Tests at Colliders

Summer School on the SME
June 17, 2018
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I. Top quark production and decay
II. Neutral meson oscillations
1. In principle, one has access (statistically) to a large number of frames as the produced particles and their decay products can be moving relativistically in all directions.

2. Precision measurements – interferometric processes like meson oscillations (colliders and meson factories)

3. Directly probe Lorentz violation in sectors of the Standard Model otherwise unavailable – top quark, Higgs boson, etc. (still have important indirect effects in loop diagrams)
Standard Model Extension

1. Add all possible Lorentz-violating terms to the Standard Model Lagrangian that are gauge-invariant, etc.

2. The resulting theory is to be viewed as an effective field theory presumably up to the Planck scale where a more fundamental theory takes over curing any problems with microcausality, etc.

3. Phenomenological $\rightarrow$ consequences for experiment.

$\rightarrow$ Part 2: Lorentz-violating supersymmetric model: add all possible supersymmetric but Lorentz-violating terms to a supersymmetric field theory.
Standard Model Extension

Add terms to the Standard Model that are gauge invariant, but whose coefficients do not transform according under Lorentz transformations.

\[ \mathcal{L}_a' \equiv a_\mu \overline{\psi} \gamma^\mu \psi \quad , \quad \mathcal{L}_b' \equiv b_\mu \overline{\psi} \gamma_5 \gamma^\mu \psi \]

Field redefinitions:
\[ \chi = \exp(ia \cdot x) \psi \]

Modified fermion propagator:
\[ S_F(x - x') = \int_{C_F} \frac{d^4p}{(2\pi)^4} e^{-ip \cdot (x - x')} \frac{1}{p_\mu \gamma^\mu - a_\mu \gamma^\mu - b_\mu \gamma_5 \gamma^\mu - m} \]

Usual methods to determine Feynman rules can be used to determine the propagator.
Lorentz Violation and the Top-Quark

- The top quark was discovered in 1995 at Fermilab. Its most important properties:
  - mass
  - production cross sections (pair production and single-top)
  - event kinematics
  - overall consistency with the Standard Model

- The top quark is the only quark with mass at the electroweak scale – order one Yukawa coupling.

- The only quark which decays before it hadronizes – allows certain tests largely independent of long distance physics

- Can presently only be studied directly at hadron colliders – LHC, Fermilab Tevatron

- New physics may appear in the top quark interactions – production and decay
Lorentz violation in SM Quark Sector

Dimensionless, CPT-even

\[ \mathcal{L}_{\text{quark}}^{\text{CPT-even}} = \frac{1}{2} i (c_Q)_{\mu \nu AB} \overline{Q}_A \gamma^\mu \not{D}^\nu Q_B + \frac{1}{2} i (c_U)_{\mu \nu AB} \overline{U}_A \gamma^\mu \not{D}^\nu U_B + \frac{1}{2} i (c_D)_{\mu \nu AB} \overline{D}_A \gamma^\mu \not{D}^\nu D_B, \]

Dimensionful, CPT-odd

\[ \mathcal{L}_{\text{quark}}^{\text{CPT-odd}} = -(a_Q)_{\mu \nu AB} \overline{Q}_A \gamma^\mu Q_B - (a_U)_{\mu \nu AB} \overline{U}_A \gamma^\mu U_B - (a_D)_{\mu \nu AB} \overline{D}_A \gamma^\mu D_B. \]

Dimensionless, CPT-even

\[ \mathcal{L}_{\text{Yukawa}}^{\text{CPT-even}} = -\frac{1}{2} \left| (H_U)_{\mu \nu AB} \overline{Q}_A \phi^c \sigma^{\mu \nu} U_B \right| (H_D)_{\mu \nu AB} \overline{Q}_A \phi \sigma^{\mu \nu} D_B \right| + \text{h.c.} \]

A, B are generational indices = 1, 2, 3

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<th>Helicity</th>
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<th>( T_3 )</th>
<th>( Y_{\text{UB}} )</th>
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Top-Quark Lorentz violation

SME contains various Lorentz violating coefficients. These can be inserted, according to Feynman rules, into the relevant diagrams.

Many possible ways LV could contribute (light quarks and leptons, gluon, electroweak interactions). Consider only those which involve the top quark.

Berger, Kostelecky and Liu
Top-Quark Production

Top quark production is dominated by $qq\bar{q}$ annihilation at the Tevatron (ppbar collider).

Gluon fusion is dominant at the LHC because of the higher energy and because it is a pp collider.
Third generation coefficients

\[
\begin{align*}
(a_L)_\mu &= (a_Q)_{\mu 33}, \\
(a_R)_\mu &= (a_U)_{\mu 33}, \\
(c_L)_{\mu \nu} &= (c_Q)_{\mu \nu 33}, \\
(c_R)_{\mu \nu} &= (c_U)_{\mu \nu 33}, \\
H'_{\mu \nu} &= \langle \phi \rangle (H_U)_{\mu \nu 33}, \\
\widetilde{H}'_{\mu \nu} &= \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} H'_{\rho \sigma},
\end{align*}
\]

SME has CPT-odd and CPT-even coefficients of Lorentz violation (ignore flavor violation – mixing between generations)

**CPT odd**

\[
\begin{align*}
a_\mu &= \frac{1}{2} [(a_L)_\mu + (a_R)_\mu] \\
b_\mu &= \frac{1}{2} [(a_L)_\mu - (a_R)_\mu]
\end{align*}
\]

**CPT even**

\[
\begin{align*}
c_{\mu \nu} &= \frac{1}{2} [(c_L)_{\mu \nu} + (c_R)_{\mu \nu}] \\
d_{\mu \nu} &= \frac{1}{2} [(c_L)_{\mu \nu} - (c_R)_{\mu \nu}] \\
H_{\mu \nu} &= \text{Re} \ H'_{\mu \nu} - \text{Im} \widetilde{H}'_{\mu \nu}
\end{align*}
\]
Vertex Correction

\[ \Gamma'_1 \equiv c^{\mu\nu} \gamma_\mu + d^{\mu\nu} \gamma_5 \gamma_\mu \]

vector coupling \( c_{\mu\nu} \)

left-handed coupling \( (c_L)_{\mu\nu} \)

\[ c_{\mu\nu} = \frac{1}{2}[(c_L)_{\mu\nu} + (c_R)_{\mu\nu}] \]

\[ d_{\mu\nu} = \frac{1}{2}[(c_L)_{\mu\nu} - (c_R)_{\mu\nu}] \]
Factorization

Narrow width approximation – spin sums performed for production and decay process
Lorentz violation in top production (qqbar)

Production + decay matrix elements (LV coefficients contracted with physical momenta)

\[|\mathcal{M}|^2 = P \bar{F} F + (\delta P_p) \bar{F} F + (\delta P_v) F \bar{F} + P(\delta F) \bar{F} + PF(\delta F).\]

\[\delta P_p = \frac{g_s^4}{18 E^4} c_{\mu \nu} \left[ (p_q \cdot p_t) (p^\mu_q p^\nu_t) + (p_q \cdot p_{\bar{t}}) (p^\mu_t p^\nu_q) + (p_{\bar{q}} \cdot p_t) (p^\mu_t p^\nu_{\bar{q}}) + (p_{\bar{q}} \cdot p_{\bar{t}}) (p^\mu_{\bar{q}} p^\nu_{\bar{t}}) \right].\]

\[\delta P_v = \frac{g_s^4}{18 E^4} c_{\mu \nu} \left[ - (p_q \cdot p_{\bar{q}}) (p^\mu_t p^\nu_{\bar{t}} + p^\mu_{\bar{t}} p^\nu_t) - (p_t \cdot p_{\bar{t}} + m_t^2) (p^\mu_q p^\nu_{\bar{q}} + p^\mu_{\bar{q}} p^\nu_q) + (p_q \cdot p_t) p^\mu_q p^\nu_{\bar{t}} + (p_{\bar{q}} \cdot p_{\bar{t}}) p^\mu_{\bar{q}} p^\nu_t + (p_{\bar{q}} \cdot p_t) p^\mu_{\bar{q}} p^\nu_{\bar{t}} + (p_q \cdot p_{\bar{t}}) p^\mu_q p^\nu_{\bar{t}} \right].\]
Lorentz violation in top decay

Decay in terms of four momenta

\[
\delta F = 2g_W^4 \left[ \frac{1}{(m_t^2 - M_t^2)^2 + (M_t \Gamma_t)^2} \right] \left[ \frac{1}{(m_{t\nu}^2 - M_{t\nu}^2)^2 + (M_{t\nu} \Gamma_{t\nu})^2} \right] (c_L)_{\mu\nu} \\
\times \left[ (p_b \cdot p_t)(p_\nu^\mu p_\nu^\nu + p_{\ell}^\mu p_{\ell}^\nu) + (p_b \cdot p_{\ell})(p_\nu^\mu p_{\ell}^\nu + p_{\ell}^\mu p_{\ell}^\nu) - (p_b \cdot p_{\nu})(p_b^\mu p_{\nu}^\nu + p_b^\mu p_{\nu}^\nu) - (p_t \cdot p_{\nu})(p_b^\mu p_{\nu}^\nu + p_b^\mu p_{\nu}^\nu) \\
+ (p_t \cdot p_{\ell})(p_b^\mu p_{\nu}^\nu + p_b^\mu p_{\nu}^\nu) + (p_{\nu} \cdot p_{\ell})(p_b^\mu p_{\nu}^\nu + p_b^\mu p_{\nu}^\nu) \right]
\]

\[
\delta F = 2g_W^4 \left[ \frac{1}{(m_t^2 - M_t^2)^2 + (M_t \Gamma_t)^2} \right] \left[ \frac{1}{(m_{t\nu}^2 - M_{t\nu}^2)^2 + (M_{t\nu} \Gamma_{t\nu})^2} \right] (c_L)_{\mu\nu} \\
\times \left[ (p_\ell \cdot p_{\bar{b}})(p_{\bar{q}}^\mu p_{\bar{q}}^\nu + p_{\bar{q}}^\mu p_{\bar{q}}^\nu) + (p_\ell \cdot p_{\bar{q}})(p_{\bar{q}}^\mu p_{\bar{q}}^\nu + p_{\bar{q}}^\mu p_{\bar{q}}^\nu) - (p_{\bar{b}} \cdot p_{\bar{q}})(p_{\bar{b}}^\mu p_{\bar{q}}^\nu + p_{\bar{b}}^\mu p_{\bar{q}}^\nu) - (p_{\bar{b}} \cdot p_{\bar{q}})(p_{\bar{b}}^\mu p_{\bar{q}}^\nu + p_{\bar{q}}^\mu p_{\bar{q}}^\nu) \\
+ (p_{\bar{b}} \cdot p_{\bar{q}})(p_{\bar{b}}^\mu p_{\bar{q}}^\nu + p_{\bar{q}}^\mu p_{\bar{q}}^\nu) + (p_{\bar{q}} \cdot p_{\bar{q}})(p_{\bar{b}}^\mu p_{\bar{q}}^\nu + p_{\bar{b}}^\mu p_{\bar{q}}^\nu) \right].
\]
D0 Search at Tevatron

- 5.3 fb\(^{-1}\) data from Aug. 2002 to Jun. 2009
- qqbar production dominant
- Top quarks (and antiquarks) decaying into Wb with one W decaying leptonically (l=e,µ) and one W decaying hadronically Leptonic decays are cleaner but hadronic decays occur more frequently
- Selection criteria includes a b-tag
- Sidereal (23 hr 56 min 4.1s) or twice-sidereal dependence
  Twice-sidereal signals can appear because c-coefficient is rank 2.

\[
\sigma(t) \approx \sigma_{\text{ave}} [1 + f_{\text{SME}}(t)]
\]
\[
f_{\text{SME}}(t) = [(c_Q)_{\mu\nu} + (c_U)_{\mu\nu}] R^\mu_{\alpha}(t) R^\nu_{\beta}(t) A^\alpha_{\beta}\]
\[
+ (c_Q)_{\mu\nu} R^\mu_{\alpha}(t) R^\nu_{\beta}(t) A^\alpha_{\beta}.
\]
Sidereal dependence

![Graphs showing sidereal dependence](image)

**FIG. 1:** The dependence of $R$, as defined in Eq. (6), on sidereal phase for (a) $e^+\rightarrow3$-jets $t\bar{t}$ candidates, and (b) $\mu^+\rightarrow3$-jets $t\bar{t}$ candidates.

$$R_i \equiv \frac{1}{f_S} \left( \frac{N_i}{N_{tot}} \frac{\mathcal{L}_i}{\mathcal{L}_{int}} - 1 \right)$$
Limits on Lorentz Violation in the Top Sector

| TABLE III: Limits on SME coefficients at the 95% C.L., assuming \((c_U)_{\mu\nu} = 0\). |
|--------------------|-----------------|-----------------|
| Coefficient       | Value ± Stat. ± Sys. | 95% C.L. Interval |
| \((c_Q)_{XX33}\)  | -0.12 ± 0.11 ± 0.02 | [-0.34, +0.11]   |
| \((c_Q)_{YY33}\)  | 0.12 ± 0.11 ± 0.02  | [-0.11, +0.34]    |
| \((c_Q)_{XY33}\)  | -0.04 ± 0.11 ± 0.01  | [-0.26, +0.18]    |
| \((c_Q)_{XZ33}\)  | 0.15 ± 0.08 ± 0.02    | [-0.01, +0.31]     |
| \((c_Q)_{YZ33}\)  | -0.03 ± 0.08 ± 0.01   | [-0.19, +0.12]     |

| TABLE IV: Limits on SME coefficients at the 95% C.L., assuming \((c_Q)_{\mu\nu} = 0\). |
|--------------------|-----------------|-----------------|
| Coefficient       | Value ± Stat. ± Sys. | 95% C.L. Interval |
| \((c_U)_{XX33}\)  | 0.10 ± 0.09 ± 0.02    | [-0.08, +0.27]    |
| \((c_U)_{YY33}\)  | -0.10 ± 0.09 ± 0.02    | [-0.27, +0.08]    |
| \((c_U)_{XY33}\)  | 0.04 ± 0.09 ± 0.01     | [-0.14, +0.22]    |
| \((c_U)_{XZ33}\)  | -0.14 ± 0.07 ± 0.02    | [-0.28, +0.01]     |
| \((c_U)_{YZ33}\)  | 0.01 ± 0.07 ± < 0.01   | [-0.13, +0.14]     |

| TABLE V: Limits on SME coefficients at the 95% C.L., assuming \(c_{\mu\nu} = 0\). |
|--------------------|-----------------|-----------------|
| Coefficient       | Value ± Stat. ± Sys. | 95% C.L. Interval |
| \(d_{X}\)        | -0.11 ± 0.10 ± 0.02  | [-0.31, +0.09]   |
| \(d_{Y}\)        | 0.11 ± 0.10 ± 0.02    | [-0.09, +0.31]    |
| \(d_{XY}\)       | -0.04 ± 0.10 ± 0.01    | [-0.24, +0.16]    |
| \(d_{XZ}\)       | 0.14 ± 0.07 ± 0.02     | [-0.01, +0.29]    |
| \(d_{YZ}\)       | -0.02 ± 0.07 ± < 0.01  | [-0.16, +0.13]    |
Top production at LHC (gluon fusion)

- Nonabelian vertex needs to be carefully handled to eliminate the unphysical longitudinal polarizations of the gluons
- Fadeev-Popov ghosts in the squared matrix element (nonAbelian SU(3) for gluons)
- Interference

\[
\begin{align*}
\sum |M_{ss}|^2 &= \frac{3g_s^4}{4} \frac{(t-m_t^2)(u-m_t^2)}{s^2}, \\
\sum |M_{tt}|^2 &= \frac{g_s^4}{6} \frac{(t-m_t^2)(u-m_t^2) - 2m_t^2(t+m_t^2)}{(t-m_t^2)^2}, \\
\sum |M_{uu}|^2 &= \frac{g_s^4}{6} \frac{(u-m_t^2)(t-m_t^2) - 2m_t^2(u+m_t^2)}{(u-m_t^2)^2}, \\
\sum |M_{st}|^2 &= \frac{3g_s^4}{8} \frac{(t-m_t^2)(u-m_t^2) + m_t^2(u-t)}{s(t-m_t^2)}, \\
\sum |M_{su}|^2 &= \frac{3g_s^4}{8} \frac{(u-m_t^2)(t-m_t^2) + m_t^2(t-u)}{s(u-m_t^2)}, \\
\sum |M_{tu}|^2 &= \frac{g_s^4}{24} \frac{m_t^2(s-4m_t^2)}{(t-m_t^2)(u-m_t^2)}. 
\end{align*}
\]

3 diagrams:
- s-channel
- t-channel
- u-channel
Lorentz violation in top production (gluon fusion)

- c-coefficient
- Modified spin-sum for the external t and tbar
- Feynman diagram insertions \( \times \) in the internal top quark lines and vertices
- Cross section is symmetric in c-when all diagrams are included (there is a field redefinition argument).

Partial results:

\[
\delta_v \sum |M_{ss}|^2 = \frac{3g_s^4 c_{\mu \nu}}{4s_s^2} \left[ t(p_t^\nu p_t^\nu + p_{t1}^\mu p_{t2}^\nu - p_{t1}^\mu p_{t2}^\nu - p_{t2}^\mu p_{t1}^\nu) + u(p_t^\mu p_t^\nu + p_u^\mu p_u^\nu - p_{t1}^\mu p_{t2}^\nu - p_{t2}^\mu p_{t1}^\nu) - m_t^2((p_1 - p_2)^\mu (p_1 - p_2)^\nu) \right],
\]

\[
\delta_v \sum |M_{tt}|^2 = \frac{g_s^4 c_{\mu \nu}}{3(t - m_t^2)} \left[ (t - 3m_t^2)(p_t^\mu p_t^\nu + p_{t1}^\mu p_{t1}^\nu + p_{t2}^\mu p_{t2}^\nu + p_{t2}^\mu p_{t1}^\nu) + 4m_t^2(p_{t1}^\mu p_{t1}^\nu + p_{t2}^\mu p_{t2}^\nu) \right],
\]

\[
\delta_v \sum |M_{uu}|^2 = \frac{g_s^4 c_{\mu \nu}}{3(u - m_t^2)} \left[ (u - 3m_t^2)(p_u^\mu p_u^\nu + p_{t1}^\mu p_{t1}^\nu + p_{t2}^\mu p_{t2}^\nu + p_{t2}^\mu p_{t1}^\nu) + 4m_t^2(p_{t1}^\mu p_{t1}^\nu + p_{t2}^\mu p_{t2}^\nu) \right],
\]
Single Top Quark Production

- Electroweak process suppressed relative to pair production via QCD
- Single top quark production (s & t-channels) seen at Tevatron, also tW-mode at LHC
- Sensitive to CPT violation
- Various production modes have different dependences on the SME parameters
Single Top Quark Production

s-channel

\[ q, \overline{q} \rightarrow W^+ \rightarrow t, \overline{b} \]

\[ b \rightarrow t, W^+ \rightarrow q(q') \]

t-channel

\[ b \rightarrow W^- \rightarrow t \]

tW-mode

\[ b \rightarrow W^- \rightarrow t \]

<table>
<thead>
<tr>
<th>Cross section</th>
<th>( t ) channel</th>
<th>( s ) channel</th>
<th>( tW ) mode</th>
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<tr>
<td>( \sigma_{Tevatron}^t )</td>
<td>1.15 ± 0.07 pb</td>
<td>0.54 ± 0.04 pb</td>
<td>0.14 ± 0.03 pb</td>
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<tr>
<td>( \sigma_{LHC}^t )</td>
<td>150 ± 6 pb</td>
<td>7.8 ± 0.7 pb</td>
<td>44 ± 5 pb</td>
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<tr>
<td>( \sigma_{LHC}^\overline{t} )</td>
<td>92 ± 4 pb</td>
<td>4.3 ± 0.3 pb</td>
<td>44 ± 5 pb</td>
</tr>
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</table>

same
Single Top Quark Production

The figure shows the cross-section $\sigma_{t\bar{t}}$ as a function of the square root of the center-of-mass energy $\sqrt{s}$, in [TeV]. The graph compares the theoretical predictions at NNLO with the experimental data from various collaborations, including CDF, D0, CMS, ATLAS, and their combined datasets. The data points are compared with the theoretical predictions for s-channel, t-channel, and Wt processes.
Spin Sum

\[ u^{(\alpha)}(\vec{p}) = N_u^{(\alpha)} \left( \phi^{(\alpha)} X_u^{(\alpha)} \phi^{(\alpha)} \right) \]

\[ v^{(\alpha)}(\vec{p}) = N_v^{(\alpha)} \left( \frac{X_v^{(\alpha)} \chi^{(\alpha)}}{\chi^{(\alpha)}} \right) \]

\[ \sum_{\alpha=1,2} u^{(\alpha)} \bar{u}^{(\alpha)} = \begin{pmatrix} 2m_t & b_0' - \vec{b}' \cdot \vec{\sigma} \\ -b_0' + \vec{b}' \cdot \vec{\sigma} & 0 \end{pmatrix} \]

\[ \sum_{\alpha=1,2} v^{(\alpha)} \bar{v}^{(\alpha)} = \begin{pmatrix} 0 & -b_0' + \vec{b}' \cdot \vec{\sigma} \\ b_0' - \vec{b}' \cdot \vec{\sigma} & -2m_t \end{pmatrix} \]

\[ [i\partial + (1 - \gamma_5)\not{b} - m]\psi = 0 \]

\[ \sum u\bar{u} = \not{\psi} + m + \not{b} - \frac{p \cdot b}{m^2} (1 + \gamma_5)\not{\psi} , \]

\[ \sum v\bar{v} = \not{\psi} - m - \not{b} + \frac{p \cdot b}{m^2} (1 + \gamma_5)\not{\psi} . \]

- calculate in the rest frame and (observer) boost to a moving frame
- spin sums are projection operators positive (u) and negative energy (v) solutions: in the rest frame they now have a mixture of the ‘small’ component
- in general there are four nondegenerate modes
Single Top s-Channel

\[ \sum |M_1|^2 = \frac{g_w^4}{4(s - M^2)^2} u(u - m^2 - 2b \cdot p_1) \]

1, 2, 3, 4 = \overline{q'}, q, \overline{b}, t.

-spin sum modifies the cross section in a simple way
-sidereal dependance on the CPT-violating coefficient
Single Top t-Channel

\[ \sum |M_2|^2 = \frac{g_w^4}{4(t - M^2)^2} s(s - m^2 + 2b \cdot p_3) \quad 1, 2, 3, 4 = q, b, q', t. \]

\[ \sum |M_2'|^2 = \frac{g_w^4}{4(t - M^2)^2} u(u - m^2 - 2b \cdot p_1) \quad 1, 2, 3, 4 = \bar{q}, b, \bar{q}', t \]

-again spin sum modifies the cross section in a simple way
-t-channel production has the largest cross section of the three single top production modes
Single Top $tW$ Mode

- Spin sum for top quark in the final state
- Propagator insertion on the top quark in the second diagram
- $tW$ mode has the property that top quark and antitop quark production is the same via CPT at the LHC (equal $b$ and antib in the proton)
Neutral Meson Oscillations

• Oscillation phenomena – particles with the same quantum numbers can convert into each other (neutral mesons, neutrinos, etc.)
• Quantum mechanical mixing phenomena
• Flavor eigenstates (production and decay) are not the same as mass eigenstates (propagation)

• The neutral mesons (of a given flavor) can evolve into the antimeson via the weak interactions

\[ P = K, D, B_d, B_s \]

• Flavor-changing neutral currents (FCNCs) are absent in the Standard Model (at tree level)
Corrections to the Meson Propagator

Sum the contributions to meson mixing

\[ \frac{i}{p^2 - m^2 \delta_{ij} - \Pi_{ij}(p^2)} \]

Expand the denominator

\[ p^2 - m^2 \delta_{ij} - \Pi_{ij}(p^2) \approx p^2 - \left( m\delta_{ij} + \frac{\Pi_{ij}}{2m} \right)^2 \]

Decompose into Hermitian and anti-Hermitian parts

\[ \Pi = \frac{(\Pi + \Pi^\dagger)}{2} - i \left( \frac{i(\Pi - \Pi^\dagger)}{2} \right) = \Pi^{(+)} - i\Pi^{(-)} \]

Divide by 2m

\[ \Lambda = \Delta m - \frac{i\Gamma}{2} = \frac{\Pi^{(+)}}{2m} - i\frac{\Pi^{(-)}}{2m} \]
\[
\Delta m - \frac{i\Gamma}{2} = \begin{pmatrix}
\Delta m_{11} - \frac{i\Gamma_{11}}{2} & \Delta m_{12} - \frac{i\Gamma_{12}}{2} \\
\Delta m_{21} - \frac{i\Gamma_{21}}{2} & \Delta m_{22} - \frac{i\Gamma_{22}}{2}
\end{pmatrix}
\]

CPT: \(\Pi_{ij} = \Pi_{\text{CPT}j,\text{CPT}i}\)

For \(i = j = 1\) (\(|P^0\rangle\)) and \(i = j = 2\) (\(|\bar{P}^0\rangle\)), this gives \(\Pi_{11} = \Pi_{22}\).

\[
\Lambda = \text{CPT} \left(\begin{pmatrix}
\Delta m_{11} - \frac{i\Gamma_{11}}{2} & \Delta m_{12} - \frac{i\Gamma_{12}}{2} \\
\Delta m_{12}^\ast - \frac{i\Gamma_{12}}{2} & \Delta m_{11} - \frac{i\Gamma_{11}}{2}
\end{pmatrix}\right)
\]

T: \(\Pi_{12} = \Pi_{21}\)

\[
\Lambda = \text{T} \left(\begin{pmatrix}
\Delta m_{11} - \frac{i\Gamma_{11}}{2} & \Delta m_{12} - \frac{i\Gamma_{12}}{2} \\
\Delta m_{12} - \frac{i\Gamma_{12}}{2} & \Delta m_{22} - \frac{i\Gamma_{22}}{2}
\end{pmatrix}\right)
\]
CPT and T (symmetric with equal diagonal):

\[ \Lambda = \text{CPT} + T \begin{pmatrix} \Delta m_{11} - \frac{i \Gamma_{11}}{2} & \Delta m_{12} - \frac{i \Gamma_{12}}{2} \\ \Delta m_{12} - \frac{i \Gamma_{12}}{2} & \Delta m_{11} - \frac{i \Gamma_{11}}{2} \end{pmatrix} \]

In this simple (but not physically realistic case) the eigenstates are

\[ |P_{1,2}\rangle = \frac{1}{\sqrt{2}} \left( |P^0\rangle \pm |\bar{P}^0\rangle \right) \]

Deviation from this form is evidence for T or CPT violation

Consider first T violation

\[ |P_1\rangle = p|P^0\rangle + q|\bar{P}^0\rangle \]
\[ |P_2\rangle = p|P^0\rangle - q|\bar{P}^0\rangle \]

\[ |p|^2 + |q|^2 = 1 \quad |p|^2 \neq |q|^2 \]

\[ |P_1(t)\rangle = e^{-i m_{1t} t - \Gamma_{1t} t/2} |P_1(0)\rangle \]
\[ |P_2(t)\rangle = e^{-i m_{2t} t - \Gamma_{2t} t/2} |P_2(0)\rangle \]
More generally

$$|P_1\rangle = u|P^0\rangle + v|\bar{P}^0\rangle$$

$$|P_2\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$

$$\frac{1 - \epsilon_1}{1 + \epsilon_1} = -\frac{v}{u}$$

$$\frac{1 - \epsilon_2}{1 + \epsilon_2} = \frac{q}{p}$$

One can show

$$|P_1\rangle = [2(1 + |\epsilon_1|^2)]^{-1/2}[(1 + \epsilon_1)|P^0\rangle - (1 - \epsilon_1)|\bar{P}^0\rangle]$$

$$|P_2\rangle = [2(1 + |\epsilon_2|^2)]^{-1/2}[(1 + \epsilon_2)|P^0\rangle + (1 - \epsilon_2)|\bar{P}^0\rangle]$$

CPT invariance:

$$\epsilon_1 = \epsilon_2 = \epsilon \quad , \quad \delta = 0$$

T invariance:

$$\epsilon = \frac{(\epsilon_1 + \epsilon_2)}{2} \quad , \quad \delta = \frac{(\epsilon_1 - \epsilon_2)}{2}$$

$$\delta = \epsilon_1 = -\epsilon_2 \quad , \quad \epsilon = 0$$
Summary of Neutral Meson Oscillations Theory

- Candidate systems: \( P = K, D, B_d, B_s \)

\[ i \frac{\partial \Psi}{\partial t} = \Lambda \Psi \]

- Effective 2 \( \times \) 2 Hamiltonian \( \Lambda = M - \frac{1}{2} i \Gamma \) for time evolution of the meson state

  Indirect CPT violation if \( \Delta \Lambda = \Lambda_{11} - \Lambda_{22} \neq 0 \)

- Eigenstates = physical states with definite masses, decay rates

\[ \left| P_{a,b} \right\rangle \propto (1 + \epsilon_P \pm \delta_P \left| P^0 \right\rangle \pm (1 - \epsilon_P \pm \delta_P \left| P^0 \right\rangle \]

— CP, T violation with CPT preserved: \( \epsilon_P \) calculable in usual Standard Model

— CP, CPT violation with T preserved: \( \delta_P \propto \Delta \Lambda \) zero in usual standard model, calculable in Standard Model extension
• In the early days of particle physics, we only knew of the $u,d,s$ quarks.
• Smallness of the effect required a $c$ quark to cancel to contribution in the loop from the up quark.

CKM factors:
- Up quark: $V_{us} V_{ud}^* = \sin \theta_C \cos \theta_C$
- Charm quark: $V_{cs} V_{cd}^* = - \sin \theta_C \cos \theta_C$
Calculation in the SME

Corrections are proportional to the $a$ coefficients

\[-a^q_{\mu} \bar{q} \gamma^{\mu} q\]

\[\Lambda \approx \beta^{\mu} \Delta a_{\mu}\]

\[\Delta a_{\mu} = r_{q_1} a^{q_1}_{\mu} - r_{q_2} a^{q_2}_{\mu}\]

This effective parameter is a combination of the parameters for the valence quarks from the SME and order one factors $r$ which account for how the quarks are bound into mesons.

The physical effects will depend on the meson momentum spectrum and angular distribution.
Sun-centered frame

LV coefficients are defined in a sun-centered celestial equatorial frame

Lab frame: $x$ (south), $y$ (east), $z$ (vertically up)
Sun centered frame: $X, Y, Z$

Earth’s sidereal angular frequency $\Omega = 2\pi/(23\text{hr} 56\text{min} 4.1\text{s})$
Colatitude of the lab: $\chi$

Most coefficients would produce sidereal signals, but the symmetric coefficients of $c_{\mu\nu}$ produce sidereal and twice-sidereal ones
Time variation of CPT observables

- Use nonrotating frame
  - Polar coordinates along beam
  \[ \beta(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \]
  \[ \beta(p) = \frac{|p|}{m\gamma(p)} \]
  \[ \gamma(p) = \sqrt{1 + \frac{|p|^2}{m_K^2}} \]
- Explicit boost and time dependence:

\[
\delta_k(p, t) = \frac{i \sin \phi}{\Delta m} \gamma(p) \times [\Delta a_0 + \beta(p) \Delta a_Z (\cos \theta \cos \chi - \sin \theta \cos \phi \sin \chi) + \beta(p) (-\Delta a_X \sin \theta \sin \phi + \Delta a_Y (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi)) \sin \Omega t + \beta(p) (\Delta a_X (\cos \theta \sin \chi + \sin \theta \cos \phi \cos \chi) + \Delta a_Y \sin \theta \sin \phi) \cos \Omega t] \]

Kostelecky, PRD61, 016002 (1999)
## Data Tables

### Table XV. Meson sector

<table>
<thead>
<tr>
<th>Combination</th>
<th>Result</th>
<th>System</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta a_1^K$</td>
<td>$(-6.3 \pm 6.0) \times 10^{-18}$ GeV</td>
<td>$K$ oscillations [117]</td>
<td></td>
</tr>
<tr>
<td>$\Delta a_2^K$</td>
<td>$(2.8 \pm 5.9) \times 10^{-18}$ GeV</td>
<td>&quot;</td>
<td>[117]</td>
</tr>
<tr>
<td>$\Delta a_1^\pi$</td>
<td>$(2.4 \pm 9.7) \times 10^{-18}$ GeV</td>
<td>&quot;</td>
<td>[117]</td>
</tr>
<tr>
<td>$\Delta a_0^K$</td>
<td>$(0.4 \pm 1.8) \times 10^{-17}$ GeV</td>
<td>&quot;</td>
<td>[118], [117]</td>
</tr>
<tr>
<td>$\Delta a_2^\pi$</td>
<td>$(-1 \pm 4) \times 10^{-17}$ GeV</td>
<td>&quot;</td>
<td>[118]</td>
</tr>
<tr>
<td>$</td>
<td>\Delta a_1^K</td>
<td>$</td>
<td>$&lt; 9.2 \times 10^{-22}$ GeV</td>
</tr>
<tr>
<td>$</td>
<td>\Delta a_2^K</td>
<td>$</td>
<td>$&lt; 9.2 \times 10^{-22}$ GeV</td>
</tr>
<tr>
<td>$</td>
<td>\Delta a_1^\pi</td>
<td>$</td>
<td>$&lt; 5 \times 10^{-21}$ GeV</td>
</tr>
<tr>
<td>$N^D(\Delta a_0^D - 0.6\Delta a_2^D)$</td>
<td>$(-2.8$ to $4.8) \times 10^{-16}$ GeV</td>
<td>$D$ oscillations [122]</td>
<td></td>
</tr>
<tr>
<td>$N^D\Delta a_1^D$</td>
<td>$(-7$ to $3.8) \times 10^{-16}$ GeV</td>
<td>&quot;</td>
<td>[122]</td>
</tr>
<tr>
<td>$N^D\Delta a_2^D$</td>
<td>$(-7$ to $3.8) \times 10^{-16}$ GeV</td>
<td>&quot;</td>
<td>[122]</td>
</tr>
<tr>
<td>$N^B(\Delta a_0^B - 0.3\Delta a_2^B)$</td>
<td>$(-3.0 \pm 2.4) \times 10^{-15}$ GeV</td>
<td>$B_s$ oscillations [123]</td>
<td></td>
</tr>
<tr>
<td>$N^B\Delta a_X$</td>
<td>$(-22 \pm 7) \times 10^{-15}$ GeV</td>
<td>&quot;</td>
<td>[123]</td>
</tr>
<tr>
<td>$N^B\Delta a_Y$</td>
<td>$(-27$ to $-4) \times 10^{-15}$ GeV</td>
<td>&quot;</td>
<td>[123]</td>
</tr>
<tr>
<td>$N^B(\Delta a_0^B - 0.3\Delta a_2^B)$</td>
<td>$(-27$ to $-4) \times 10^{-15}$ GeV</td>
<td>&quot;</td>
<td>[123]</td>
</tr>
<tr>
<td>$N^B\sqrt{(\Delta a_Y^B)^2 + (\Delta a_Y^D)^2}$</td>
<td>$(37 \pm 16) \times 10^{-15}$ GeV</td>
<td>&quot;</td>
<td>[124]</td>
</tr>
<tr>
<td>$(\Delta a_s)^B$</td>
<td>$(3.7 \pm 3.8) \times 10^{-12}$ GeV</td>
<td>$B_s$ oscillations [121]*</td>
<td></td>
</tr>
</tbody>
</table>


[118] A. Di Domenico, KLOE Collaboration, in Ref. [5], vol. IV.


Summary

+ Many Lorentz frames available simultaneously as the produced particles have different rest frames from event to event, e.g. In a high energy hadron collider (like the LHC) there tend to be large boosts which can enhance sensitivity

+ Distinctive time-dependent experimental signatures

+ Very sensitive tests are possible in meson oscillations because of their interferometric nature

+ Potential to access many LV coefficients simultaneously

+ Some Lorentz violating coefficients (e.g. top quark LV) cannot be easily accessed any other way

+CPT violation can appear as a sidereal signal in single-top production, but also as a different rate in the tW mode for quark versus antiquark
Supersymmetry and Lorentz Violation

Summer School on the SME

June 17, 2018

M. Berger
Symmetries in Particle Physics

- Spacetime symmetries and internal symmetries
- Local and global symmetries
- Exact and spontaneously broken symmetries

The Lorentz symmetry and supersymmetry are both spacetime symmetries.

1) Supersymmetry is experimentally determined to be a broken symmetry.

2) Could the Lorentz symmetry also be broken at some level?
Uses of spacetime symmetries

Why study spacetime symmetries?
-- historical significance, unification
-- physical insight, simplifies calculations (conservation laws)

Why study breaking of spacetime symmetries?
Cornerstone of modern theory
-- must be tested
-- valuable to have theoretical framework allowing violations
Probes of Planck-scale physics
-- Lorentz violation, SUSY breaking

“Planck-scale” physics = quantum gravity/string theory/etc.
Evolution of the Knowledge of Spacetime Symmetries

- **Stern and Gerlach**: Intrinsic spin, properties with respect to the rotation operator $J$ doubles the number of electron states.

- **Dirac**: particle/antiparticle, properties with respect to the Lorentz boost generator, $K$, doubling the number of electron states: electron-positron.

- **Supersymmetry**: introduces a new generator $Q$ doubling the number of states once again: electron and scalar electron (selectron).

Difference: Lorentz symmetry is exact as far as we know; supersymmetry must be broken.

If we lived at the Planck scale, we might be surprised to learn from our experiments that supersymmetry is a broken spacetime symmetry.

\[ M_{LV} \ll M_{SUSY} \ll M_{Pl} \]
Symmetries and Divergences

- **Gauge symmetry:** Gauge boson is massless and the symmetry protects the mass to all orders in perturbation theory (no quadratic divergences)

- **Chiral symmetry:** An exact chiral symmetry for a fermion implies its mass term. Even a massive fermion, say the electron, has an approximate symmetry and radiative corrections to the mass must be proportional to the electron mass itself (no linear divergence)

- **Supersymmetry:** Exact supersymmetry relates the masses and couplings of fermions and scalars so that the scalar self-energy has only a logarithmic divergence (no quadratic divergence to $m^2$)

- From power counting one expects a quadratic divergence for gauge bosons and scalars, but a linear divergence for fermions.
Supersymmetry

- Relates fermions to scalars
- Implies relationships between their masses and couplings
- Can stabilize the electroweak scale
- Possible origin of the dark matter particle
- Gauge coupling unification
Spacetime symmetry violation:
A version of the twin paradox

- Inertial frame change at turnaround point
- Earth Twin ages more than Space Twin
- Clearly asymmetrical as one twin has a frame change, and the other does not
Twin paradox on a cylinder

• No frame changes (as in the conventional twin paradox)
• Earth Twin ages more than Space Twin
• → Breaking of Lorentz and rotational symmetry by a global feature of spacetime, $R^4 \rightarrow R^3 \times S^1$
Properties of spacetime

If supersymmetry exists as a symmetry it must be a broken spacetime symmetry, and there is as yet no experimental evidence the other spacetime symmetries (Lorentz, Poincare) are broken.

Could there be some connection between the breaking of supersymmetry and a possible breaking of the Lorentz symmetry?

Spacetime compactification leads to

1. Nontrivial spacetime topology.
2. Violation of Lorentz and rotational invariance
3. Supersymmetry breaking, e.g. Scherk-Schwarz
Scale of Symmetry Breaking

- There is circumstantial evidence for weak-scale supersymmetry: dark matter, hierarchy problem, absence of substantial radiative corrections to precision electroweak measurements.

- Electroweak-scale supersymmetry is broken to accommodate the lack of observation of superpartners. There is no experimental evidence at the LHC.

- Compared to the fundamental Planck scale, this symmetry breaking is a small effect (like Lorentz violation?), and supersymmetry is an approximate symmetry.

- Supersymmetry breaking can be incorporated into a supersymmetric Standard Model in the context of field theory (adding all possible soft supersymmetry breaking terms).

- The remaining spacetime symmetries such as the Lorentz symmetry could possibly have even more suppressed violations (enforced by supersymmetry?).
**Supersymmetry and Lorentz violation**

<table>
<thead>
<tr>
<th></th>
<th>Supersymmetry</th>
<th>Lorentz symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>exact</strong></td>
<td>Fundamental Lagrangian has exact supersymmetry</td>
<td>Experiment</td>
</tr>
<tr>
<td><strong>broken</strong></td>
<td>MSSM, Experiment</td>
<td>Question: Why is Lorentz breaking so much smaller than the EW scale?</td>
</tr>
<tr>
<td></td>
<td>Question: Is SUSY breaking at the EW scale? Soft breaking</td>
<td></td>
</tr>
</tbody>
</table>

Question: Supersymmetry and relativity are both parts of overall spacetime symmetries. It is well-known that one can explicitly break supersymmetry (MSSM) without Lorentz violation. Can one break the selectively break the Lorentz symmetry?

Lorentz-violating supersymmetric model: add all possible supersymmetric but Lorentz-violating terms to a supersymmetric field theory.
Supersymmetry Algebra

- Poincare algebra (4 generators of translation and the 6 Lorentz generators):
  \[ [P_\mu, P_\nu] = 0 \]
  \[ [P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho) \]
  \[ [M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho}M_{\mu\sigma} - \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\mu\rho}M_{\nu\sigma} + \eta_{\mu\sigma}M_{\nu\rho}) \]

- 3 generators of the rotation group
  \[ J^i = \frac{1}{2} \epsilon^{ijk} M_{jk} \]
  \[ [J^i, J^j] = i\epsilon^{ijk} J^k \]

- 3 generators of Lorentz boosts
  \[ K^i = M^{0i} \]

- Supersymmetry extends the Poincare algebra to include a fermionic generator \( Q \)
  \[ [Q, P_\mu] = 0 \]
  \[ \{Q, Q\} = 2\gamma^\mu P_\mu \]
Incorporation of symmetry breaking effects phenomenologically

1. Supersymmetry breaking
   - Add fields to Supersymmetrize the Standard Model
   - Add supersymmetry breaking terms to the supersymmetric Standard Model. These soft supersymmetry breaking terms can in principle be derived from a more fundamental model of supersymmetry breaking, but often they are treated in a purely phenomenological manner.
   - The result is an effective Field Theory.
   - Supersymmetry breaking is believed to be spontaneous by nonperturbative effects (in some scenarios)

2. Lorentz breaking
   - 1) Lorentz violation can be incorporated into the Standard Model in a general but phenomenological way: Standard Model Extension: Lorentz breaking terms.
   - 2) Presumably the Lorentz symmetry breaking is spontaneous in some underlying fundamental theory.
Supersymmetric Model with Lorentz Violation

MB, Kostelecky, Phys. Rev. D65, 091701 (2002);

Add Lorentz-violating terms to the Wess-Zumino supersymmetric model in the spirit of the Standard Model Extension.

Supersymmetry requires terms involving scalars and terms involving fermions, with related coefficients (similar to masses and couplings).

Modifications: supersymmetric transformations, supersymmetry algebra, kinetic terms of the supersymmetric Lagrangian.

The requirement of supersymmetry places strong constraints on the kinds of Lorentz-violating terms that can be added.
Wess-Zumino Model

Lagrangian

\[ \mathcal{L} = \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B + \frac{1}{2} i \overline{\psi} \partial \psi \]

Transforms under the supersymmetry transformations

\[ \delta A = \overline{\epsilon} \psi \]
\[ \delta B = i \overline{\epsilon} \gamma_5 \psi \]
\[ \delta \psi = -i \partial (A + i \gamma_5 B) \epsilon \]
\[ \delta \overline{\psi} = i \overline{\epsilon} \partial (A - i \gamma_5 B) \]

Into a total derivative

\[ \delta \mathcal{L} = \frac{1}{2} \overline{\epsilon} \partial [\partial (A + i \gamma_5 B) \psi] \]
Wess-Zumino Model-2

Lagrangian

\[ \mathcal{L}_{WZ} = \mathcal{L}_{k.e.} + \mathcal{L}_m + \mathcal{L}_g \]

where

\[ \mathcal{L}_{k.e.} = \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B \]

\[ + \frac{1}{2} i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} F^2 + \frac{1}{2} G^2 \]

\[ \mathcal{L}_m = m \left( -\frac{1}{2} \bar{\psi} \gamma^\mu \gamma^\nu A F + BG \right) , \]

\[ \mathcal{L}_g = \frac{g}{\sqrt{2}} \left[ F(A^2 - B^2) + 2GAB - \bar{\psi}(A - i\gamma_5 B)\psi \right] \]
LV Wess-Zumino Model

Lagrangian

$$\mathcal{L} = \mathcal{L}_{WZ} + \mathcal{L}_{Lorentz}$$

where

$$\mathcal{L}_{Lorentz} = k_{\mu\nu} \partial^\mu A \partial^\nu A + k_{\mu\nu} \partial^\mu B \partial^\nu B$$

$$+ \frac{1}{2} k_{\mu\nu} k^\mu_\rho (\partial^\nu A \partial^\rho A + \partial^\nu B \partial^\rho B)$$

$$+ \frac{1}{2} i k_{\mu\nu} \overline{\psi} \gamma^\mu \partial^\nu \psi.$$ 

Deformed SUSY transformation:

$$\delta A = \bar{\epsilon} \psi,$$

$$\delta B = i \bar{\epsilon} \gamma_5 \psi,$$

$$\delta \psi = -(i \phi + i k_{\mu\nu} \gamma^\mu \partial^\nu)(A + i \gamma_5 B) \epsilon + (F + i \gamma_5 G) \epsilon$$

$$\delta F = -\bar{\epsilon} (i \phi + i k_{\mu\nu} \gamma^\mu \partial^\nu) \psi,$$

$$\delta G = \bar{\epsilon} (\gamma_5 \phi + k_{\mu\nu} \gamma_5 \gamma^\mu \partial^\nu) \psi.$$ 

$$\partial_\mu \rightarrow \partial_\mu + k_{\mu\nu} \partial^\nu$$
Commutator of two SUSY transformations

\[ [\delta_1, \delta_2] = 2i\bar{\epsilon}_1 \gamma^\mu \epsilon_2 \partial_\mu + 2i \kappa_{\mu\nu} \bar{\epsilon}_1 \gamma^\mu \epsilon_2 \partial^\nu \]

Yields a closed superalgebra

\[ [Q, P_\mu] = 0, \quad \{Q, \bar{Q}\} = 2\gamma^\mu P_\mu + 2k_{\mu\nu} \gamma^\mu P^\nu \]

and

\[ [Q, P^2] = 0 \]

For one scalar field and its fermionic partner, this transformation can be scaled away ... but it cannot be removed for two or more such fields simultaneously.
Superspace Formulation

1. Supersymmetry is elegantly formulated as a field theory on superspace \((x, \theta)\).

2. The Wess-Zumino model is the theory of one superfield (one fermion and one scalar). The superfield is a chiral superfield that contains a complex scalar (spin 0), a fermion (spin \(\frac{1}{2}\)), and the auxiliary fields (nonpropagating).

3. CPT-violation in a supersymmetric field theory can be understood as a CPT-violating transformation of the superfields.
Superfield Formulation

Chiral superfield is a function of four spacetime coordinates and four anticommuting coordinates:

\[ \Phi(x, \theta, \bar{\theta}) = \phi(y) + \sqrt{2}\theta \psi(y) + (\theta \theta)F(y), \quad y^\mu = x^\mu + i\theta \sigma^\mu \bar{\theta} \]

\[ = \phi(x) + i\theta \sigma^\mu \bar{\theta} \partial_\mu \phi(x) - \frac{1}{4}(\theta \theta)(\bar{\theta} \bar{\theta})\partial_\mu \partial^\mu \phi(x) \]

\[ + \sqrt{2}\theta \psi(x) + i\sqrt{2}\theta \sigma^\mu \bar{\theta} \partial_\mu \psi(x) + (\theta \theta)F(x) \]

The conjugate of the chiral superfield:

\[ \Phi^*(x, \theta, \bar{\theta}) = \phi^*(z) + \sqrt{2}\bar{\theta} \bar{\psi}(z) + (\bar{\theta} \bar{\theta})F^*(z) \quad z^\mu = x^\mu - i\theta \sigma^\mu \bar{\theta} \]

\[ = \phi^*(x) - i\theta \sigma^\mu \bar{\theta} \partial_\mu \phi^*(x) - \frac{1}{4}(\theta \theta)(\bar{\theta} \bar{\theta})\partial_\mu \partial^\mu \phi^*(x) \]

\[ + \sqrt{2}\bar{\theta} \bar{\psi}(x) + i\sqrt{2}\theta \sigma^\mu \bar{\theta} \partial_\mu \bar{\psi}(x) + (\bar{\theta} \bar{\theta})F^*(x) \]

The Wess-Zumino Lagrangian can be derived from the superspace integral:

\[ \int d^4\theta \Phi^* \Phi + \int d^2\theta \left[ \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + h.c. \right] \]
The action of the supersymmetry generators on the superfield is
\[ \delta_S \Phi(x, \theta, \bar{\theta}) = -i(\epsilon Q + \bar{\epsilon} \bar{Q}) \Phi(x, \theta, \bar{\theta}) \]
\[ = \left[ \epsilon^\alpha \partial_\alpha + \bar{\epsilon}_\dot{\alpha} \bar{\partial}_{\dot{\alpha}} + i \theta \sigma^\mu \bar{\epsilon} \partial_\mu - i \epsilon \sigma^\mu \bar{\theta} \partial_\mu \right] \Phi(x, \theta, \bar{\theta}) \]

In components the Lagrangian is
\[ \mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \frac{i}{2} [ (\partial_\mu \psi) \sigma^\mu \bar{\psi} + (\partial_\mu \bar{\psi}) \bar{\sigma}^\mu \psi ] + \mathcal{F}^* \mathcal{F} \]
\[ + m \left[ \phi \mathcal{F} + \phi^* \mathcal{F}^* - \frac{1}{2} \psi \psi - \frac{1}{2} \bar{\psi} \bar{\psi} \right] \]
\[ + g \left[ \phi^2 \mathcal{F} + \phi^2 \mathcal{F}^* - \phi (\psi \psi) - \phi^* (\bar{\psi} \bar{\psi}) \right] \]

As is well-known, the chiral superfield can be represented as
\[ X = (\theta \sigma^\mu \bar{\theta}) \partial_\mu \]
\[ U_x \equiv e^{iX} = 1 + i(\theta \sigma^\mu \bar{\theta}) \partial_\mu - \frac{1}{4} (\theta \theta) (\bar{\theta} \bar{\theta}) \partial_\mu \partial^\mu \]
\[ \Phi(y, \theta) = U_x \Psi(x, \theta) \]
\[ \Phi^*(z, \theta) = U_x^* \Psi^*(x, \theta) \]
Lorentz-violating superfield transformations

In an analogous way one can define two Lorentz-violating extensions of the Wess-Zumino model.

Define

\[ X \equiv (\theta \sigma^\mu \bar{\theta}) \partial_\mu \]
\[ Y \equiv k_{\mu\nu}(\theta \sigma^\mu \bar{\theta}) \partial^\nu \]
\[ K \equiv k_\mu(\theta \sigma^\mu \bar{\theta}) \]

so that

\[ U_x \equiv e^{iX} = 1 + i(\theta \sigma^\mu \bar{\theta}) \partial_\mu - \frac{1}{4}(\theta \theta)(\bar{\theta} \bar{\theta}) \partial^\mu \partial_\mu \]
\[ U_y \equiv e^{iY} = 1 + ik_{\mu\nu}(\theta \sigma^\mu \bar{\theta}) \partial^\nu - \frac{1}{4}k_{\mu\nu}k^{\mu\rho}(\theta \theta)(\bar{\theta} \bar{\theta}) \partial^\nu \partial_\rho \]
\[ T_k \equiv e^{-K} = 1 - k_\mu(\theta \sigma^\mu \bar{\theta}) + \frac{k^2}{4}(\theta \theta)(\bar{\theta} \bar{\theta}) \]
CPT-Even Model with Lorentz Violation

Following the SM extension, consider adding Lorentz violating terms to the Wess-Zumino model.

Apply the derivative operator effects the substitution

\[ \partial_\mu \rightarrow \partial_\mu + k_{\mu\nu} \partial^\nu \]

And give the superfields:

\[ \Phi_y(x, \theta, \bar{\theta}) = U_y U_x \psi(x, \theta) \]

\[ \Phi_y^*(x, \theta, \bar{\theta}) = U_y^* U_x^* \psi^*(x, \bar{\theta}) = U_y^{-1} U_x^{-1} \psi^*(x, \bar{\theta}) \]

The Lorentz-violating Lagrangian is then given by

\[ \int d^4 \theta \Phi_y^* \Phi_y + \int d^2 \theta \left[ \frac{1}{2} m \Phi_y^2 + \frac{1}{3} g \Phi_y^3 + h.c. \right] \]

\[ = \int d^4 \theta \left[ U_y^* \Phi^* \right] [U_y \Phi] + \int d^2 \theta \left[ \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3 + h.c. \right] \]
CPT-Even Model

The resulting Lagrangian in components is (as before)

\[ \mathcal{L}_{\text{CPT-even}} = (\partial_\mu + k_{\mu\nu}\partial^{\nu})\phi^*(\partial^{\mu} + k^{\mu\rho}\partial_\rho)\phi \]
\[ + \frac{i}{2} [((\partial_\mu + k_{\mu\nu}\partial^{\nu})\psi)\sigma^{\mu}\bar{\psi} + ((\partial_\mu + k_{\mu\nu}\partial^{\nu})\bar{\psi})\bar{\sigma}^{\mu}\psi] + F^*F \]
\[ + m [\phi F + \phi^* F^* - \frac{1}{2} \psi \bar{\psi} - \frac{1}{2} \bar{\psi} \psi] \]
\[ + g [\phi^2 F + \phi^* 2 F^* - \phi (\psi \bar{\psi}) - \phi^* (\bar{\psi} \psi)] \]

The presence of the terms in the extension forces a relationship on the coefficients for Lorentz violation, analogous to the common mass and common couplings that are a standard consequence of supersymmetric theories.

The fermion propagator can be written as

\[ iS_F(p) \]
\[ = \frac{i}{p_\mu (\gamma^\mu + k_{\mu\nu} \gamma^{\nu}) - m} \]
\[ = \frac{i}{p^2 + 2p_\mu p^{\nu} k_{\mu\nu} + k_{\mu\rho} k^{\rho}_{\nu} p^{\mu} p^{\nu}} \]

The scalar propagator has the same form for the denominator.
CPT-Even model (cont.)

The modified superfields can be understood by defining left-chiral and right-chiral coordinates as

\[ x^\mu_\pm = x^\mu \pm i\theta\sigma^\mu\bar{\theta} \pm ik^{\mu\nu}\theta\sigma^\nu\bar{\theta} \]

The chiral superfield is then simply

\[ \Phi_y(x, \theta, \bar{\theta}) = \Phi(x, \theta, \bar{\theta}; \partial_\mu \rightarrow \partial_\mu + k_{\mu\nu}\partial^\nu) \]
\[ = \phi(x_+) + \sqrt{2}\theta\psi(x_+) + (\theta\theta)F(x_+) \]

A superfield covariant derivative can also be introduced in analogy with the usual case with the substitution

\[ \partial_\mu \rightarrow \partial_\mu + k_{\mu\nu}\partial^\nu \]

The modified superalgebra is

\[ [P_\mu, Q] = 0, \quad \{Q, \overline{Q}\} = 2\sigma^\mu P_\mu + 2k_{\mu\nu}\sigma^\mu P^\nu \]
Summary

• Explicit Lorentz violation can be added to the Wess-Zumino model.

• By virtue of the Lorentz violation, manifest through the presence of the Lorentz-violating coefficient, the superalgebra

\[ [P_\mu, Q] = 0, \quad \{Q, \overline{Q}\} = 2\sigma^\mu P_\mu + 2k_{\mu\nu}\sigma^\mu P^\nu. \]

lies outside the usual list of possible supersymmetric extensions of the Poincare algebra.

• Superfield description permits one to survey the possible types of Lorentz violation permitted by supersymmetry.

• Realistic models should incorporate supersymmetry breaking as well as Lorentz violation. Since the source of spacetime symmetry breaking is not understood, there may be some relationship between the breaking of supersymmetry and possible breaking of the Lorentz symmetry.