Lorentz Tests with Astrophysical Photons

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Outline

• Astrophysical Signatures of Lorentz-invariance Violation: Introduction and Motivation
• Connection to the SME: Phenomenological Description
• Techniques and Results: Overview of Current Research
• Outlook: Improved Sensitivity of Future Instruments
Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the “light medium,” suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the “Principle of Relativity”) to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity $c$ which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell’s theory for stationary bodies.
Part I

ASTROPHYSICAL SIGNATURES
Quantum Gravity Theories

- Fundamental symmetry of Special Relativity: **Lorentz Invariance**.
  - Foundation of the Standard Model of particle physics.
  - Tested to a great degree at observable length (or energy) scales.

- **Current theory of gravity: General Relativity** and Einstein’s field equations.
  - Tested at small and large scales.
  - Black holes; gravitational waves; cosmology.
  - Incompatible with Quantum Mechanics.

- **Theories of Quantum Gravity:**
  - May violate Lorentz Invariance and CPT symmetry at the Planck scale.

- **Lorentz Invariance violation:** Speed of light depends on observer, or:
  - Wavelength,
  - Polarization,
  - Propagation direction.
Astrophysical Observation

• Effects expected to be small.
  – Suppressed by $E/E^d_{\text{Planck}}$ for some $d > 0$.

• Unique strengths of astrophysical observations:
  – Access to extremely high energies:
    • Cosmic rays up to EeV;
    • Photons up to 100s of TeV.
  – Very long baselines, cosmological distances.
    • Miniscule differences of the speed of light accumulate over distances.

• Main challenge:
  – Unknown or uncertain emission properties.

• Observables:
  – Time of flight;
  – Polarization;
  – Anisotropy of the above.
**Time-of-Flight Measurements**

- **Does group velocity depend on wavelength?**
- **Emission time of individual photons unknown.**
  - Observe sources with variable flux, e.g. Active Galactic Nuclei (AGN);
  - Observe transient events, e.g. Gamma-ray Bursts (GRBs).
- **Simple approach:** Assume brightness changes independent of photon energy.
  - **What if that’s not true?**
    - Can still constrain arrival time differences ⇒ set limits on LIV parameters.
    - Unlikely that source-intrinsic and LIV effects cancel.
- **Sufficient photon statistics:** Model lag energy spectrum:
  \[
  \Delta t_{\text{obs}}(E) = \Delta t_{\text{int}}(E) + \Delta t_{\text{LV}}(E).
  \]
Birefringence

• Does group velocity depend on energy and linear polarization?
  – **Polarization** dependence ⇒ linear polarization angle changes between source and observer.
  – **Energy** dependence ⇒ effect will depend on energy.

• **Assumption:** linear polarization at source independent of energy.
  – Polarization angle $\chi_{\text{source}}(E) = \text{const.} \Rightarrow \chi_{\text{obs}} = f(E)$.
  – Measure energy-dependent polarization.

• **What if that’s not true?**
  – It’s unlikely that source and LIV effects will cancel.
  – Constrain LIV parameters around 0.
  – Observe multiple sources.
The Power of Polarization

• Assume source at distance $L$.
• Time-of-Flight sensitivity determined by ability to measure photon arrival times $\delta t \approx \delta \nu L$.
  – Sensitivity $\propto \delta t / L$.
  – Typically dominated by photon statistics.
  – Time scale seconds ... days.
• Polarimetric quantity of interest is phase difference $\delta \phi \approx E \delta \nu L$.
  – Sensitivity $\propto \delta \phi / (EL)$.

Assumptions:
• Bandwidth $E_2 / E_1 = 10$.
• Source distance $z = 1$.

[Sensitivity $g_{\phi \phi}$(Kislat&Krawczynski, PRD (2015))](#)

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Part II

CONNECTION TO THE SME
SME Photon Sector

- Basically following [Kostelecký&Mewes, PRD (2009)].
- Lagrange density:
  \[ \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}A_\lambda (\hat{k}_{AF})_\kappa F_{\mu\nu} - \frac{1}{4}F_{\kappa\lambda} (\hat{k}_F)^{\kappa\lambda\mu\nu} F_{\mu\nu} \]
- CPT-odd operators:
  \[ (\hat{k}_{AF})_\kappa = \sum_{d=\text{odd}} (k_{AF}^{(d)})^\alpha_1\ldots\alpha_{(d-3)} \partial_{\alpha_1} \ldots \partial_{\alpha_{(d-3)}} \]
- CPT-even operators:
  \[ (\hat{k}_F)^{\kappa\lambda\mu\nu} = \sum_{d=\text{even}} (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}} \partial_{\alpha_1} \ldots \partial_{\alpha_{(d-4)}} \]
- Operators are ordered by mass dimension \( d \).
- Size estimate of coefficients:
  For example \( (k_{AF}^{(d)})^{\alpha_1\ldots\alpha_{(d-3)}} \), \( (k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\ldots\alpha_{(d-4)}} \propto M_{\text{Planck}}^{4-d} \).
Vacuum Dispersion Relation

• Dispersion relation derived from Lagrangian:

\[ p^0 \approx \left( 1 - \zeta^0 \pm \sqrt{(\zeta^1)^2 + (\zeta^2)^2 + (\zeta^3)^2} \right) p \]

with

\[ \zeta^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{n}) c^{(d)}_{(l)jm}, \]

\[ \zeta^1 \pm i\zeta^2 = \sum_{djm} \omega^{d-4} \pm_2 Y_{jm}(\hat{n}) \left( k^{(d)}_{(E)jm} \mp i k^{(d)}_{(B)jm} \right), \]

\[ \zeta^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{n}) k^{(d)}_{(V)jm}. \]

where \( \hat{n} = -\hat{p} \) is the direction from observer to source.

• Vacuum coefficients related to SME operators by spherical decomposition:

\[ c^{(d)}_{(l)jm}, k^{(d)}_{(E)jm}, k^{(d)}_{(B)jm} \Leftrightarrow (k^{(d)}_F)_{\kappa\lambda\mu\nu\alpha_1...\alpha_{(d-4)}} \]

\[ k^{(d)}_{(V)jm} \Leftrightarrow (k^{(d)}_{AF})_{\kappa}^{\alpha_1...\alpha_{(d-3)}} \]

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Let’s recap ...

• 2 additional terms in the Lagrangian:
  – Coefficients of mass dimension $d$:
    \[
    (k_{A^F}^{(d)})_{\ell}^{\alpha_1...\alpha_{(d-3)}} \Rightarrow \text{CPT odd},
    \]
    \[
    (k_{F}^{(d)})_{\kappa\lambda\mu\nu\alpha_1...\alpha_{(d-4)}} \Rightarrow \text{CPT even}.
    \]

• Spherical decomposition ⇒ Vacuum coefficients:
  Non-birefringent
  \[
  \begin{pmatrix}
  c_{(I)jm}^{(d)} \\
  k_{(E)jm}^{(d)} \\
  k_{(B)jm}^{(d)}
  \end{pmatrix}
  \]
  CPT–even
  Birefringent
  \[
  \begin{pmatrix}
  k_{(I)jm}^{(d)} \\
  k_{(B)jm}^{(d)} \\
  k_{(V)jm}^{(d)}
  \end{pmatrix}
  \]
  CPT–odd

• Experiments constrain:
  \[
  p^0 \approx \left(1 - \zeta^0 \pm \sqrt{(\zeta^1)^2 + (\zeta^2)^2 + (\zeta^3)^2}\right) p
  \]
Counting Coefficients I

• So do we have to constrain an infinite number of coefficients?

• Coefficients of higher mass dimension are more strongly suppressed
  ⇒ Restrict summation to some $d_{\text{max}}$ to characterize low-energy effects.

• Symmetry arguments
  ⇒ $j \leq d - 2, -j \leq m \leq j$.

• For a given mass dimension $d$, number of complex coefficients:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{(I)jm}^{(d)}$</td>
<td>$(d - 1)^2$</td>
</tr>
<tr>
<td>$k_{(E)jm}^{(d)}$</td>
<td>$(d - 1)^2 - 4$</td>
</tr>
<tr>
<td>$k_{(B)jm}^{(d)}$</td>
<td>$(d - 1)^2 - 4$</td>
</tr>
<tr>
<td>$k_{(V)jm}^{(d)}$</td>
<td>$(d - 1)^2$</td>
</tr>
</tbody>
</table>
Counting Coefficients II

• Complex coefficients \( c_{(l)jm}, k_{(E)jm}, k_{(B)jm}, k_{(V)jm} \Rightarrow 2 \times \text{real coefficients.} \)
• However: \( p \) is real \( \Rightarrow \zeta^0, (\zeta^1)^2 + (\zeta^2)^2, \zeta^3 \in \mathbb{R}. \)
• Spherical harmonics \( 0Y_{jm}(\hat{n}) \Rightarrow \)
  \[
  c_{(l)j,-m}^{(d)} = (-1)^m(c_{(l)jm}^{(d)})^*
  \\
  k_{(V)j,-m}^{(d)} = (-1)^m(k_{(l)jm}^{(d)})^*
  
\]
In particular: \( c_{(l)j0}^{(d)}, k_{(V)j0}^{(d)} \in \mathbb{R}. \)
  \( \Rightarrow (d - 1)^2 \text{ real coefficients each.} \)
• Spherical harmonics \( \pm 2Y_{jm}(\hat{n}) \Rightarrow \)
  \[
  k_{(E)j,-m}^{(d)} = (-1)^m(k_{(E)jm}^{(d)})^*
  \\
  k_{(B)j,-m}^{(d)} = (-1)^m(k_{(B)jm}^{(d)})^*
  
\]
In particular: \( k_{(E)j0}^{(d)}, k_{(B)j0}^{(d)} \in \mathbb{R}. \)
  \( \Rightarrow (d - 1)^2 - 4 \text{ real coefficients each.} \)
Can only be tested through **time-of-flight measurements**.

From dispersion relation:

$$\delta v \simeq -c^0 = - \sum_{djm} E^{d-4} Y_{jm}(\hat{n}) c^{(d)}_{(1)jm}.$$  

Integrate along photon trajectory ⇒ **arrival time difference** $\delta t(E_1, E_2)$.

Photons traveling over cosmological distance ⇒ $E = E(z)$.

For a source at **redshift** $z$, observed photon energies $E_1$ and $E_2$:

$$\delta t = t_2 - t_1 \approx \int_0^z \frac{v_1 - v_2}{H_{z'}} dz' \approx (E_2^{d-4} - E_1^{d-4}) \int_0^z \frac{(1 + z')^{d-4}}{H_{z'}} dz' \sum_{jm} Y_{jm}(\hat{n}) c^{(d)}_{(1)jm}$$

with

$$H_z = H_0 \left[ \Omega_r (1 + z)^4 + \Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_\Lambda \right]^{1/2}.$$  

**Signature:**

$$\sum_{jm} Y_{jm}(\hat{n}) c^{(d)}_{(1)jm} > 0 \Rightarrow \text{Higher energy photons arrive later}$$
Electromagnetic wave completely described by 4 **Stokes parameters**:

\[
\begin{align*}
I &= s^0 = \langle E_x^2 + E_y^2 \rangle \\
Q &= s^1 = \langle E_x^2 - E_y^2 \rangle = I \Pi \cos 2\psi \\
U &= s^2 = \langle 2E_xE_y \cos \delta \rangle = I \Pi \sin 2\psi \\
V &= s^3 = \langle 2E_xE_y \sin \delta \rangle
\end{align*}
\]

- Define Stokes 3-vector to describe polarization:
  \[
  \mathbf{s} = (s^1, s^2, s^3)^T.
  \]
- Coefficients \( k^{(d)}_{(E)jm}, k^{(d)}_{(B)jm}, k^{(d)}_{(V)jm} \) define **birefringence axis**:
  \[
  \mathbf{\zeta} = (s^1, \zeta^2, \zeta^3)^T.
  \]
- Birefringence \( \iff \) **Rotation** of \( \mathbf{s} \) around \( \mathbf{\zeta} \):
  \[
  \frac{ds}{dt} = 2E\mathbf{\zeta} \times \mathbf{s}.
  \]
- CPT-even \( \iff \) \( \mathbf{\zeta} \) in \( s^1-s^2 \) plane \( \Rightarrow \) Linear and circular polarization **become elliptical**;
  CPT-odd \( \iff \) \( \mathbf{\zeta} \) in \( s^3 \) plane \( \Rightarrow \) Linear polarization **angle rotates**, circular remains circular.
Consider only linearly polarized light.

Assumptions:
- Source at redshift $z$.
- Identical polarization angle $\psi$ at the source.
- Observed photon energies $E_1$ and $E_2$.

Observed polarization angles:

$$\delta \psi = \psi_2 - \psi_1 \approx (E_2^{d-3} - E_1^{d-3}) \int_0^z \frac{(1 + z')^{d-4}}{H_{z'}} \, dz' \sum_{jm} Y_{jm}(\hat{n})k^{(d)}_{(V)jm}$$

Note the larger power of $E_i$ compared to dispersion measurements $\delta t$.

Measure polarization over energy band $E_1 \leq E \leq E_2$:

$$\Pi_{eff} \leq \sqrt{\langle s^1 \rangle^2 + \langle s^2 \rangle^2}$$

$\Rightarrow$ Observed polarization basically vanishes if $\delta \psi \geq \pi$ [Kostelecký&Mewes, PRL (2013)].
Birefringence in CPT-even Models

- Parameters $k_{Ejm}^{(d)}$ and $k_{Bjm}^{(d)}$: linearly polarized eigenmodes.
- In Stokes space: $s$ traces a cone around $\zeta$.
  - Assuming initially linear polarization with angle $\psi_0$.
  - Initial angle between $s$ and $\zeta$: $\Psi = \psi_0 - \psi_a$,
    where $\psi_a$ polarization angle of faster eigenmode.
  - Opening angle of the cone: $4\Psi$.
- X-ray and $\gamma$-ray polarimeters: not sensitive to circular polarization ($s^3$).
- Effective polarization in energy band $E_1 \ldots E_2$ [Kostelecký&Mewes, PRL (2013)]:
  \[
  \Pi_{\text{eff}} \leq \sqrt{1 - (1 - \langle \cos 2\delta \psi \rangle^2) \sin^2 2\Psi}.
  \]
  ⇒ **Observed linear polarization** essentially vanishes for $\delta \psi \geq \pi$ and $\Psi = \pm \pi/4$.
- Single source: light could be in eigenmode $\Psi = 0, 2\pi$. 

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Reference Frame

- General vacuum models are anisotropic ⇒ need a reference frame.

Relation to astronomical coordinates:
  - Codeclination: \( \theta = \pi - \delta \).
  - Right ascension: \( \phi = \alpha \).

\[ \eta \approx 23^\circ \]

[Bluhm et al. PRD (2003)]
Quiz Time!

1) What are the lowest-\(d\) coefficients that can be constrained with astrophysical time of flight measurements? Why?
   - \(d = 5\). At \(d = 4\) there is only anisotropy, no energy dependence.

2) What is the best way to constrain the coefficients \(k_{(V)jm}^{(5)}\)?
   - Astrophysical polarization measurements (gamma-ray, X-ray, optical).

3) How many sources need to be observed to constrain all coefficients \(k_{(E)jm}^{(6)}\) and \(k_{(B)jm}^{(6)}\)?
   - There are \(2 \times ((d - 1)^2 - 4) = 42\) real coefficients \(\Rightarrow \geq 42\) sources.

4) Can you think of a way to measure the coefficients \(k_{(V)jm}^{(3)}\)?
   - Correlations of E and B mode CMB polarization.
Part III

TECHNIQUES AND RESULTS
Part III A

TECHNIQUES AND RESULTS: TIME-OF-FLIGHT
Photons with energies $E_1$ and $E_2$ arrive at $t_1$ and $t_2$. 
Assumption: photons were emitted simultaneously.

Time lag:

$$t_2 - t_1 \approx (E_2^{d-4} - E_1^{d-4}) \int_0^z \frac{(1 + z')^2}{Hz} \sum_{jm} Y_{jm}(\hat{n}) c_{(I)jm}^{(d)} \gamma_{(I)}^{(d)}(\hat{n}) \vartheta_{(I)}^{(d)}(z,\hat{n})$$

For a variable astrophysical source: measure $\vartheta_{(I)}^{(d)}$.

Candidates:
- Gamma-ray bursts (GRBs),
- Flaring Active Galactic Nuclei (AGNs).

Requirement: Known redshift $z$. 
Time of Flight: GRB Spectral Lag

- **Gamma-ray Bursts:**
  - Short transient events ($\leq 100$s), non-repetitive;
  - Often at high redshift (e.g. GRB 090423 at $z = 8.2$ [Tanvir et al, Nature (2009)], *Swift* mean at $z \sim 1.7$ [Le & Mehta, ApJ (2017)];
  - Can be extremely bright;
  - Spectral peak $\sim 1$MeV.

- **Extract photon arrival times $t(E)$:**
  - Measure GRB light curve in several energy bins.
  - Model independent: **Cross-correlation function** extracts relative arrival times in energy bins.
Spectral Lag of GRB 160625B

- Extremely bright 2nd sub-burst.
- First GRB with a spectral lag transition:
  - Above ~8MeV spectral lag decreases.
- Spectral lag model:
  \[ \Delta t_{\text{obs}}(E) = \Delta t_{\text{int}}(E) + \Delta t_{\text{LV}}(E). \]
- Intrinsic lag:
  \[ \Delta t_{\text{int}}(E) = \tau \left[ \left( \frac{E}{\text{keV}} \right)^{\alpha} - \left( \frac{E_0}{\text{keV}} \right)^{\alpha} \right] \]
  Fit parameters: \( \tau > 0, \alpha > 0 \).
- LIV lag according to SME with \( d = 6, 8, 10 \):
  - Two cases each: positive & negative lag.
- Constraints:
  - \(-2.8 \times 10^{-16}\text{GeV}^{-2} < \gamma_{(6)}^{(I)} < 3.4 \times 10^{-15}\text{GeV}^{-2}\)
  - \(-1.5 \times 10^{-13}\text{GeV}^{-4} < \gamma_{(8)}^{(I)} < 2.0 \times 10^{-12}\text{GeV}^{-4}\)
  - \(-9.1 \times 10^{-11}\text{GeV}^{-6} < \gamma_{(10)}^{(I)} < 1.0 \times 10^{-9}\text{GeV}^{-6}\)

The DisCan Method I

• **Basic idea:**
  – **Transient event** (AGN flare or GRB) energy independent.
  – **Observed light curve** “smeared” due to photon dispersion.

• **Cancel** LIV-induced dispersion:
  – Correct photon arrival times by subtracting LIV time delay:
    \[ t_0 = t - \theta^{(d)}_{(l)}(z, \hat{n}) \times (E^{d-4} - \langle E^{d-4} \rangle). \]

• Find best fit value \( \hat{\theta}^{(d)}_{(l)}(z, \hat{n}) \). Maximize **Shannon information** (summation over all photons):
  \[
  S = \sum_i \frac{w_i}{W} \ln \frac{w_i}{W} \quad \text{with} \quad W = \sum_i w_i,
  \]
  with the weight
  \[
  w_i = 2(t_{i+1} - t_{i-1})^{-1}.
  \]

“Observed” Fermi AGN light curve. (Simulation)  
After dispersion cancellation.

The *DisCan Method I*

- **Basic idea:**
  - **Transient event** (AGN flare or GRB) energy independent.
  - **Observed light curve** “smeared” due to photon dispersion.

- **Cancel** LIV-induced dispersion:
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    \[ t_0 = t - \vartheta^{(d)}_{(l)}(z, \hat{n}) \times (E^{d-4} - \langle E^{d-4} \rangle). \]

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  \]
  with the weight
  \[
  w_i = 2(t_{i+1} - t_{i-1})^{-1}.
  \]

---

6-year gamma-ray light curve of 3C 454.3 from Fermi/LAT
[Kislat&Krawczynski, PRD (2015)].
Instruments


Gamma-ray Burst Monitor (GBM): CsI and BGO scintillator. Energy range: 5keV – 30MeV. Area: 130cm². Field of view: all sky (not occulted by Earth).
Part III B

TECHNIQUES AND RESULTS: ANISOTROPY
• Constraining time lags from a single source $k$

$$\Rightarrow \gamma_k = \gamma^{(d)}_1(\hat{n}_k) = \sum_{jm} Y_{jm}(\hat{n}_k)c^{(d)}_{(l)jm}.$$ 

• Let’s say, we obtain limits from $N$ sources:
  – For simplicity, they shall be symmetric, i.e. $|\gamma_k| < \gamma_{k,\text{max}}$.
  – Assume Gaussian distribution of $\gamma_k \Rightarrow$ Distributions of $c^{(d)}_{(l)jm}$.

• Linear transformation ($N \times (d - 1)^2$ dimensional) $\gamma_k \rightarrow c^{(d)}_{(l)jm}$. Let’s look at a 2D example!
Anisotropy I

• Constraining time lags from a single source $k$

\[ \Rightarrow \gamma_k = \gamma^{(d)}_{(l)}(\hat{n}_k) = \sum_{jm} Y_{jm}(\hat{n}_k)c^{(d)}_{(l)jm}. \]

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Constraining time lags from a single source $k$

$$\Rightarrow \gamma_k = \gamma_{(l)}^{(d)}(\hat{n}_k) = \sum_{jm} Y_{jm}(\hat{n}_k)c_{(l)jm}^{(d)}.$$ 

Let’s say, we obtain limits from $N$ sources:
- For simplicity, they shall be symmetric, i.e. $|\gamma_k| < \gamma_{k,\text{max}}$.
- Assume Gaussian distribution of $\gamma_k \Rightarrow$ Distributions of $c_{(l)jm}^{(d)}$.

Linear transformation ($N \times (d - 1)^2$ dimensional) $\gamma_k \rightarrow c_{(l)jm}^{(d)}$. Let’s look at a 2D example!
Anisotropy II

• Tasks:
  – Combine $N$ distributions of $\gamma_k$ to calculate $(d-1)^2$-dimensional distribution of $c_{(l)jm}^{(d)}$.
  – Calculate $((d-1)^2 - 1)$-fold integrals to project onto the $c_{(l)jm}^{(d)}$-axes.

• Given $N$ measurements $\Rightarrow N$ equations:

\[ \sum_{jm} Y_{jm}(\hat{n}_k)c_{(l)jm}^{(d)} = 0 \pm \gamma_{k,\text{max}}. \]

$\Rightarrow$ Linear system of equations for the components $c_i$ of $c_{(l)jm}^{(d)}$:

\[ H \cdot c = 0 \pm \gamma_{\text{max}}. \]

$H$: $(d-1)^2 \times N$ matrix.

• Identical to a linear least-squares fit with parameter covariance matrix

\[ V(c) = (H^TV(y)H)^{-1}. \]

• 95% upper limits:

\[ c_{i,\text{max}} \approx 1.96 \times \sqrt{V_{ii}(c)}. \]
All Parameters $c^{(6)}_{(1)jm}$

- 34 sources in total:
  - 25 Fermi AGNs (1 – 100GeV),
  - 4 Fermi GRBs (1 – 30GeV),
  - 1 GRB observed with RHESSI (Reuven Ramaty High Energy Solar Spectroscopic Imager, 4keV – 1MeV),
  - 4 TeV AGNs (H.E.S.S., MAGIC, Whipple).
### All Parameters $c^{(6)}_{(I)jm}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^{(6)}_{(I)00}$</td>
<td>$-2.705 \times 10^{-14}$ to $3.925 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)10}$</td>
<td>$-3.753 \times 10^{-14}$ to $2.889 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)11}$</td>
<td>$-2.816 \times 10^{-14}$ to $3.574 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)11}$</td>
<td>$-3.299 \times 10^{-15}$ to $5.984 \times 10^{-15}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)20}$</td>
<td>$-4.232 \times 10^{-14}$ to $3.032 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)21}$</td>
<td>$-1.590 \times 10^{-14}$ to $1.043 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)21}$</td>
<td>$-4.412 \times 10^{-14}$ to $3.288 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)22}$</td>
<td>$-2.353 \times 10^{-14}$ to $3.113 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)22}$</td>
<td>$-5.144 \times 10^{-14}$ to $6.634 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)30}$</td>
<td>$-4.823 \times 10^{-14}$ to $6.435 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)31}$</td>
<td>$-2.439 \times 10^{-14}$ to $1.798 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)31}$</td>
<td>$-2.822 \times 10^{-14}$ to $2.078 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)32}$</td>
<td>$-3.125 \times 10^{-14}$ to $3.855 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)32}$</td>
<td>$-2.171 \times 10^{-14}$ to $1.624 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)33}$</td>
<td>$-3.693 \times 10^{-14}$ to $2.943 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)40}$</td>
<td>$-4.216 \times 10^{-14}$ to $5.656 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)41}$</td>
<td>$-2.313 \times 10^{-14}$ to $2.739 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)41}$</td>
<td>$-9.021 \times 10^{-15}$ to $1.131 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)41}$</td>
<td>$-2.953 \times 10^{-14}$ to $3.904 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)41}$</td>
<td>$-4.650 \times 10^{-15}$ to $6.846 \times 10^{-15}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)42}$</td>
<td>$-2.489 \times 10^{-14}$ to $1.961 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)42}$</td>
<td>$-7.276 \times 10^{-15}$ to $1.014 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)43}$</td>
<td>$-1.246 \times 10^{-14}$ to $1.343 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)43}$</td>
<td>$-3.919 \times 10^{-14}$ to $2.923 \times 10^{-14}$</td>
</tr>
<tr>
<td>$c^{(6)}_{(I)44}$</td>
<td>$-1.801 \times 10^{-14}$ to $1.427 \times 10^{-14}$</td>
</tr>
</tbody>
</table>

Units: GeV$^{-2}$

[ Kislat & Krawczynski, PRD (2015) ]
Cancellation of Linear Polarization I

- **Observed polarization** angles in CPT-odd models:

\[ \psi_2 - \psi_1 \approx \left( E_2^{d-3} - E_1^{d-3} \right) \int_0^z \frac{(1 + z')^{d-4}}{H_z'} \, dz' \sum_{jm} Y_{jm}(\hat{n}) k^{(d)}_{(V)jm} \cdot \gamma^{(d)}_{(V)\hat{n}}. \]

- Assuming uniform detector response and constant polarization at the source in \( E_1 \) ... \( E_2 \):
  - Complete cancellation of linear polarization for \( \delta \psi = n\pi \),
  - Partial cancellation for other values.

![Graph showing the cancellation of linear polarization](image)
X-ray & Gamma-ray Polarimetry

- X-ray & Gamma-ray polarization measurements of GRBs with
  - INTEGRAL (100keV – 1MeV) [Laurent et al, PRD (2011)].
- High-energy polarization ⇒ very sensitive to LIV parameters.
- Main challenge: instruments not optimized for polarization measurements.
  - Large systematic uncertainties.
  - Large statistical uncertainties.

| $\sum_{jm} Y_{jm}(\theta, \phi) k_{(V)jm}^{(5)}$ | $< 2 \times 10^{-34}$ GeV$^{-1}$ | $< 7 \times 10^{-35}$ GeV$^{-1}$ | $< 2 \times 10^{-34}$ GeV$^{-1}$ | $< 1 \times 10^{-34}$ GeV$^{-1}$ |
| $\sum_{jm} Y_{jm}(\theta, \phi) k_{(V)jm}^{(7)}$ | $< 2 \times 10^{-28}$ GeV$^{-3}$ | $< 4 \times 10^{-28}$ GeV$^{-3}$ | $< 2 \times 10^{-27}$ GeV$^{-3}$ | $< 8 \times 10^{-28}$ GeV$^{-3}$ |
| $\sum_{jm} Y_{jm}(\theta, \phi) k_{(V)jm}^{(9)}$ | $< 2 \times 10^{-22}$ GeV$^{-5}$ | $< 2 \times 10^{-21}$ GeV$^{-5}$ | $< 2 \times 10^{-20}$ GeV$^{-5}$ | $< 5 \times 10^{-21}$ GeV$^{-5}$ |
| $k_{(V)00}^{(5)}$ | $< 8 \times 10^{-34}$ GeV$^{-1}$ | $< 2 \times 10^{-34}$ GeV$^{-1}$ | $< 9 \times 10^{-34}$ GeV$^{-1}$ | $< 4 \times 10^{-34}$ GeV$^{-1}$ |
| $k_{(V)00}^{(7)}$ | $< 8 \times 10^{-28}$ GeV$^{-3}$ | $< 1 \times 10^{-27}$ GeV$^{-3}$ | $< 8 \times 10^{-27}$ GeV$^{-3}$ | $< 3 \times 10^{-27}$ GeV$^{-3}$ |
| $k_{(V)00}^{(9)}$ | $< 8 \times 10^{-22}$ GeV$^{-5}$ | $< 7 \times 10^{-21}$ GeV$^{-5}$ | $< 7 \times 10^{-20}$ GeV$^{-5}$ | $< 2 \times 10^{-20}$ GeV$^{-5}$ |

6/19/2018 Fabian Kislat – Lorentz Tests with Astrophysical Photons
Cancellation of Linear Polarization II

- **Observed polarization** angles in CPT-odd models:

  \[ \psi_2 - \psi_1 \approx (E_2^{d-3} - E_1^{d-3}) \int_0^z \frac{(1 + z')^{d-4}}{H z'} \mathrm{d}z' \sum_{jm} Y_{jm}(\hat{n})k_{(V)jm}^{(d)} \cdot \gamma_{(V)}^{(d)}(\hat{n}) \]

- **Measure polarization in energy band** \( E_1 \ldots E_2 \) with **instrument response function** \( T(E) \):

  \[ Q(\vartheta^{(d)}_{(V)}) = \int_{E_1}^{E_2} \cos \left( 2\vartheta^{(d)}_{(V)}(E^{d-3} - E_1^{d-3}) \right) T(E) \mathrm{d}E \]

Oscillation of the Stokes parameter \( Q \) as a function of wavelength due to Lorentz violation, folded with telescope filter transmissivity \( T(E) \) [Kislat&Krawczynski, PRD (2017)].
Cancellation of Linear Polarization II

- Starting from the following assumptions at the source (maximal polarization):
  - 100% linear polarization,
  - $\psi = \text{const.}$

- Given transmissivity $T(E)$ or effective area $A_{\text{eff}}(E)$ (*right top*):

$$Q \left( \vartheta^{(d)}_{(V)} \right) = \int_{E_1}^{E_2} \cos \left( 2 \vartheta^{(d)}_{(V)} (E^{d-3} - E_1^{d-3}) \right) T(E) \, dE.$$  

- Partial cancellation of polarization. *Right bottom*: maximum possible observable polarization $\Pi_{\text{max}}$.
  - Ringing when $\delta \psi \sim n\pi$.

- Measure linear polarization $\Pi_{\text{observed}}$.
- Find maximum value $\vartheta_{\text{max}}$ for which $\Pi_{\text{max}}(\vartheta_{\text{max}}) \geq \Pi_{\text{observed}}$. 

---

[Kislat&Krawczynski, PRD (2017)]
Optical Polarimetry

- Steward Observatory AGN monitoring program:
  - Monitors 30+ AGN since 10 years.
  - Support of the Fermi mission.
  - Spectroscopic and spectropolarimetric data public: http://james.as.arizona.edu/~psmith/Fermi/

- Bandwidth: 4000Å – 7550Å.
  - Polarimetry: achromatic waveplate + Wollaston prism.
- Ideal for $d \leq 5$. 

![Graph showing polarization angle versus wavelength for 3C 454.3]
Constraints for all $d = 5$ Coefficients
### Constraints for all $d = 5$ Coefficients

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
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<tbody>
<tr>
<td>$-5.1 \times 10^{-24} &lt; k_{V0}^{(5)}$</td>
<td>$k_{V0}^{(5)} &lt; 5.1 \times 10^{-24}$</td>
<td></td>
</tr>
<tr>
<td>$-6.1 \times 10^{-24} &lt; k_{V10}^{(5)}$</td>
<td>$k_{V10}^{(5)} &lt; 6.1 \times 10^{-24}$</td>
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<tr>
<td>$-2.5 \times 10^{-24} &lt; \text{Re}(k_{V11}^{(5)})$</td>
<td>$\text{Re}(k_{V11}^{(5)}) &lt; 2.5 \times 10^{-24}$</td>
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<tr>
<td>$-7.1 \times 10^{-24} &lt; \text{Im}(k_{V11}^{(5)})$</td>
<td>$\text{Im}(k_{V11}^{(5)}) &lt; 7.1 \times 10^{-24}$</td>
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<td>$-7.0 \times 10^{-24} &lt; k_{V20}^{(5)}$</td>
<td>$k_{V20}^{(5)} &lt; 7.0 \times 10^{-24}$</td>
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<tr>
<td>$-2.8 \times 10^{-24} &lt; \text{Re}(k_{V21}^{(5)})$</td>
<td>$\text{Re}(k_{V21}^{(5)}) &lt; 2.8 \times 10^{-24}$</td>
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<td>$-2.5 \times 10^{-24} &lt; \text{Re}(k_{V22}^{(5)})$</td>
<td>$\text{Re}(k_{V22}^{(5)}) &lt; 2.5 \times 10^{-24}$</td>
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<tr>
<td>$-1.4 \times 10^{-24} &lt; \text{Im}(k_{V22}^{(5)})$</td>
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<td>$-3.6 \times 10^{-24} &lt; k_{V30}^{(5)}$</td>
<td>$k_{V30}^{(5)} &lt; 3.6 \times 10^{-24}$</td>
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<td>$-2.3 \times 10^{-24} &lt; \text{Re}(k_{V31}^{(5)})$</td>
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<tr>
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<td>$\text{Re}(k_{V32}^{(5)}) &lt; 5.5 \times 10^{-24}$</td>
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<tr>
<td>$-8.0 \times 10^{-25} &lt; \text{Re}(k_{V33}^{(5)})$</td>
<td>$\text{Re}(k_{V33}^{(5)}) &lt; 8.0 \times 10^{-25}$</td>
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</table>
Part III D

TECHNIQUES AND RESULTS: CMB
CMB Polarization

- At $d = 3: k^{(3)}_{(\nu)jm} \Rightarrow$ energy-independent anisotropic birefringence.
- CMB polarization patterns: E and B mode.
- CMB isotropy $\Rightarrow$ no polarization.
  - Density fluctuations $\Rightarrow$ E mode polarization.
    $\Rightarrow$ Correlation $TE$ between temperature and E mode.
  - Primordial gravitational waves $\Rightarrow$ B mode polarization.
  - Generally weak B mode.
    $\Rightarrow$ No $TB$ or $EB$ correlation.

[Krauss et al, Science (2010)]
CMB Polarization

- At $d = 3$: $k_{(v)jm}^{(3)} \Rightarrow$ energy-independent anisotropic birefringence.
- CMB polarization patterns: E and B mode.
- CMB isotropy $\Rightarrow$ no polarization.
  - Density fluctuations $\Rightarrow$ E mode polarization. $\Rightarrow$ Correlation $TE$ between temperature and E mode.
  - Primordial gravitational waves $\Rightarrow$ B mode polarization.
  - Generally weak B mode. $\Rightarrow$ No $TB$ or $EB$ correlation.
- Lorentz violation $\Rightarrow$ birefringence $\Rightarrow$ E$\rightarrow$B and B$\rightarrow$E rotation.
- Search CMB polarization maps for:
  - Unusually large B mode;
  - $TB$ and $EB$ correlation.
- Constraints on all $k_{(v)jm}^{(3)}$ coefficients $O(10^{43}\text{GeV})$. 

Part IV

OUTLOOK
Possible Future Observations

- Optical
- X-rays
- Compton Telescopes
- Ground-based TeV Instruments
- Fermi

Graph showing different data points and trends for future observations.
HAWC & CTA

- Ground-based high-energy gamma-ray telescopes.
  - HAWC: Detector air shower secondary particles in water Cherenkov tanks.
  - CTA: Array of imaging atmospheric Cherenkov telescopes.

- Highest energy photons: 10GeV – 50TeV
- CTA effective area: $10^6$ m$^2$.
- HAWC field of view: 50°.
- Target coefficients: Large $d$ non-birefringent.
AMEGO

- **All-sky Medium Energy Gamma-ray Observatory.**
  - Probe-class instrument concept, white paper for Decadal Review.
- **Compton** and pair production telescope.
- **Energy range:** 300keV – 10GeV.
- Compton scattering: preferentially **perpendicular to polarization**.
  - Sensitive range: 300keV – 2MeV.
- **Field of view:** 70°.
- Time frame: Late 2020s.

[McEnery et al.]
X-Calibur

- Balloon-borne instrument.
- Scattering **hard X-ray polarimeter**.
- Energy range: 20keV – 50keV.
  - Expected flight duration: 14 – 30 days.
  - Primary target: Vela X-1.
- **Future plans:** new focusing mirror.
  - Spare ASTRO-H HXT mirror.
  - 5× effective area.
  - Energy range up to 70keV.
IXPE

- **Imaging X-ray Polarimetry Explorer.**
  - NASA Small Explorer mission.
- **Photo-electric effect polarimeter.**
  - Gas Pixel Detectors (GPDs).
  - Measure photo electron track, parallel to polarization direction.
  - Image location of photon interaction.
- **Energy range:** 2 – 8keV.
- **Sensitivity:** 3% MDP at 1mCrab in 1Ms.

[Weisskopf et al.]
Summary

• Astrophysical tests in the photon sector provide access to
  – Extremely long baselines;
  – Very high photon energies.

• Polarization measurements are many orders of magnitude more sensitive than time of flight.

• Extremely strong constraints up to $d = 6$ exist.

• Future instruments will provide:
  – Larger effective area at very high energies;
  – High energy polarimetric capabilities.
BONUS SLIDES
The *DisCan* Method II

- **What is the uncertainty** on $\hat{\vartheta}^{(d)}_{(I)}(z, \hat{n})$?
- **Apply DisCan method to randomized light curves** [Kislat & Krawczynski, PRD (2015)]:
  - Preserve photon arrival times;
  - Preserve energy spectrum of the source;
  - Permute $t \leftrightarrow E$ assignments.
- **Result:** observation free of any LIV or source intrinsic time lags.
- **If** $\hat{\vartheta}^{(d)}_{(I)}(z, \hat{n})$ **compatible with 0:** find 95% upper and lower confidence limits.

---

Left: Maximizing Shannon information $S$.

Right: Best fit value $\hat{\vartheta}^{(d)}_{(I)}(z, \hat{n})$ from randomized light curves.