Gravitational Tests 1: Theory to Experiment

Jay D. Tasson

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outline

• sources of basic information
• theory to experiment
  – intro to GR
  – Lagrangian expansion in gravity
  – addressing the fluctuations
  – presentation of tools
outline part 2

• independent discussions of useful issues
  – experimental example: lab test
  – experimental example: orbital test
  – effective LV: when prosaic effects are observable via SME methods
  – what happens to antimatter gravity when you have CPT violation?
  – there is also a PPN formalism for testing gravity, how is it related to the SME?
  – coordinate and field redefinitions: when LV is not really LV
  – the challenge of getting a hamiltonian in gravity and LV
outline

• sources of basic information

• theory to experiment
  – intro to GR
  – Lagrangian expansion in gravity
  – addressing the fluctuations
  – presentation of tools
overview of Lorentz violation/SME

http://iopscience.iop.org/0034-4885/77/6/062901/

– simple examples
– general overview
– video abstract
simple examples: inclined plane

OBSERVER AND PARTICLE TRANSFORMATIONS AND NEWTON’S LAWS

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A frequently confused point in studies of symmetry violation is the distinction between observer and particle transformations. In this work, we consider a model in which a coefficient in the Standard-Model Extension leads to violations of rotation invariance in Newton’s second law. The model highlights the distinction between observer and particle transformations.

1. Introduction

The Standard-Model Extension (SME) provides a general field-theoretic framework for studying Lorentz violation, including rotation-invariance violation. To highlight the basic ideas of Lorentz-symmetry breaking, we consider rotation-invariance violation in Newton’s second law:

\[
F_j = m_{jk} a_k. \tag{1}
\]

Here \(m_{jk}\) is a symmetric direction-dependent inertial mass (we consider conventional gravitational mass). This yields a valid and more general form that Newton himself could have chosen.

Our effective inertial mass can be generated as a low-energy limit of the SME:

\[
m_{jk} = m (\delta_{jk} + 2c_{jk}). \tag{2}
\]
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The Standard-Model Extension (SME) provides a general field-theoretic framework for studying Lorentz violation,\(^1\) including rotation-invariance violation. To highlight the basic ideas of Lorentz-symmetry breaking, we consider rotation-invariance violation in Newton’s second law:\(^2\)

\[ F_j = m_{j,k} a_k. \] (1)

Here \(m_{j,k}\) is a symmetric direction-dependent inertial mass (we consider conventional gravitational mass). This yields a valid and more general form that Newton himself could have chosen.

Our effective inertial mass can be generated as a low-energy limit of the SME:\(^3\)

\[ m_{j,k} = m (\delta_{j,k} + 2c_{j,k}). \] (2)
1. Introduction

The Standard Model (SME) framework considers that Lorentz violation should be observable, as shown in Fig. 1. This produces an observably different acceleration. Solving for the motion of the particle subject to the constraint yields

\[ a_y = -(1 - 2c_{xx} - 2c_{yy})g \sin^2 \theta + O(c^2), \]
\[ a_x = (1 - 2c_{xx} - 2c_{yy})g \sin \theta \cos \theta + O(c^2). \]  

Here the component along the ramp is

\[ a_R = (1 - 2c_{xx} - 2c_{yy})g \sin \theta + O(c^2), \]

which is different from the first case, revealing observable Lorentz violation.
outline

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Signals for Lorentz Violation in Post-Newtonian Gravity

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(Dated: IUHET 489, March 2006; Physical Review D, in press)

The pure-gauge sector of the minimal Standard-Model Extension is studied in the limit of Riemann spacetime. A method is developed to extract the modified Einstein field equations in the limit of small metric fluctuations about the Minkowski vacuum, while allowing for the dynamics of the 20 independent coefficients for Lorentz violation. The linearized effective equations are solved to obtain the post-newtonian metric. The corresponding post-newtonian behavior of a perfect fluid is studied and applied to the gravitating many-body system. Illustrative examples of the methodology are provided using bumblebee models. The implications of the general theoretical results are studied for a variety of existing and proposed gravitational experiments, including lunar and satellite laser ranging, laboratory experiments with gravimeters and torsion pendula, measurements of the spin precession of orbiting gyroscopes, timing studies of signals from binary pulsars, and the classic tests involving the perihelion precession and the time delay of light. For each type of experiment considered, estimates of the attainable sensitivities are provided. Numerous effects of local Lorentz violation can be studied in existing or near-future experiments at sensitivities ranging from parts in $10^9$ down to parts in $10^{15}$.

I. INTRODUCTION

At the classical level, gravitational phenomena are well described by general relativity, which has now survived nine decades of experimental and theoretical scrutiny. In the quantum domain, the Standard Model of particle physics offers an accurate description of matter and nongravitational forces. These two field theories provide a comprehensive and successful description of nature. However, it remains an elusive challenge to find a consistent quantum theory of gravity that would merge them into a single underlying unified theory at the Planck scale.

Since direct measurements at the Planck scale are infeasible, experimental clues about this underlying theory are scant. One practical approach is to search for properties of the underlying theory that could be manifest as suppressed new physics effects, detectable in sensitive experiments at attainable energy scales. Promising candidate signals of this type include ones arising from microscale violations of Lorentz symmetry [1-3]. Predictions of realistic theories involving relativity modifications are therefore expressible in terms of the SME by specifying the SME coefficient values. In fact, the explicit form of all dominant Lorentz-violating terms in the SME is known [4]. These terms consist of Lorentz-violating operators of mass dimension three or four, coupled to coefficients with Lorentz indices controlling the degree of Lorentz violation. The subset of the theory containing these dominant Lorentz-violating terms is called the minimal SME.

Since Lorentz symmetry underlies both general relativity and the Standard Model, experimental searches for violations can take advantage either of gravitational or of nongravitational forces, or of both. In the present work, we initiate an SME-based study of gravitational experiments searching for violations of local Lorentz invariance. To restrict the scope of the work to a reasonable size while maintaining a good degree of generality, we limit attention here to the pure-gravity sector of the minimal SME in Riemann spacetime. This neglects possible complexities associated with matter-sector effects and with
Matter-gravity couplings and Lorentz violation

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The gravitational couplings of matter are studied in the presence of Lorentz and CPT violation. At leading order in the coefficients for Lorentz violation, the relativistic quantum Hamiltonian is derived from the gravitationally coupled minimal Standard-Model Extension. For spin-independent effects, the nonrelativistic quantum Hamiltionan and the classical dynamics for test and source bodies are obtained. A systematic perturbative method is developed to treat small metric and coefficient fluctuations about a Lorentz-violating and Minkowski background. The post-newtonian metric and the trajectory of a test body freely falling under gravity in the presence of Lorentz violation are established. An illustrative example is presented for a bumblebee model. The general methodology is used to identify observable signals of Lorentz and CPT violation in a variety of gravitational experiments and observations, including gravimeter measurements, laboratory and satellite tests of the weak equivalence principle, antitematter studies, solar-system observations, and investigations of the gravitational properties of light. Numerous sensitivities to coefficients for Lorentz violation can be achieved in existing or near-future experiments at the level of parts in $10^4$ down to parts in $10^9$. Certain coefficients are uniquely detectable in gravitational searches and remain unmeasured to date.

I. INTRODUCTION

General Relativity (GR) is known to provide an accurate description of classical gravitational phenomena over a wide range of distance scales. A foundational property of the gravitational couplings of matter in GR is local Lorentz invariance in freely falling frames. The realization that a consistent theory of quantum gravity at the Planck scale $m_P \approx 10^{19} \text{ GeV}$ could induce tiny manifestations of Lorentz violation at observable scales [1] has revived interest in studies of Lorentz symmetry, with numerous sensitive searches for Lorentz violation being undertaken in recent years [2].

Gravitational signals of Lorentz violation are more challenging to study than ones in Minkowski spacetime for several reasons, including the comparative weakness of gravity at the microscopic level and the impossibility of screening gravitational effects on macroscopic scales. Both for purely gravitational interactions and for matter-gravity couplings, Lorentz violations can be classified and enumerated in effective field theory [3]. Several searches for purely gravitational Lorentz violations in this context have recently been performed [4-6] using a treat-

the equation for the trajectory of a test body moving under gravity in the presence of Lorentz violation, allowing also for Lorentz-violating effects from the composition of the test and source bodies and for effects from possible additional long-range modes associated with Lorentz violation. We also seek to explore the implications of our analysis in a wide variety of experimental and observational scenarios, identifying prospective measurable signals and thereby enabling more complete searches using matter-gravity couplings.

Despite the current lack of a satisfactory quantum theory of gravity, established gravitational and particle phenomena at accessible energy scales can successfully be analyzed using the field-theoretic combination of GR and the Standard Model (SM). This combination therefore serves as a suitable starting point for a comprehensive effective field theory describing observable signals of Planck-scale Lorentz and CPT violation in gravity and particle physics [9]. The present paper adopts this general framework, known as the gravitational Standard-Model Extension (SME) [3], to analyze Lorentz violation in matter-gravity couplings. Each term violating Lorentz symmetry in the SME Lagrange density is a scalar den-
Gravity is a universal but comparatively weak force. This makes it challenging to study and today, some 350 years after Newton’s *Principia*, our experimental understanding of gravity remains in some respects remarkably limited. On the scale of the solar system, we are confident that Newton’s law describes the dominant physics and that Einstein’s General Relativity provides accurate relativistic corrections. However, on larger scales we lack a complete and compelling understanding, as evidenced by dark energy. On smaller scales below about 10 microns, it is presently unknown whether gravity obeys Newton’s law, and forces vastly stronger than the usual inverse-square behavior remain within the realm of possibility.

Perhaps the most crucial underlying principle of General Relativity is the Einstein equivalence principle. Two of its ingredients are the weak equivalence principle, which essentially states that gravity is flavor independent, and local Lorentz invariance, which states that rotations and boosts are local symmetries of nature. Developing a deep understanding of gravity at all scales therefore depends on strong experimental support for these principles. The weak equivalence principle has been widely tested, but tests of local Lorentz invariance have been largely limited to the pure-matter sector or to matter-gravity couplings [1, 2]. Here, we undertake to address this gap by focusing on violations of local Lorentz symmetry in the pure-gravity sector.

Effective field theory is a powerful and unique tool for investigating physics at attainable scales when defining expressions for \( d = 5 \) and 6 and investigating prospective experimental constraints. Operators of higher mass dimension \( d \) involve more derivatives, which translate to corrections to the Newton force law varying as \( 1/r^{d-1} \). Short-range tests of gravity therefore offer the sharpest sensitivities to effects from operators with \( d > 4 \) and are our focus in what follows. Moreover, as discussed below, the predicted signals contain novel features that to date are unexplored in experiments.

We focus here on spontaneous violation of Lorentz symmetry [13] in spacetime theories of gravity, since the alternative of explicit Lorentz violation is generically incompatible with conventional Riemann geometry and technically unnatural in such theories [3]. Spontaneous Lorentz violation occurs when an underlying action with local Lorentz invariance involves gravitational couplings to tensor fields \( \tilde{h}_{\alpha\beta} \) that acquire nonzero background values \( \tilde{F}_{\alpha\beta} \) [14]. The field fluctuations \( \tilde{h}_{\alpha\beta} = h_{\alpha\beta} - \tilde{F}_{\alpha\beta} \) include massless Nambu-Goldstone and massive modes that affect the physics. The presence of nonzero backgrounds means the resulting gravitational phenomenology violates local Lorentz invariance, and so the backgrounds \( \tilde{F}_{\alpha\beta} \) are called coefficients for Lorentz violation [15].

In typical post-newtonian applications, the coefficients \( \tilde{F}_{\alpha\beta} \) are assumed small on the relevant physical scale and constant in asymptotically flat coordinates, and the analysis is performed at linear order in the metric fluctuation \( h_{\alpha\beta} \) and the coefficients \( \tilde{F}_{\alpha\beta} \). Elimination of the fluctuations \( \tilde{h}_{\alpha\beta} \) may be achieved by imposing the
outline

• sources of basic information

• theory to experiment
  – intro to GR
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Who are you?

Rate your background in gravitational physics (before Robert's talk):

A) have a fresh working knowledge of GR: recently done research in the area, recently took/taught a course, etc.

B) familiar with the common constructs of GR: recognize the Einstein equations, the geodesic equation, etc.

C) little familiarity beyond perhaps a conceptual picture
relativistic gravity basics (in words)

Newton: gravity is a force

Einstein: gravity is a relativistic theory (like Maxwell's E&M) viewable as free particles moving on a curved spacetime warped by other mass-energy – rubber sheet analogy
relativistic gravity basics (in words)

Newton: gravity is a force

Einstein: gravity is a relativistic theory (like Maxwell's E&M) viewable as free particles moving on a curved spacetime warped by other mass-energy – rubber sheet analogy
relativistic gravity basics – classical particles

- classical matter action

\[ S = \int -m \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \, d\tau \]

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

e.g. \( h_{00} = 2U + \ldots \)

- matter equation of motion

\[ \ddot{x}^\mu = -\Gamma^\mu_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta \quad \Gamma^\mu_{\alpha\beta} = g^{\mu\nu} (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta}) \]

- nonrelativistic limit

\[ \dddot{x} = \dddot{g} \]

- once can also do a post-newtonian expansion in powers of

\[ \nu \sim \sqrt{U} \]
relativistic gravity basics – fermion fields

- fermions live on a Minkowski tangent space linked via the vierbein

\[ g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab} \]

\[
ge_\mu^a \approx \eta_\mu^a + \frac{1}{2} h_\mu^a + \chi_\mu^a
\]

- gravitationally coupled Dirac Lagrangian

\[ \mathcal{L}_{SM} = \frac{1}{2} ie_\mu^a \bar{\psi} \gamma^a \overleftrightarrow{D_\mu} \psi - \bar{\psi} m \psi \]

- covariant derivative for gravity as well as U1
relativistic gravity basics – field equations

- action
  \[ S_G = \frac{1}{16\pi G_N} \int d^4x \varepsilon e R \]

- Riemann curvature tensor
  \[ R^\kappa_{\lambda\mu\nu} = (\partial_\mu \Gamma^\kappa_{\nu\lambda} + \Gamma^\kappa_{\mu\alpha} \Gamma^\alpha_{\nu\lambda}) - (\mu \leftrightarrow \nu) \]
  - vierbein determinant

- Ricci tensor
  \[ R_{\mu\nu} = R^\kappa_{\mu\kappa\nu} \]
  - 2 derivatives of metric

- curvature scalar
  \[ R = g^{\mu\nu} R_{\mu\nu} \]

- Einstein tensor
  \[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \]

- field equations via variation w.r.t. the metric (cf Maxwell)
  \[ G_{\mu\nu} = 8\pi G_N T_{\mu\nu} \]
Q1: Newtonian example
which of the following is a likely consequence of Lorentz violation in gravity?

1) The force changes when you rotate the observer.
2) The force changes when you rotate the tennis ball.
3) The force changes when you rotate the Earth and the tennis ball about a common axis.
4) The force changes when you rotate the Earth.
5) None of the above.
A1: Newtonian example

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4) The force changes when you rotate the Earth.
5) None of the above.
A1: Newtonian example

which of the following is a likely consequence of Lorentz violation in gravity?

\[ \vec{F} = m \vec{g}(1 + LV) \]

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3) The force changes when you rotate the Earth and the tennis ball about a common axis.
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SME → develop this idea systematically via effective field theory → many signals → most gravitational experiments
idea

• expand about the SM coupled to GR

• 4 areas
  1) minimal gravity sector
  2) minimal matter sector with gravitational couplings
  3) nonminimal gravity sector
  4) gravitationally coupled nonminimal matter sector

\[ \mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{m \text{ matter}} + \mathcal{L}_{nm \text{ matter}} + \frac{e}{16\pi G_N} (\mathcal{L}_{GR} + \mathcal{L}_{m \text{ grav}} + \mathcal{L}_{nm \text{ grav}}) \]
idea

• expand about the SM coupled to GR

• 4 areas

  1) minimal gravity sector
  2) minimal matter sector with gravitational couplings
  3) nonminimal gravity sector
  4) gravitationally coupled nonminimal matter sector

\[ \mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_\text{m matter} + \mathcal{L}_\text{nm matter} + \frac{e}{16\pi G_N} \left( \mathcal{L}_\text{GR} + \mathcal{L}_\text{m grav} + \mathcal{L}_\text{nm grav} \right) \]
Q2: sectors?

- In which of the 4 areas would following term be classified? \( \mathcal{L} \supset \frac{1}{16\pi G_N} e^{-S^{\mu\nu}} R_{\mu\nu} \)

  1) minimal gravity sector
  2) minimal matter sector with gravitational couplings
  3) nonminimal gravity sector
  4) gravitationally coupled nonminimal matter sector
A2: sectors?

• In which of the 4 areas would following term be classified?

\[ \mathcal{L} \supset \frac{1}{16\pi G_N} e^{\tilde{S}_{\mu\nu}} R_{\mu\nu} \]

1) minimal gravity sector
2) minimal matter sector with gravitational couplings
3) nonminimal gravity sector
4) gravitationally coupled nonminimal matter sector
Introduction: minimal SME

What is the minimal SME?
- power-counting renormalizable \( (d \leq 4) \)
- flat spacetime
- gauge invariance
- translation invariance
- Lorentz violating

Leading-order (renormalizable?) remnants

\[
\text{known physics} \quad \text{SM} + \text{GR} \quad + \quad \circ \quad + \quad \cdot \quad + \quad \cdot \quad + \quad \ldots = \quad \text{quantum gravity}
\]

Higher-order nonrenormalizable remnants

courtesy of M. Mewes
show the Lagrangian, guess the effects
minimal gravity sector

gravity sector: dynamics of gravitational field alone

conventional pure-gravity

\[ \mathcal{L}_{\text{GR}} = R \]

minimal Lorentz violation

\[
\mathcal{L}_{\text{m grav}} = \left( k^{(4)} \right)_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} \]

\[
\mathcal{L}_{\text{m grav}} = -u R + s^{\mu\nu} R^T_{\mu\nu} + t^{\kappa\lambda\mu\nu} C_{\kappa\lambda\mu\nu}
\]

- minimal post-Newtonian effects originate from \( s^{\mu\nu} \)
- experiments that involve the gravitational field are relevant
- particle species independent by definition
matter sector vs. gravity sector

conventional gravitationally coupled matter sector

\[
\mathcal{L}_{\text{SM}} = \frac{1}{2} i e \mu_{a} \bar{\psi} \gamma^{a} \overrightarrow{D}_{\mu} \psi - \bar{\psi} m \psi
\]

\[
L_{\text{SM}} = -m \sqrt{-g_{\mu\nu} u^{\mu} u^{\nu}}
\]

\[
\approx \frac{1}{2} m \dot{x}^{2} - U
\]

Lorentz violation

\[
\mathcal{L}_{\text{m matter}} = -\frac{i}{2} e^{\mu}_{a} \bar{\psi} \left( c_{\nu\lambda} e^{\nu a} e^{\lambda b} \gamma^{b} + e_{\nu} e^{\nu a} \ldots \right) \overrightarrow{D}_{\mu} \psi - \bar{\psi} \left( a_{\mu} e^{\mu}_{a} \gamma^{a} \ldots \right) \psi
\]

\[
L_{\text{m matter}} = -m \sqrt{-\left( g_{\mu\nu} + 2 c_{\mu\nu} \right) u^{\mu} u^{\nu} + \left( a_{\text{eff}} \right)_{\mu} u^{\mu}}
\]

\[
\left( a_{\text{eff}} \right)_{\mu} = a_{\mu} - m e_{\mu}
\]

- source-dependent field distortions
- test-particle dependent responses
- additional coefficient structures
nonminimal gravity sector

gravity sector: dynamics of gravitational field alone

conventional pure-gravity

$$\mathcal{L}_{\text{GR}} = R$$

$$\vec{g} \sim \frac{1}{r^2}$$

$$\vec{g} \sim \frac{1}{r^2} + \frac{1}{r^4}$$

$$\mathcal{L}_{\text{nm grav}}^{(6)} = (k_1^{(6)})_{\alpha \beta \gamma \delta \kappa \lambda} \{D^\kappa, D^\lambda\} R^{\alpha \beta \gamma \delta}$$

$$+ (k_2^{(6)})_{\alpha \beta \gamma \delta \kappa \lambda \mu \nu} R^{\kappa \lambda \mu \nu} R^{\alpha \beta \gamma \delta}$$

range (along with time and direction) dependence

→ short range experiments
phenomenology

Lagrange density

testable relations among observables

measurements of/constraints on SME coefficients
outline

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What is the functional form of $\mathcal{L}_m$? What is the functional form of $s_{\mu\nu}$? What is the functional form of $R_{\mu\nu}$?

Lagrange density

testable relations among observables

measurements of constraints on SME coefficients

phenomenology
Q3: functional form of coefficients?

What is typically assumed about the functional form of the coefficients for LV in the flat spacetime SME?

A) They are infinite, maximally destroying Lorentz symmetry.

B) They are obtained separately for each case as a solution to appropriate equations of motion.

C) They are constant.

D) They are zero
A3: functional form of coefficients?

What is typically assumed about the functional form of the coefficients for LV in the flat spacetime SME?

A) They are infinite, maximally destroying Lorentz symmetry.

B) They are obtained separately for each case as a solution to appropriate equations of motion.

C) They are constant.

D) They are zero
why constant coefficients in flat spacetime?

• maintain energy and momentum conservation (cf Kostelecký's Cutlass$^1$)
• could be leading term of a series expansion of some other function
• could arise via explicit Lorentz breaking
  – LV is a predetermined property of the spacetime
• be regarded as the vev of spontaneous breaking
  – LV arises dynamically
  – fluctuations are not considered
  – unspecified dynamics coefficient field → general results

\[ a_\mu = \overline{a}_\mu + \hat{a}_\mu \]

why constant coefficients in flat spacetime?

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Explicit Lorentz-symmetry breaking and/or ignoring the fluctuations is typically in conflict with the Riemann geometry on which our gravity theory is based!

\[ a_\mu = \overline{a}_\mu + \hat{a}_\mu \]
geometric consistency example

- key issue: Bianchi identities
- consider a one coefficient example $\mathcal{L} = R + s^{\mu\nu} R_{\mu\nu}$
- field equations
  \[
  G^{\mu\nu} = (T^{Rs})^{\mu\nu}
  \]

\[
(T^{Rs})^{\mu\nu} = \frac{1}{2} s^{\alpha\beta} R_{\alpha\beta} g^{\mu\nu} + D_\alpha D^{(\mu} s^{\alpha\nu)} - \frac{1}{2} D^2 s^{\mu\nu} - \frac{1}{2} g^{\mu\nu} D_\alpha D_\beta s^{\alpha\beta}
\]

- apply covariant to the LHS
  $D_\mu G^{\mu\nu} = 0$ identity!
- apply covariant to the RHS
  $D_\mu (T^{Rs})^{\mu\nu} = \text{nonzero}$
what went wrong?

• we didn't treat $s_{\mu\nu}$ as dynamical
• if we did, there would be more action

$$\mathcal{L} = \frac{e}{16\pi G_N} (\mathcal{L}_{GR} + \mathcal{L}_{m\text{ grav}} + \mathcal{L}_s)$$

• and hence more Einstein equation

$$G^{\mu\nu} = (T^{Rs})^{\mu\nu} + T_s^{\mu\nu}$$

• and the contribution from the dynamics would be just what's needed to satisfy Bianchi

$$D_\mu((T^{Rs})^{\mu\nu} + T_s^{\mu\nu}) = 0$$

how can we fix it?

• specify $\mathcal{L}_s$ → specific model → loss of generality
• identify the necessary contributions generally from a class of well-behaved models
some simplifying procedures & assumption

• write coefficient fields as vev (coefficient) + fluctuations
  \[ a_\mu = \bar{a}_\mu + \tilde{a}_\mu \]
  \[ s^{\mu\nu} = \bar{s}^{\mu\nu} + \tilde{s}^{\mu\nu} \]
• notation warning: avoid things like \( \tilde{b}^{\mu} \) (why?)

• work to first order in the coefficients

• work at linear order in the fluctuations

• assume the vevs are constant in asymptotically inertial
  Cartesian coordinates
  \[ \partial_\alpha \bar{s}_{\mu\nu} = 0 \]
proceeding generally with $s$

- Bianchi $\rightarrow \partial^\mu (T_s)_{\mu\nu} = -\bar{S}^{\alpha\beta} \partial_\beta R_{\alpha\nu}$

- assume no independently conserved energy momentum and solve for $(T_s)_{\mu\nu}$

- now we know the full Einstein eqn, but it has unknown fluctuations in it

\[
G^{\mu\nu} = (T^R_s)_{\mu\nu} + T^s_{\mu\nu}
\]

\[
(T^R_s)_{\mu\nu} = \frac{1}{2} S^{\alpha\beta} R_{\alpha\beta} g^{\mu\nu} + D_\alpha D^{(\mu} S^{\alpha\nu)} - \frac{1}{2} D^2 s^{\mu\nu} - \frac{1}{2} g^{\mu\nu} D_\alpha D_\beta S^{\alpha\beta}
\]

\[
s^{\mu\nu} = \bar{s}^{\mu\nu} + \bar{\tilde{s}}^{\mu\nu}
\]

- in a specific model $\rightarrow$ solve for $\tilde{s}^{\mu\nu}$ via its equation of motion
- generally $\rightarrow$ write the generic form the solution would take and demand diffeomorphism invariance

\[
\tilde{s} \sim \bar{s} h
\]
nonminimal gift

\[ \mathcal{L}_{\text{nm grav}}^{(6)} = (k_1^{(6)})_{\alpha \beta \gamma \delta \kappa \lambda} \{D^\kappa, D^\lambda\} R^{\alpha \beta \gamma \delta} + (k_2^{(6)})_{\alpha \beta \gamma \delta \kappa \lambda \mu \nu} R^{\kappa \lambda \mu \nu} R^{\alpha \beta \gamma \delta} \]

• for a part of this, the fluctuations are not relevant in the linearized theory
• which part?
proceeding generally with the matter sector

- fluctuation problems in 2 places

\[ L_{m\text{ matter}} = -m \sqrt{-(g_{\mu\nu} u + 2c_{\mu\nu})u^{\mu}u^{\nu} + (a_{\text{eff}})_{\mu}u^{\mu}} \]

- vary with respect to \( x \)

\[ \ddot{x}^{\mu} = -\Gamma^{\mu}_{\alpha\beta}\dot{x}^{\alpha}\dot{x}^{\beta} + \partial^{\mu}\dot{c}_{\alpha\beta}u^{\alpha}u^{\beta} + \ldots \]

- vary with respect to the metric

\[ T^{\mu\nu} = -\int d\tau \frac{mu^{\mu\nu}u^{\nu}\delta^{4}(x-x'(\tau))}{e\sqrt{1-2(c_{\alpha\beta}^{S} + \dot{c}_{\alpha\beta}^{S})u^{\alpha}u^{\beta}}} \]

- if minimal gravity couplings:
  — Bianchi satisfied to post newtonian order 3 w/o fluctuations
  — to find \( \dot{c}_{\mu\nu}^{T} \) to order 3 impose \( dP_{\mu}/dt = 0 \) on system

- if nonminimal couplings:
  — impose diffeomorphism invariance on general \( (\dot{a}_{\text{eff}})^{S}_{\mu} \)
  — use \( dP_{\mu}/dt = 0 \) to get \( h_{\mu\nu} \)
phenomenology

Lagrange density

testable relations among observables

measurements of/
constraints on SME coefficients
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• sources of basic information

• theory to experiment
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  – presentation of tools
\[ g_{00} = -1 + 2U + 3\bar{s}^{00}U + \bar{s}^{jk}U^{jk} - 4\bar{s}^{0j}V^j + O(4), \]  
\[ g_{0j} = -\bar{s}^{0j}U - \bar{s}^{0k}U^{jk} - \frac{7}{2}(1 + \frac{1}{28}\bar{s}^{00})V^j + \frac{3}{4}\bar{s}^{jk}V^k \]
\[ -\frac{1}{2}(1 + \frac{15}{4}\bar{s}^{00})W^j + \frac{5}{4}\bar{s}^{jk}W^k \]
\[ + \frac{9}{4}\bar{s}^{kl}X^{klj} - \frac{15}{8}\bar{s}^{klk}X^{jkl} - \frac{3}{8}\bar{s}^{kl}Y^{klj}, \]  
\[ g_{jk} = \delta^{jk} + (2 - \bar{s}^{00})\delta^{jk}U \]
\[ + (\bar{s}^{lm}\delta^{jk} - \bar{s}^{jl}\delta^{mk} - \bar{s}^{kl}\delta^{jm} + 2\bar{s}^{00}\delta^{jl}\delta^{km})U^{lm}. \]

\[ U = G \int d^3x' \frac{\rho(\tilde{x}', t)}{R}, \]
\[ U^{jk} = G \int d^3x' \frac{\rho(\tilde{x}', t)R^j R^k}{R^3}, \]
\[ V^j = G \int d^3x' \frac{\rho(\tilde{x}', t)\nu^j(\tilde{x}', t)}{R}, \]
current minimal gravity

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{s}^{XY}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>$\bar{s}^{XZ}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>$\bar{s}^{YZ}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>$\frac{\bar{s}^{XX} - \bar{s}^{YY}}{2}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>$\frac{\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}}{2}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>$\bar{s}^{TT}$</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>$\bar{s}^{TX}$</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>$\bar{s}^{TY}$</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>$\bar{s}^{TZ}$</td>
<td>$10^{-8}$</td>
</tr>
</tbody>
</table>
minimal matter sector

\[
(h_a^{(1,1)})_{00} = \frac{2}{m} \left[ 2\alpha(\bar{a}_\text{eff})_0 U + \alpha(\bar{a}_\text{eff})_j V^j - \alpha(\bar{a}_\text{eff})_j W^j \right],
\]

\[
(h_a^{(1,1)})_{0j} = \frac{1}{m} \left[ \alpha(\bar{a}_\text{eff})_j U + \alpha(\bar{a}_\text{eff})_k U^{jk} - \alpha(\bar{a}_\text{eff})_0 V^j - \alpha(\bar{a}_\text{eff})_0 W^j \right],
\]

\[
(h_a^{(1,1)})_{jk} = \frac{2}{m} \left[ -\alpha(\bar{a}_\text{eff})_0 U \delta^{jk} + \alpha(\bar{a}_\text{eff})_0 U^{jk} \right]. \tag{88}
\]

\[
\bar{x}^\mu = -\Gamma_{(0,1)}^{\mu} \alpha_\beta u^\alpha u^\beta
\]

\[
-\Gamma_{(1,1)}^{\mu} \alpha_\beta u^\alpha u^\beta + 2\eta^{\mu\gamma}(\bar{c}^T)_{(\gamma\delta)} \Gamma_{(0,1)}^{\delta} \alpha_\beta u^\alpha u^\beta
\]

\[
+2(\bar{c}^T)_{(\alpha\beta)} \Gamma_{(0,1)}^{\alpha} \gamma_\delta u^\beta u^\gamma u^\delta u^\mu + \partial^\mu(\bar{c}^T)_{\alpha\beta} u^\alpha u^\beta
\]

\[
-2\eta^{\mu\gamma} \partial_\alpha(\bar{c}^T)_{(\gamma\beta)} u^\alpha u^\beta - \partial_\gamma(\bar{c}^T)_{(\alpha\beta)} u^\alpha u^\beta u^\gamma u^\mu
\]

\[
- \frac{1}{m_T} \left[ \partial^\mu(\bar{a}_\text{eff})_{(\alpha} - \eta^{\mu\beta} \partial_\alpha(\bar{a}_\text{eff})_{\beta)} \right] u^\alpha, \tag{78}
\]

\[
(\bar{a}_\text{eff})^{(1,1)}_\mu = \frac{1}{2} \alpha h_{\mu\nu}(\bar{a}_\text{eff}^B)^\nu - \frac{1}{4} \alpha(\bar{a}_\text{eff}^B)_\mu h_{\nu\nu} + \partial_\mu \Psi \tag{87}
\]
minimal matter sector

\[ H^{(1,1)}_a = \tilde{a}_0 - \bar{a}^j h_{j0} + (\tilde{a}_j - \frac{1}{2} \bar{a}_j h_{00} - \frac{1}{2} \bar{a}^k h_{jk}) \gamma^0 \gamma^j \]  

(49)

\[ H^{(1,1)}_{\text{NR, } a_{\text{eff}}} = (\tilde{a}_{\text{eff}})_0 + (\bar{a}_{\text{eff}})_k h^{0k} - \frac{1}{m} (\bar{a}_{\text{eff}})^j h_{jk} p^k \]

\[ + \frac{1}{m} \left( (\tilde{a}_{\text{eff}})_j - \frac{1}{2} (\bar{a}_{\text{eff}})_j h_{00} \right) p^j. \]

(63)
countershaded Lorentz violation

\[ a_\mu = \bar{a}_\mu + \frac{1}{2} \alpha \bar{a}_\nu h_{\mu\nu} - \frac{1}{4} \alpha \bar{a}_\mu h^{\nu}\_\nu \]

- \( \bar{a}_\mu \) for matter is unobservable in flat-spacetime tests.
- Observable \( \bar{a}_\mu \) effects are suppressed by the gravitational field.
- \( \bar{a}_\mu \) could be large (~1eV) relative to existing matter-sector bounds \( b_\mu < 10^{-30} \)
current \((\bar{a}_{\text{eff}})_\mu\) limits

- Data Tables: Kostelecký & Russell, arXiv:0801.0287v7

- gravity summary

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Electron</th>
<th>Proton</th>
<th>Neutron</th>
</tr>
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<tbody>
<tr>
<td>(\alpha(\bar{a}_{\text{eff}})_T)</td>
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</tr>
<tr>
<td>(\alpha(\bar{a}_{\text{eff}})_X)</td>
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<td>(10^{-5}) GeV</td>
</tr>
<tr>
<td>(\alpha(\bar{a}_{\text{eff}})_Y)</td>
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<td>(10^{-6}) GeV</td>
<td>(10^{-4}) GeV</td>
</tr>
<tr>
<td>(\alpha(\bar{a}_{\text{eff}})_Z)</td>
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</tbody>
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“...the displayed sensitivity for each coefficient assumes for definiteness that no other coefficient contributes.”

- 12 independent coefficients
- constraints: 2 at \(10^{-11}\) GeV
  
  2 at \(10^{-6}\) GeV

  4 at \(10^{-1}\) GeV

  as summarized in JT arXiv:1308.1171

- 4 unconstrained combinations require gravitational experiments with charged matter to separate
current \((\bar{a}_{\text{eff}})_{\mu}\) limits

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“...the displayed sensitivity for each coefficient assumes for definiteness that no other coefficient contributes.”

• 12 independent coefficients
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• 4 unconstrained combinations require gravitational experiments with charged matter to separate
current $\tilde{c}_{\mu\nu}$ limits

- Data Tables: Kostelecký & Russell, arXiv:0801.0287v7
- limits likely to be improved via gravity experiments

<table>
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- most gravitation experiments with ordinary matter are sensitive to various combination of many of the above coefficients
nonminimal gravity sector

\[ U(r, T) = \frac{G_N M}{r} \left( 1 + \frac{k(\hat{r}, T)}{r^2} \right) \]  \quad (12)

arXive:1412.8363, PRD '15

Search for Lorentz violation in short-range gravity

J.C. Long and V. Alan Kostelecký

Physics Department, Indiana University, Bloomington, IN 47405, U.S.A.

Abstract

Search for Lorentz invariance violation through tests of the gravitational inverse square law at short-ranges

Cheng-Gang Shao\(^1\), Yu-Jie Tan\(^1\), Wen-Hai Tan\(^1\), Shan-Qing Yang\(^1\), Jun Luo\(^1\),\(^*\) and Michael Edmund Tobar\(^2\)\(^\dagger\)

\(^1\)Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, People’s Republic of China

\(^2\)School of Physics, University of Western Australia, Crawley, WA 6009, Australia

(Dated: April 14, 2015)

A search for sidereal variations in the non-Newtonian force between two tungsten plates separated at millimeter ranges sets experimental limits on Lorentz invariance violation involving quadratic couplings of Riemann curvature. We show that the Lorentz invariance violation force between two finite flat plates is dominated by the edge effects, which includes a suppression effect leading to lower limits than previous rough estimates. From this search, we determine the current best constraints of the Lorentz invariance violating coefficients at a level of \(10^{-8} \text{ m}^2\).

PACS numbers: 04.80.-y, 04.25.Nx, 04.80.Cc
experiments

• lab tests (Earth source)
  - gravimeter
  - Weak Equivalence Principle (WEP)
• space-based WEP
• exotic tests
  - charged matter
  - antimatter
  - higher-generation matter
• solar-system tests
  - laser ranging
  - perihelion precession
• pulsar tests
• light-travel tests
  - time delay
  - Doppler shift
  - red shift
• clock tests
  - null redshift
  - comagnetometers
• short range tests
• gravity probe B
• ...

Q4: experiments?

Experimentalist U. Falwell has a weak equivalence principle experiment in her lab. Which SME coefficients is she likely able to place interesting constraints on (minimal gravity sector, nonminimal gravity sector, matter sector)?

In which paper is she likely to find help with the analysis?

What modifications might she make to her experiment to see other kinds of SME coefficients?
A4: experiment

answer next time
summary

• sources of basic information
• theory to experiment
  – intro to GR
  – Lagrangian expansion in gravity
  – addressing the fluctuations
  – presentation of tools