(\zeta - 1)(\zeta^{-1} - 1)(\zeta^{r'} - 1)(\zeta^{-r'} - 1)$$

to

$$1 = (\zeta^a - 1)(\zeta^{-a} - 1)(\zeta^{ar} - 1)(\zeta^{-ar} - 1)[(\zeta - 1)(\zeta^{-1} - 1)(\zeta^{r'} - 1)(\zeta^{-r'} - 1)]^{-1}$$

**\*p. 356, line -11.** Change “first” to “second.”