1 Review & Motivations

The SME with gravity includes a gravity sector, a Standard Model (SM) sector, and a Lorentz Violating (LV) sector. It is defined in terms of an observer-scalar Lagrangian,

\[ \mathcal{L} = \mathcal{L}_{\text{gravity}} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{LV}} + \cdots \]  

(1)

With gravity, there are two local symmetries that are relevant: local Lorentz symmetry and diffeomorphisms. Local Lorentz transformations (LLTs) act on tensor components, e.g., \( T_{abc \ldots} \), defined with respect to a local Lorentz frame (using Latin letters), while diffeomorphisms ( diffs) act on tensor components, e.g., \( T_{\lambda\mu\nu \ldots} \), defined with respect to the spacetime coordinate frame (using Greek letters).
Tensors components with respect to these frames are connected by a vierbein $e_\mu^a$,

$$T_{\lambda \mu \nu \cdots} = e_\lambda^a e_\mu^b \cdots T_{abc \cdots}$$  \hspace{1cm} (2)

In particular the metric obeys,

$$g_{\mu \nu} = e_\mu^a e_\nu^b \eta_{ab}.$$  \hspace{1cm} (3)

With covariant derivatives, a spin connection $\omega_\mu^{ab}$ must be introduced, which obeys $\omega_\mu^{ab} = -\omega_\mu^{ba}$. In Riemann spacetime, $\omega_\mu^{ab}$ is determined by the vierbein, and it does not have independent degrees of freedom. The curvature is the geometrical quantity that characterizes the spacetime in a Riemann geometry. However, in Riemann-Cartan geometry, the spin connection fields $\omega_\mu^{ab}$ are independent degrees of freedom, and the geometry is described by both curvature and torsion.

The SME allows Riemann-Cartan geometry, and the independent fields are then $e_\mu^a$, which has 16 components, and $\omega_\mu^{ab}$, which has 24 components. Both types of fields can propagate as independent degrees of freedom. The SME Lagrangian is formed as an observer-scalar combination of the gravitational fields, SM fields, and the SME coefficients.

For example, a fermion would have terms,

$$\mathcal{L}_{\text{fermion}} = e_\mu^a \bar{\psi} \gamma^a D_\mu \psi - m \bar{\psi} \psi + a_\mu e_\mu^a \bar{\psi} \gamma^a \psi + b_\mu e_\mu^a \bar{\psi} \gamma^5 \gamma^a \psi + \cdots,$$  \hspace{1cm} (4)

as well as torsion couplings, e.g.,

$$\cdots + k_{\alpha \beta \gamma} T^{\alpha \beta \gamma} \bar{\psi} \psi + k_{\alpha \beta \gamma} T^{\alpha \beta \gamma} \bar{\psi} e^\delta_a \gamma^a \psi + \cdots.$$  \hspace{1cm} (5)

The pure gravity sector contains terms of the form

$$\mathcal{L}^{\text{LV}}_{e,\omega} = (k_T)^{\mu \nu} T^{\mu \nu} + (k_R)^{\kappa \mu \nu} R_{\kappa \mu \nu} + (k_T T)^{\alpha \beta \gamma} T_{\alpha \beta \gamma} T^{\mu \nu} + (k_D T)^{\kappa \mu \nu} D_\kappa T^{\mu \nu} + \cdots.$$  \hspace{1cm} (6)

In all of these terms, the interpretation is that the SME coefficients arise as vacuum expectation values (vevs) from spontaneous Lorentz violation. They therefore are dynamical in origin, unlike the case of explicit breaking where the backgrounds have no physical explanation. Theories with spontaneous Lorentz violation maintain many of the same features as GR, including diffeomorphism and local Lorentz invariance at a fundamental level prior to the symmetry breaking.

1.1 Ideas from String Theory

One of the original motivations for considering spontaneous Lorentz breaking comes from string theory. In the context of string field theory, it was found that mechanisms occur in which tensor fields can acquire vacuum expectation values. In this case, the details of the fundamental theory remain unknown. Nonetheless the fundamental
theory at the Planck scale is understood to be fully Lorentz invariant, and it is only
the lower energy solutions that exhibit symmetry breaking. At the level of effective
field theory, this allows couplings between the vacuum values and particle fields.
The SME coefficients are modeled on this type of structure, with the coefficients
representing possible vacuum values.

The generic form of the terms that can appear in a particle expansion in string field
theory with spontaneous Lorentz breaking is

\[ L \sim \frac{\lambda}{M^k} \langle T \rangle \Gamma \bar{\psi}(i\partial)^k \psi. \] (7)

Here, \( M \) is the symmetry-breaking mass scale, which is presumably the Planck scale,
\( \lambda \) is a coupling constant, \( \psi \) is a particle (in this case fermion) field, \( \Gamma \) is a generalized
Dirac matrix, and \( \langle T \rangle \) is the vacuum value of a tensor field. Notice that the general
term includes derivative operators of arbitrary dimension (given in terms of an integer
\( k \)) and that these types of terms are suppressed by additional factors of \( M \).

The terms in the SME are modeled on this type of structure. The SME coefficients
can be viewed as a combination of the coupling \( \lambda \), the powers of \( M \), and the tensor
vev \( \langle T \rangle \). At the level of effective field theory it is not required that the full mechanism
giving rise to the vevs \( \langle T \rangle \) be contained in the theory. However, since the theory is
assumed to arise from a process of spontaneous Lorentz breaking, it is important that
any features such as Nambu-Goldstone or Higgs modes be properly accounted for.

2 Spontaneous Lorentz Violation

Spontaneous symmetry breaking occurs when the Lagrangian and equations of mo-
tion have a symmetry, but the solutions do not. It is usually the ground-state or
vacuum solution that breaks the symmetry due to the appearance of fixed vacuum
expectation values. Spontaneous symmetry breaking is a central feature of the SM,
where the electroweak interactions are broken by the appearance of a constant scalar
vev. At the level of the Lagrangian, the theory includes a potential \( V \) for the scalar
field, which has a degenerate space of potential vacuum solutions. When an actual
vacuum solution occurs, it is chosen spontaneously from this degenerate space, and
the symmetry is then broken. In the case of the electroweak model, it is a scalar
field that acquires a constant vev, so there is no spontaneous breaking of Lorentz
symmetry or diffeomorphisms.

2.1 Implementation & Consequences

For spontaneous Lorentz breaking, a tensor field must acquire a vacuum value. Thus,
\( \langle T_{\mu\nu\cdots} \rangle \neq 0 \) holds in the spacetime frame, or \( \langle T_{abc\cdots} \rangle \neq 0 \) holds in a local frame. In
this case, a potential \( V(T) \) defined in terms of the tensor and metric field must have
a structure that gives rise to \( \langle T_{\mu\nu\cdots} \rangle \neq 0 \) at its minimum.
Labeling the vacuum values in the local frame as

\[ \langle T_{abc...} \rangle = t_{abc...}, \]  

then the vacuum values in the spacetime frame can be written as

\[ \langle T_{\lambda\mu\nu...} \rangle = \langle e^{a}_{\lambda} \rangle \langle e^{b}_{\mu} \rangle \cdots t_{abc...} \equiv t_{\lambda\mu\nu...}. \]  

Notice that the vierbein acquires a vev \( \langle e^{a}_{\lambda} \rangle \) as well. In terms of the local background, a local-frame scalar can be formed as

\[ t^2 = t_{abc...} \eta^{am} \eta^{bn} \cdots t_{mn...}. \]  

One possibility for the potential \( V \) can then be formed as quadratic terms of the form

\[ V \sim (T_{\lambda\mu\nu...} g^\lambda_{\alpha} g^\mu_{\beta} \cdots T_{\alpha\beta\gamma...} \pm t^2)^2. \]  

This then has a vev when \( \langle T_{abc...} \rangle = t_{abc...} \) or \( \langle T_{\lambda\mu\nu...} \rangle = t_{\lambda\mu\nu...} \). Examples of this type of potential are considered later.

In gauge theories, spontaneous symmetry breaking has well known consequences. First, the Goldstone theorem states that when a global continuous symmetry is spontaneously broken, massless modes known as Nambu-Goldstone (NG) modes should appear. Second, if the symmetry is a local symmetry a Higgs mechanism can occur. In this case, the NG modes can be reinterpreted in such a way that gives rise to massive gauge fields. This is the mechanism that gives the W and Z bosons a mass in the electroweak model. Third, depending on the form of the potential \( V \), additional massive modes called Higgs modes can appear as well. In the electroweak model this is the Higgs boson.

With a potential \( V \), the generic behavior is that the NG modes are excitations that stay in the degenerate space that minimizes the vacuum. Thus, these excitations obey \( V' = 0 \). In contrast, the Higgs modes do not stay in the minimum of the potential, and instead obey \( V' \neq 0 \).

In the case of spontaneous Lorentz violation, the question naturally arises whether these features occur as well in this context. If massless NG modes appear, these would involve long-range interactions and any theory that predicts such modes would need to account for them either as known long-range interactions or unknown long-range interactions. In either case, there would be Important consequences for phenomenology.

3 Nambu-Goldstone Modes

With spontaneous Lorentz breaking, the nonzero vevs,

\[ \langle T_{abc...} \rangle \neq 0, \]  

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\[ \langle T_{\lambda\mu\nu\cdots} \rangle \neq 0, \quad (13) \]
\[ \langle e_\mu^a \rangle \neq 0, \quad (14) \]

indicate that both LLTs and diffs are spontaneously broken. In a gravitational theory, it is not possible to spontaneously break LLTs without also spontaneously breaking diffs, and vice versa.

The number of NG modes depends on the number of broken symmetries, though for spontaneously broken spacetime symmetries there is not a one-to-one relation. Thus, for the broken LLTs there can be up to six NG modes, while for the broken diffs there can be up to four NG modes. In total there can be as many as ten NG modes when Lorentz and diffeomorphism invariance are spontaneously broken.

Notice that the vierbein \( e_\mu^a \) has 16 components, whereas in GR the metric has ten components but only up to six independent degrees of freedom. Four of the metric components are gauge due to the diffeomorphism invariance in GR. Since a vierbein formalism in GR is also diffeomorphism invariant, the vierbein has four gauge degrees of freedom as well. In addition, the local Lorentz invariance allows six more degrees of freedom to be gauged away. This leaves only up to six physical modes in the vierbein, which matches the case of the metric.

When local Lorentz symmetry and diffeomorphisms are spontaneously broken, the ten NG modes can take the place of the ten gauge degrees of freedom in the vierbein. This is achieved by making gauge choices that remove the gauge degrees of freedom from the other sectors of the theory, while leaving the excitations in the vierbein corresponding to the broken symmetries. These excitations remain in the potential minimum, obeying \( V' = 0 \), as expected for NG modes.

If all ten NG modes were to appear in a gravitational theory with spontaneous Lorentz breaking, then at least several of these modes would propagate as ghosts (excitations with negative kinetic energy terms). Thus, in searching for viable models of spontaneous Lorentz breaking, it is likely that the vevs that appear will only partially break the ten symmetries, producing a smaller number of NG modes. Alternatively, by choosing suitable kinetic terms, it is possible for some of the NG modes to remain auxiliary and not propagate as physical modes. This would eliminate potential ghost modes as well.

### 3.1 Perturbative Treatment

In a perturbative treatment about a Minkowski background, the metric can be written as

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (15) \]

Since in this case, the vacuum value for the metric is \( \eta_{\mu\nu} = \langle e_\mu^a \rangle \langle e_\nu^b \rangle \eta_{ab} \), the vierbein vacuum can be identified as

\[ \langle e_\mu^a \rangle = \delta_\mu^a. \quad (16) \]
This allows the distinction between Greek and Latin indices to be dropped in a leading-order treatment. Thus, the vierbein with lowered indices takes the form

\[ e_{\mu\nu} = \eta_{\mu\nu} + (\frac{1}{2}h_{\mu\nu} + \chi_{\mu\nu}). \] (17)

In this expression, there are ten symmetric components given in terms of \( h_{\mu\nu} = h_{\nu\mu} \), and six antisymmetric components \( \chi_{\mu\nu} = -\chi_{\nu\mu} \).

The NG modes have the form of excitations away from the vacuum solutions that stay in the potential minimum. Writing the excitations for a generic tensor as

\[ \delta T_{\lambda\mu\nu\cdots} = (T_{\lambda\mu\nu\cdots} - t_{\lambda\mu\nu\cdots}), \] (18)

with \( T_{\lambda\mu\nu\cdots} = e^\alpha e^\beta \cdots t_{\alpha\beta\cdots} \), gives the NG modes at leading order as

\[ \delta T_{\lambda\mu\nu\cdots} \simeq (\frac{1}{2}h_{\lambda\alpha} + \chi_{\lambda\alpha})t^\alpha_{\mu\nu\cdots} + (\frac{1}{2}h_{\mu\alpha} + \chi_{\mu\alpha})t^\alpha_{\lambda\mu\nu\cdots} + \cdots . \] (19)

Evidently, the combination \((\frac{1}{2}h_{\mu\nu} + \chi_{\mu\nu})\) contains the NG modes. These modes can be separated off from the gravitational excitations as virtual symmetry transformations. Under LLTs,

\[ h_{\mu\nu} \rightarrow h_{\mu\nu}, \] (20)
[\[ \chi_{\mu\nu} \rightarrow \chi_{\mu\nu} - \epsilon_{\mu\nu}, \] (21)

while under diffs,

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu, \] (22)
[\[ \chi_{\mu\nu} \rightarrow \chi_{\mu\nu} - \frac{1}{2}(\partial_\mu \xi_\nu - \partial_\nu \xi_\mu). \] (23)

The six Lorentz coefficients \( \epsilon_{\mu\nu} \) and four diffs coefficients \( \xi_\mu \) can be promoted, respectively, to six Lorentz NG modes and four diffs NG modes. The specifics of how the NG modes propagate and interact depends on a number of factors, including the form of the potential \( V \), the geometry (Minkowski, Riemann, or Riemann-Cartan), and the kinetic terms for the tensor \( T_{\lambda\mu\nu\cdots} \). The question of whether ghost modes are present depends heavily on these features as well. Hence, the overall viability of a model with spontaneous Lorentz breaking must be considered on a case-by-case basis.

### 4 Higgs Mechanism

Since there are two types of NG modes, Lorentz NG modes and diffeomorphism NG modes, there are potentially two types of Higgs mechanisms. In the Higgs mechanism, which occurs when the broken symmetry is local, the degrees of freedom associated with the NG modes get reinterpreted as degrees of freedom associated with the gauge fields. The end result is that the gauge fields acquire a mass. In theories with spontaneous Lorentz breaking in Riemann-Cartan spacetime, there are two sets of gauge fields. The metric or vierbein is the carrier of the gravitational force and is tied to the diffeomorphism invariance. The spin connection is the second gauge field, which is associated with the spin and torsion, and it is tied to the local Lorentz invariance. Thus, with two potential Higgs mechanisms it is possible to consider both the metric and the spin connection as gauge fields that could possibly acquire a mass.
4.1 Metric & Spin Connection

In the Higgs mechanism the mass term for the gauge field originates from the kinetic term for the field that acquires a vev. The covariant derivatives include couplings between the gauge field and the field with a vev. When the vev forms, the quadratic kinetic terms become quadratic mass terms for the gauge field. For the case of the metric, an examination of the kinetic term for the tensor that acquires a vev shows that the resulting quadratic mass term includes terms of the form,

\[ (D_\mu t_{\alpha\beta\gamma\ldots})^2 \sim (\Gamma^\lambda_{\mu\alpha} t_{\lambda\beta\gamma\ldots})^2 + \cdots. \]  (24)

Note, however, that with the gravitational covariant derivative it is the connection \( \Gamma^\lambda_{\mu\alpha} \) that appears, not the metric. Thus, the resulting mass term involves quadratic terms for \( \Gamma^\lambda_{\mu\alpha} \), which depends on derivatives of the metric, not just the metric itself. Terms of this form modify the propagation of gravity, but do not give rise to a conventional Higgs mechanism. Thus, there is no conventional Higgs mechanism associated with spontaneous diffeomorphism breaking that results in a massive metric field.

The situation is different, however, for the broken local Lorentz invariance. In this case, the spin connection \( \omega^{ab}_{\mu} \) is the gauge field. In a local Lorentz frame, when a covariant derivative acts on a tensor with local Lorentz indices, it couples the background vev with the spin connection. With a quadratic kinetic term for the tensor, a mass term with quadratic powers of the spin connection can appear, for example, of the form

\[ (D_\mu t_{\alpha\beta\gamma\ldots})^2 \sim (\omega^\rho_{\mu\alpha} t_{\rho\beta\gamma\ldots})^2 + \cdots. \]  (25)

Here, these terms give rise to quadratic terms for the spin connection that can act as mass terms. Thus, a Higgs connection for the spin connection is a possibility. Of course, for this to work there need to be suitable kinetic terms for the spin connection that are compatible with the mass terms that form. This is only possible in models that allow a propagating spin connection, which requires dynamical torsion as well. Thus, it is only in a Riemann-Cartan spacetime that a Higgs mechanism for the spin connection is possible.

Attempts have been made at model building, where various kinetic terms giving rise to a propagating spin connection have been considered along with different types of potentials for a background tensor. However, these models are complicated since the spin connection \( \omega^{ab}_{\mu} \) has 24 components. Many of these are ghost modes if they are permitted to propagate. As a result, it remains an open question whether a viable model exists in which a ghost-free propagating spin connection acquires a mass as a result of spontaneous Lorentz breaking.

5 Massive Higgs Modes

In models with spontaneous Lorentz breaking, the NG modes are excitations that stay in the minimum of the potential and obey \( V' = 0 \). At the same time, there
can be excitations that go up the potential well and obey \( V' \neq 0 \). These types of excitations are massive Higgs modes. In a gravitational theory with a potential that induces spontaneous Lorentz breaking, these types of excitations can occur as well. Interestingly, the massive modes that arise involve the metric. Thus, while there is no conventional Higgs mechanism that gives the metric a mass, there are massive Higgs excitations that can occur that include contributions involving the metric. This provides an alternative route for massive metric modes to appear in theories with spontaneous Lorentz breaking.

With a potential for a tensor \( T_{\lambda \mu \nu} \ldots \) that has the form,

\[
V \sim (T_{\lambda \mu \nu} \ldots g^{\lambda \alpha} g^{\mu \beta} \ldots T_{\alpha \beta \gamma} \ldots \pm t^2)^2,
\]

an expansion about the vacuum value \( t_{\lambda \mu \nu} \ldots \) can be performed. Expanding

\[
T_{\lambda \mu \nu} = t_{\lambda \mu \nu} + (\delta T)_{\lambda \mu \nu} \ldots,
\]

\[
g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu},
\]

the massive excitations take the form

\[
V' = t^{\lambda \mu \nu \ldots}((\delta T)_{\lambda \mu \nu} \ldots - \frac{1}{2} h_{\lambda \alpha} t_{\mu \nu}^{\alpha} \ldots - \ldots).
\]

The massive excitations are combinations of \((\delta T)_{\lambda \mu \nu} \ldots \) and \( h_{\mu \nu} \). Thus, some of the the metric excitations \( h_{\mu \nu} \) can acquire mass terms. This will affect the propagation of gravitons. However, specific results are highly model dependent and need to be looked at on a case-by-case basis.

### 6 Examples

Specific examples of gravitational theories with spontaneous Lorentz breaking include Bumblebee models, where it is a vector field that acquires a vev, Cardinal models with a symmetric two-tensor vev, and Phon models with an antisymmetric two-tensor vev. These models can be considered in either Riemann spacetime or Riemann-Cartan spacetime and can have a variety of possible kinetic terms and potential terms. Thus, the examples considered here are just a small sampling of the possibilities.

#### 6.1 Bumblebee Models

Bumblebee models are gravitational theories with a vector field \( B_\mu \) and a potential \( V = V(B^\mu B_\mu \pm b^2) \) that induces spontaneous Lorentz breaking. The vector acquires a vev denoted as \( \langle B_\mu \rangle = b_\mu \), which obeys \( \pm b^2 = b_\mu \eta^{\mu \nu} b_\nu \). The vev can be timelike, spacelike, or lightlike. A significant feature of Bumblebee models is that they do not have a U(1) gauge symmetry due to the presence of the potential \( V \). The generic form of the Lagrangian is

\[
\mathcal{L} = \frac{1}{16\pi G} (R - 2\Lambda) + \mathcal{L}_B - V + \mathcal{L}_{\text{int}},
\]
where $\mathcal{L}_B$ contains the kinetic terms for $B_\mu$ and $\mathcal{L}_{\text{int}}$ contains interaction terms with matter fields. In a purely vector-tensor model of modified gravity, the term $\mathcal{L}_{\text{int}}$ can be omitted.

There are various possibilities for the form of the kinetic term. These include very general expressions involving a mixture of covariant derivative terms and possible gravitational coupling, e.g.,

$$L_B = \sigma_1 B^\mu B^\nu R_{\mu\nu} + \sigma_2 B^\mu B_\mu R - \frac{1}{4} \tau_1 B_\mu B^\mu B_{\mu\nu} + \frac{1}{2} \tau_2 D_\mu B_\nu D^\mu B^\nu + \frac{1}{2} \tau_3 (D_\mu B^\mu)^2. \tag{31}$$

The constants $\sigma_1, \sigma_2, \tau_1, \tau_2, \tau_3$ are arbitrary, allowing linear combinations of the different terms. The tensor $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. Models of this type typically have $\mathcal{L}_{\text{int}} = 0$.

Alternatively, a Maxwell kinetic term can be used for $B_\mu$. For example, the Kostelecký-Samuel (KS) model uses

$$L_B = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{32}$$

$$\mathcal{L}_{\text{int}} = B_\mu J^\mu, \tag{33}$$

where $J^\mu$ is a conserved matter current. The Maxwell term eliminates a potential ghost mode from the kinetic term and leads to a form of Bumblebee electrodynamics.

### 6.2 Physical Interpretations

As a concrete example, consider the KS Bumblebee model in Riemann spacetime (with no torsion) and with a smooth quadratic potential,

$$V = \frac{1}{2} \kappa (B_\mu B^\mu \pm b^2)^2. \tag{34}$$

Assume a purely timelike vev $b_\mu = (b, 0, 0, 0)$, which breaks 3 Lorentz boosts and time diffeomorphisms. Thus, up to four NG modes are expected. They obey $V' = \kappa (B_\mu B^\mu \pm b^2) = 0$. Since the model is in Riemann spacetime, there is no Higgs mechanism. However, the theory can have a massive mode, which is an excitation obeying $V' = \kappa (B_\mu B^\mu \pm b^2) \neq 0$.

The equations of motion for the KS Bumblebee model are the Einstein equations,

$$G^{\mu\nu} = 8\pi G (T_M^{\mu\nu} + T_B^{\mu\nu}), \tag{35}$$

where $T_M^{\mu\nu}$ is the matter energy-momentum tensor and $T_B^{\mu\nu}$ is the Bumblebee energy-momentum given by

$$T_B^{\mu\nu} = B^{\mu\alpha} B_\alpha^\nu - \frac{1}{4} g^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta} - V g^{\mu\nu} + 2V' B^{\mu\nu} B^\nu, \tag{36}$$
and the Bumblebee equation of motion,

$$D_\mu (J^\mu - 2 V' B^\mu) = 0.$$  \hspace{1cm} (37)

Observe that the massive mode $V' \neq 0$ acts as a source of energy and charge density. With conserved matter couplings obeying $D_\mu J^\mu = 0$, it is possible to restrict the initial value space in such a way that the massive mode decouples.

For the NG modes, it is found that the diff NG mode does not propagate and is instead auxiliary. However, the Lorentz NG modes do propagate. To examine these modes, a linearized treatment can be used. It sets

$$B_\mu = b_\mu + \epsilon_\mu,$$  \hspace{1cm} (38)

where the excitations $\epsilon_\mu$ obey the condition

$$b^\mu (\epsilon_\mu - \frac{1}{2} b_{\mu \nu} b^\nu) = 0,$$  \hspace{1cm} (39)

which in the absence of gravity reduces to an axial gauge condition. Examining the equations of motion reveals that in this limit two massless transverse modes for the Bumblebee propagate as physical modes. They effectively have the form of massless photons in a fixed axial gauge in a gravitational field.

The idea that photons can be described as NG modes in flat spacetime (in the absence of gravity) has been around since the 1960s when Bjorken and Nambu proposed models along these lines. The KS model is different in that it includes gravity, and the presence of the potential $V$ destroys any possible interpretation of the theory as a U(1) gauge theory. The KS model also includes signatures of Lorentz violation through matter couplings with the vev $b_\mu$. In general, the theory also has modified forms of the static Newtonian and Coulomb potentials. However, in a large mass limit where $\kappa b^2 \to \infty$, solutions from Einstein-Maxwell theory are recovered, including the usual forms of the Newtonian and Coulomb potentials.

### 6.3 Higgs Model

To examine the possibility of a Higgs mechanism, consider a Bumblebee model in Riemann-Cartan spacetime. In this case, the spin connection is dynamical, and a vierbein can be used to look at the excitations carried by it in $B_\mu = e_\mu^a b_a$. In terms of this, the field strength tensor is $B_{\mu \nu} = (e_\mu^b \omega_\nu^a - e_\nu^b \omega_\mu^a) b_a$. The Maxwell kinetic term for the Bumblebee field in this case becomes

$$-\frac{1}{4} B_{\mu \nu} B^{\mu \nu} \simeq -\frac{1}{4} (\omega_{\mu \rho \nu} - \omega_{\nu \rho \mu})(\omega^{\mu \sigma \nu} - \omega^{\nu \sigma \mu}) b_\rho b_\sigma.$$  \hspace{1cm} (40)

Expanding this out gives various quadratic products which serve as mass terms in the effective Lagrangian for the propagating spin connection. With these types of terms, $\omega_{\lambda \mu \nu}$ can acquire a mass. However, the model building is tricky. Finding an appropriate kinetic term for the spin connection that does not include ghosts remains elusive.
6.4 Tensor Models

The Cardinal model involves a symmetric two-tensor field in Minkowski spacetime that undergoes spontaneous Lorentz violation. The resulting NG modes can be shown to behave like massless gravitons. A bootstrap method can then be used to generate nonlinear terms. It can then be shown that the low energy limit of this theory includes GR.

Similarly, a model with an antisymmetric two-tensor, known as a Phon model, can be developed in curved spacetime. It too undergoes a process of spontaneous Lorentz breaking and has both NG modes and massive modes. Phon models can be explored as potential theories of modified gravity.

7 SME Gravity Sector

The gravity sector of the SME uses effective field theory and includes background fields in the context of a phenomenological framework. In this context, the origin of the SME coefficients is not specified. Nonetheless, the SME coefficients are assumed to have arisen from a process of spontaneous Lorentz breaking. This requires that the theory also includes NG modes and potentially massive modes as well. Techniques accounting for these additional modes have been developed and are incorporated into the framework derived in a post-Newtonian limit. It is this limit that is primarily used to investigate signatures of Lorentz violation in gravity experiments. Other talks will describe the post-Newtonian limit and gravitational tests of Lorentz invariance.

8 References

The original papers describing spontaneous Lorentz breaking in gravity are in [1]. These discuss the original motivations from string theory. The Bumblebee models are described here, including the KS model. Ref. [2] develops the gravity sector of the SME. A useful phenomenological framework in a post-Newtonian limit is developed in [3, 4]. The processes of spontaneous Lorentz and diffeomorphism breaking and the behavior of the NG and massive modes are described in [5]. Various aspects of Bumblebee models are described in [5, 6]. The Cardinal and Phon models are described, respectively, in [7] and [8]. Many additional references can be found as well in these papers.


