High-Energy SME Phenomenology

There are a lot of interesting LV effects at high energies. Practically, that means using astrophysical data, usually. Space provides the highest available energies (and the longest times/distances).

A lot of the best data in cosmic ray photons (Cosmic ray protons/nuclei are higher energy but much less well understood.) In many cases, we can learn a lot about photons and the particles that emit them.

Most important at high $E$ are $c$, $d$, and $kF$. Dimension 3 coefficients have their effects suppressed relatively by $mp_F$; e.g., and $g$ are nonrenormalizable in the full SME. $kF$ can be absorbed into $c$ or is contained by birefringence.

But $\Gamma_\mu^\nu = c_\mu^\nu \gamma^\nu + d_\mu^\nu \gamma^\nu \gamma^5$ affects both fermion propagation and fermion-photon interactions. However, the changes to the vertex are very simple:

$$H = H(\vec{p} - e\vec{A}) = H(\vec{p}) \quad \vec{p} = \text{mechanical momentum}$$

$$\nabla = \frac{\partial}{\partial \vec{x}} = -i \left[ \vec{A}, H \right]$$

$$= -i (\vec{A} H - H \vec{A}) = -i \left( \frac{\partial H}{\partial \vec{p}} \right)$$

Only $[\vec{x}_i, \vec{p}_j] = i\delta_{ij}$ contributes. (Compare $\nabla_3 = \frac{\partial}{\partial \vec{k}}$ for the group velocity.)

Let's work with this a bit:

$$\frac{\partial H}{\partial \vec{p}} = \frac{\partial H}{\partial \vec{p}} = -\frac{1}{e} \frac{\partial H}{\partial \vec{A}}$$

But $\frac{\partial H}{\partial \vec{A}}$ is just what gives the coefficient of the vertex $\Gamma_\mu^\nu$.

\(\vec{A}\) always couples to $e\nabla$, by gauge invariance. (\(L, H\) only depend not $\vec{p}$, not $\vec{p}$ alone.)

The single-particle $L$ is always

$$L = L_0 + q \vec{A}_0 - q \vec{\nabla} \cdot \vec{A}$$

($L$ and $S = \int d^4x L$ are scalars, by $L$ is not.)

This is great; when the path of electrons is known (and it can be found in, e.g., synchrotron systems), the radiation is determined just by the path.

Explicit consideration of the LV is no longer needed.

The important expression for $\vec{\nabla}$ is

$$\nabla = v \equiv -c_0 - c_0 j_0 \vec{p}_j - c_0 k_0 \vec{p}_k - \frac{m^2}{2p}$$

(with $c_0 = c_0 + c_j$)
Let's look at a typical process: \( \gamma \rightarrow e^- + e^+ \). This is forbidden by LI in vacuum. (In the field of a nucleus to take up some momentum, this is how high-energy photons really interact in matter.)

The way to think of why it's forbidden:

\[
E_\gamma = p \quad \text{(total momentum)}
\]

\[
E_{e^-} + E_{e^+} = \sqrt{m^2 + p_-^2} + \sqrt{m^2 + (\frac{p_+}{2} - \frac{p_\gamma}{2})^2} > p.
\]

The photon has too little energy to produce the pair.

It can become allowed if the energy-momentum relations are different. With only an electron c:

\[
E_{e^-} + E_{e^+} = \sqrt{m^2 + (1 + 2\delta)p^2} + \sqrt{m^2 + (1 + 2\delta_+)p_+^2} \quad \text{(assume p >> m)}
\]

\[
\delta_- = 1 - c_{00} - c_{03} \hat{p}_- - c_{02} \hat{p}_+ \quad \delta_+ = 1 - c_{00} - c_{03} \hat{p}_+ - c_{02} \hat{p}_- \hat{p}_j
\]

\( \delta_- \) and \( \delta_+ \) are generally unequal; the particles move in different directions. However, you can check that, at threshold, \( \hat{p}_- = \hat{p}_+ \) — and, in fact, \( \hat{p}_- = \hat{p}_+ = \hat{P}_2 \).

Even above threshold, \( \hat{p}_- \) and \( \hat{p}_+ \) are almost always very close. This can be seen as relativistic beaming; anything emitted the photon is beamed into a narrow pencil of angles around \( \hat{p} \).

\[
E_{e^-} + E_{e^+} \approx (p_- + \frac{m^2}{2p_-} + \delta_\gamma p_-) + (p_+ + \frac{m^2}{2p_+} + \delta_\gamma p_+) \quad \text{at threshold}
\]

\[
= (p_- + p_+) + \frac{2m^2}{p} + \delta_\gamma p.
\]

\[
\therefore \quad \delta = -\frac{2m^2}{p^2}
\]

The process is allowed in a direction \( \hat{p} \) if

\[
p^2 > -\frac{2m^2}{\delta_\gamma (\hat{p})}, \quad \text{so} \quad \delta_\gamma (\hat{p}) \text{ must be negative.}
\]

Getting the rate is not so hard:

\[
\Gamma = \frac{1}{4 M^2} \frac{1}{\Gamma_{\text{kinematics}}}
\]

\( \Gamma_{\text{kinematics}} \) is zero without LV, so it is \( O(\gamma) \). Then \( c \) can be ignored in \( M \) to get \( \Gamma \) to \( O(c) \).
Including the necessary terms, \( \Gamma \approx \frac{e^2}{\pi} \frac{1}{s} \). If there is also photon LV, \( s \) is the difference of \( \gamma \) and \( e^- \) limiting speeds in the \( \hat{p} \) direction.

This is fact, by the way. If \( \gamma \rightarrow e^- + e^+ \) were allowed, no photons above threshold would reach us from beyond the solar system. Consequently, and photon we observe coming at us with momentum \( \hat{p} \) constrains \( \gamma(\hat{p}) > -\frac{2m^2}{\hat{p}^2} \).

It's not always so easy, however. Compare

\[
L_c = \bar{\psi} \left[ \left( \gamma^\mu + e^\mu \gamma_5 \right) \left( i \partial_5 - m \right) \right] \psi \quad \text{to} \quad L_d = \bar{\psi} \left[ \left( \gamma^\mu + d^\mu \gamma_5 \gamma_\nu \right) \left( i \partial_5 - m \right) \right] \psi
\]

The dispersion relations look similar, with \( d \):

\[
E = \sqrt{m^2 + (1+2s)p^2}, \quad \gamma(\hat{p}) = s \left[ d_{00} + d_{03} \hat{p}_3 + d_{33} \hat{p}_3 \hat{p}_3 \right]
\]

\( s \) is the helicity times fermion number.

You might think \( \gamma \rightarrow e^- + e^+ \) is somewhat suppressed. The energies of different helicities are shifted up or down.

\[
E \approx (1 + s)p + \frac{m^2}{2p}
\]

So only some final states satisfy the kinematics (maybe one of 4?).

In fact, just above threshold, the rate is zero, even if it seems kinematically allowed. The reason is in the \( M^2 \); it vanishes for collinear particles.

\( J \) is a \( J = 1 \) state. \( J \) is almost conserved. The rate for a process that violated angular momentum conservation would be \( O(d^2) \). So the final \( e^- + e^+ \) state also has \( J = 1 \). Both daughter particles have the same helicity as the parent \( \gamma \).

\[
E_{e^-} + E_{e^+} = (1+\delta)p + \frac{m^2}{p} + (1-\delta)p + \frac{m^2}{p} = p + \frac{2m^2}{p} > p
\]

So the process is disallowed kinematically for the allowed dynamically.

People got really confused about this.

Above threshold, the \( e^- \) and \( e^+ \) are not perfectly collinear, so the \( \gamma \) isn't needed to have the same helicity.
There are other complications with $M$, particularly when looking at processes that occur even without 'LV'. The trickiest part is getting the right external spinors $u, \bar{u}, v, \bar{v}$ for the fermions involved.

You have to use canonically normalized spinors, so that the incoming and outgoing currents (related to time derivatives) are correct in a cross section or decay rate.

This means you need to do a spinor-space transformation so that there are no nonstandard $D^0$ terms, i.e., $c^{\alpha 0} d^{\alpha 0}$ ($e^0$, $f^0$, or $g^0 \bar{v}$). This breaks the usual $\eta$ and $\pi$-symmetry and tracelessness of $c^{\alpha 0}$ and $d^{\alpha 0}$, but it's needed for the right answers. The transformations can give the largest contributions to the rates.

In connection with the superluminal neutrino "discovery" (with corrections from others) calculated the effect of $c^{\alpha 0}$ on $\pi^+ \rightarrow \mu^+ + \nu$. The required modifications to $\mu$ were way too large to have gone unnoticed. (Others pointed out the Cerenkov-like $\nu \rightarrow \gamma + Z \rightarrow \nu + e^- + e^+$ also ruled the effect out.) The results were later combined with MINOS data for a new bound.

Returning to astrophysics, there are other processes that generate TeV photons besides inverse Compton upscattering — $\pi^0 \rightarrow \gamma + \gamma$. This is observed from H clouds near energetic sources.

$\pi^0 \rightarrow \gamma + \gamma$ can only occur if there is not too much Lorentz violation.

$$E_{\pi} \approx \sqrt{m_{\pi}^2 + (1 + 2\delta) \hat{p}_{\pi}^2}$$

$$\delta(p) = -\frac{1}{2} \left[ k_{\pi^{00}} + k_{\pi^{(0j)} \hat{k}_j} + k_{\pi^{ij} k_{ij}} \hat{p}_k \right]$$

If $\delta$ is too negative, $E_{\pi} < p$, and $p$ is the minimum energy of the two photons. The process will cease at $p = \frac{m_{\pi}^2}{2\delta(p)}$.

If $\delta$ is positive though, what happens then? $\pi^0 \rightarrow 2\gamma$ is always allowed, but it is superseded by $\pi^0 \rightarrow N + \bar{N}$, because the $\pi NN$ coupling is so much stronger. This happens at $p^2 = \frac{2m_N^2 - \frac{3}{2}m_{\pi}^2}{\delta(p)}$

So observation of $\pi^0 \rightarrow \gamma + \gamma$ in the $\hat{p}$ direction gives a two-sided bound on $\delta_{\pi}(\hat{p})$. 