To really understand a QFT, we need to understand quantum corrections. The first QFT (QED at tree level) ran into trouble in this regard. Many things at $O(\hbar)$ were found to be infinite. It took 20 years to solve this problem.

In the language of Feynman graphs:

\[ \text{is } O(\hbar^0) \quad \text{is } O(\hbar). \]

The number of loops is the power of $\hbar$. Each loop has a momentum to be integrated over; this is a sum over intermediate quantum states.

Renormalization of the SME has proceeded as follows:

- QED
- pure Yang–Mills
- chiral gauge theories (subsumes the first two)
- $\sigma^4$–theory
- Yukawa theory (subsumes $\sigma^4$)

Still missing:

- scalar gauge theories

Scalar QED is complicated enough without LV vertex corrections:

\[ \\text{at } O(\hbar) \]

All the usual QED diagrams are present, plus extra ones involving the $\sigma^4$ vertex.

Other things are also missing:

- higher order results — higher in both $\hbar$ and the LV
  Only $\sigma^4$–theory has been done to all orders, because the LV can be moved away.
- gauge invariant results
The renormalization group is often seen as the most important consequence of radiative corrections. The RG controls whether operators grow or shrink in importance at low energy. At a basic level, RG calculations in the SME can be done just like in normal QFTs. However, there are complications.

\[ \mathcal{L} = \bar{\psi} \left[ (\gamma^\mu + i \gamma^\mu (m + M_1)) \psi - \frac{i}{4} F_{\mu \nu} F^{\mu \nu} + \text{more} \right] - \frac{1}{4} k_F \bar{\psi} \gamma^\mu P^\sigma \gamma_\sigma \psi \] (mor?)

Feynman rules

\[ \begin{array}{c}
\begin{array}{c}
\text{m m} \rightarrow \alpha M_1 \\
\text{m m} \rightarrow \alpha k_F
\end{array}
\end{array} \]

You can get β-functions by usual means:

\[ Z_\psi \text{ from } \]

\[ Z_A \text{ from } \]

There are a lot of places the SME coefficients can enter.

\[ \Delta \bar{\psi} \psi_A \text{ from } \]

The correction \( \Delta \psi_A \) (i.e. β-function coefficient) to \( \psi_A \) looks like the infinite part of \( \Delta \psi_A = 2(\frac{1}{2} Z_\psi) - \frac{1}{2} Z_A \).

The calculations can be long, but at this level, they are straightforward.
Now come the complications. So far as I know, these are not unique, but in the SME they are more prominent.

In a gauge theory, coefficients appear in multiple places. In QCD,

\[ \alpha g^2 \quad \text{There is only one gauge coupling.} \]

In the SME, the \(\Gamma\) appear in both the vertex and propagator. The \(\beta\)-function for \(\Gamma\) coefficients should be the same whether calculated for vertex or propagator. Ward identities guarantee this, but showing it explicitly is tricky. We should get the same coefficient from \(Z\) only as from the vertex with \(\Delta\Gamma d A = Z_d - Z\). This generally involves combining all the \(\gamma\)-s into an integral

\[
\sim - \int \frac{d^4 k}{(2\pi)^4} \frac{i}{(\not k - m)} \frac{1}{(\not k' - m)} \frac{i}{(\not p' - k') - m} \frac{i}{(\not p - k - \not q'} - m) \left( \not q' \right)
\]

As \(q' \to 0\), there are obvious similarities, but there are many integrals that must be collected and shown to give equal results.

In renormalized perturbation theory, these results manifest themselves as relations among counterterms.

The second tricky aspect is more subtle. In a pure Yang-Mills theory, \(\beta_g\) is independent of the renormalization scheme to two-loop order—but not beyond. In the SME, scheme dependence shows up at one loop. In the cases I am familiar with, this happens because there are redundancies in the Lagrangian.

In SME QED, the published \(\beta_\gamma\) is zero. This is the case because, when working with \(\gamma\), it is impossible to get a physical effect before \(\mathcal{O}(f^2)\). However, \(f^2\) may be mapped onto a \(\mathcal{O}(-\frac{1}{2} f^2\not f\not f)\).
\[ L' = \bar{\psi} \left[ (i \gamma^\mu + i c_{00} \sigma^\mu \sigma_0) - m \right] \psi \]

The thing multiplying \( \sigma_0 \) — \( i \gamma_0 \sigma^0 \sigma_0 \) — anticommutes with \( \sigma_1, \sigma_2, \sigma_3 \). It looks just like a \( \gamma_0 \), except it's not normalized.

However, in any reasonable interacting theory, \( \beta_c \neq 0 \). (In fact, with only c--type LV, a combination \( c_{\mu\nu} - c_{\nu\mu} \) from different sectors runs to zero at low energy. The theory evolves toward the LI fixed point.)

If we calculate with \( f \) by making it into a c, then finding \( \beta_c \), then going back to the original Dirac algebra, \( f \) will run.

\[ \frac{\beta_f}{\beta_c} = \frac{1}{2} \frac{\beta_c}{c} \]

It's not so simple to find quantum corrections as to just stick \( f(Q^2) \) into a process at scale \( Q^2 \). The reason is the redundancy of the description.