Tests of Lorentz symmetry with clocks and trapped ions

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Third IUCSS Summer School

IUCSS

UNIVERSITY OF DELAWARE

Joint Quantum Institute
Outline

• Introduction: AMO precision measurements

• Lorentz violation with atoms and ions

• How to calculate LV matrix elements (electron-photon sector)

• Lorentz violations searches with trapped ions

• Atomic clocks

• Future: prospects for significant improvements
Great times to search for Lorentz violation and other new physics with atoms and molecules!
Advances in AMO Physics:
New world of ultracold

1997 Nobel Prize
Laser cooling and trapping
Steve Chu
Claude Cohen-Tannoudji
Bill Phillips

2001 Nobel Prize
Bose-Einstein Condensation
Eric Cornell
Wolfgang Ketterle
Carl Wieman

300K
500nK
Trapping neutral atoms and ions

Optical Lattices: crystals of light

Trapped ion chains

http://www.uibk.ac.at/th-physik/qo/research/trappedions.html
How to “count” optical frequencies:

Laser-based precision spectroscopy and the optical frequency comb technique

2005 Nobel Prize

Theodor Hänsch

John Hall

Frequency comb “teeth” can mesh with both optical and microwave frequencies

Optical Frequency

5x10^{14} Hz

10^{10} Hz

(Not to scale)
Quantum Control: measuring and manipulation of individual quantum systems

2012 Nobel prize

Serge Haroche

David Wineland

Picture of a string of ions

\[ \Psi = | -1/2 \rangle + | +1/2 \rangle \]

Making quantum superposition of two ions

\[ \text{Ca}^+ \]

3d\[5/2\]
Among many applications:
New capabilities for tests of fundamental physics
Beyond the Standard Model
Search for New Physics with Atoms and Molecules

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\textsuperscript{8}University of Nevada, Reno, Nevada, USA

This article reviews recent developments in tests of fundamental physics using atoms and molecules, including the subjects of parity violation, searches for permanent electric dipole moments, tests of the $CPT$ theorem and Lorentz symmetry, searches for spatiotemporal variation of fundamental constants, tests of quantum electrodynamics, tests of general relativity and the equivalence principle, searches for dark matter, dark energy and extra forces, and tests of the spin-statistics theorem. Key results are presented in the context of potential new physics and in the broader context of similar investigations in other fields. Ongoing and future experiments of the next decade are discussed.
Test of Lorentz symmetry with atomic systems

Lorentz violation with

Structure within the Atom

Quark
Size $< 10^{-19} \text{ m}$

Nucleus
Size $\approx 10^{-14} \text{ m}$

Atom
Size $\approx 10^{-10} \text{ m}$

Electron
Size $< 10^{-18} \text{ m}$

Neutron and Proton
Size $\approx 10^{-15} \text{ m}$

If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.
Standard Model Extension

Spin ½ Dirac fermion $\psi$ with mass $m$

\[ L = \frac{1}{2} i \overline{\psi} \gamma^{\nu} \partial^\nu \psi - \overline{\psi} m \psi \]

\[ L = \frac{1}{2} i \overline{\psi} \Gamma^{\nu} \partial^\nu \psi - \overline{\psi} M \psi \]

$\gamma^\nu$ are Dirac matrices

Kostelecky & Lane, Phys. Rev. D 60, 116010 (1999)
Standard Model Extension
Spin $\frac{1}{2}$ Dirac fermion $\psi$ with mass $m$

$$L = \frac{1}{2} i \bar{\psi} \Gamma_v \partial^v \psi - \bar{\psi} M \psi$$

$$\Gamma_v := \gamma_v + c_{\mu \nu} \gamma^\mu + d_{\mu \nu} \gamma_5 \gamma^\mu$$

$$M := m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H_{\mu \nu} \sigma^{\mu \nu}$$

Testing Lorentz violation: experiments set limits on the coefficients

$$a_\mu, b_\mu, H_{\mu \nu}, c_{\mu \nu}, d_{\mu \nu}$$

for all particles

Kostelecky & Lane, Phys. Rev. D 60, 116010 (1999)
This talk: setting limits on $c_{\mu\nu}$ coefficient for electrons and light

$C_{\mu\nu}$ is a tensor in a four-dimensional spacetime

4-vectors and tensors:

4-vector: $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$

Example of 4-tensor: electromagnetic field tensor

$$F_{\mu\nu} = \begin{bmatrix}
0 & E_x/c & E_y/c & E_z/c \\
-F_x/c & 0 & -B_z & B_y \\
-E_y/c & B_z & 0 & -B_x \\
-E_z/c & -B_y & B_x & 0
\end{bmatrix}.$$
Setting limits on $c_{\mu \nu}$ Lorentz-violating coefficient

$C_{\mu \nu}$ is a tensor in a four-dimensional spacetime

$$c_{\mu \nu} = \begin{cases} 
C_{TT} \\
C_{TJ} \\
C_{JK} 
\end{cases} \quad \mu, \nu = \begin{cases} 
T \\
J = 1, 2, 3 
\end{cases}$$

"timelike" component
three "spacelike" components
The basic idea of atomic physics tests of Lorentz invariance:

Atomic energy levels are affected differently by Lorentz violation: transition frequency will change when experimental set up rotates or moves.

Animation is from Alan Kostelecký web site: http://www.physics.indiana.edu/~kostelec/mov.html
Violation of Lorentz Symmetry with bound electrons

The $c_{\mu\nu}$ tensor modifies in the electronic QED Lagrangian

Atomic energy levels shift!

$$\delta H = - \left( C_0^{(0)} - \frac{2U}{3c^2} c_{00} \right) \frac{p^2}{2} - \frac{1}{6} C_0^{(2)} T_0^{(2)}$$

$U$ is the Newtonian potential $- MG/r$

$C_0^{(0)} \equiv c_{00} + \frac{2}{3} c_{jj}$ Scalar shift due to Lorentz violation

$C_0^{(2)} \equiv c_{11} + c_{22} - 2c_{33}$ Quadrupole shift due to Lorentz violation

$T_0^{(2)} \equiv p^2 - 3p_3^2$
How to calculate the shift of atomic energy levels due to

\[ \delta H = - \left( C_0^{(0)} - \frac{2U}{3c^2} c_{00} \right) \frac{p^2}{2} - \frac{1}{6} C_0^{(2)} T_0^{(2)} \]

Need to calculate the expectation values of the \( p^2 \) and \( T_0^{(2)} \) operators.

\[ T_0^{(2)} \equiv p^2 - 3p_3^2 \]

Need to calculate atomic wave functions.

Note: We can use virial theorem for \( p^2 \):

\[ E = -\frac{1}{2} \left< \Psi \left| \sum_i \hat{p}^2(i) \right| \Psi \right> \]
High-precision atomic calculations

Why is it so difficult?

\[ |\Psi_v\rangle = \Omega |\Psi^{(0)}_v\rangle \]

- **Exact wave function**
- **Many-body operator, describes excitations from lowest-order**
- **Dirac-Hartree-Fock wave function (lowest order)**

Cs: 55 electrons \rightarrow **55-fold excitations to get exact wave function**

Even for 100 function basis set \rightarrow \(100^{55}\)

**Approximate methods:** perturbation theory does not converge well, need to use all-order methods (for example coupled-cluster method)
Cs: atom with single (valence) electron outside of a closed core.

Lowest-order atomic wave function

\[ |\Psi_v^{(0)}\rangle = \alpha_v^\dagger |\Psi_{\text{core}}\rangle \]
What is important for theory accuracy?

**THE NUMBER OF VALENCE ELECTRONS:**

- **ONE** – best accuracy (Cs)
- **TWO** – very good accuracy (Sr)
- **THREE - FOUR** generally ok accuracy (Th)
- **MORE THAN FOUR** ...
What is important for theory accuracy?

THE NUMBER OF VALENCE ELECTRONS:

ONE – best accuracy
TWO – very good accuracy
THREE - FOUR generally ok accuracy
MORE THAN FOUR ... No high-precision methods are presently fully applicable
What is important for theory accuracy?

THE NUMBER OF VALENCE ELECTRONS:

- ONE – best accuracy
- TWO – very good accuracy
- THREE - FOUR generally ok accuracy
- MORE THAN FOUR ... UNDER DEVELOPMENT
“Simplest case”: one valence electron
Cs: atom with single (valence) electron outside of a closed core.

Lowest-order atomic wave function

\[ |\Psi_{v}^{(0)}\rangle = \alpha_{v}^{\dagger} |\Psi_{\text{core}}\rangle \]
Linearized coupled-cluster method

Main idea: allow single and double excitations of the initial wave functions to any orbital from finite basis set

Excitations are described by cluster excitation coefficients $\rho_{ij}, \rho_{ijkl}$.

$7s \rightarrow 8s, 9s, \ldots, 35s, 7p, 8p, \ldots, 35p, 7d, 8d, \ldots$ valence excitation
Coupled-cluster method

- Main feature: includes correlations to all orders of perturbation theory
- Implementation has to be very efficient
- Both formula derivations and required coding are very extensive
Too many terms beyond single and double excitations

\[ H \frac{1}{2} S_2^2 \mid \Psi_v^{(0)} \rightarrow a_i^+ a_j^+ a_l a_k : a_m a_n a_r a_s a_d a_c a_b a_a a_v^+ : 0_c > \]

Contract operators by Wick’s theorem

800 TERMS!
Coupled-cluster method

- Main feature: includes correlations to all orders of perturbation theory
- Implementation has to be very efficient
- Both formula derivations and required coding are very extensive

Codes that write formulas

Codes that write codes

Codes that analyse results and estimate uncertainties
Monovalent systems: very brief summary of what we calculated with all-order method

Properties

- Energies
- Transition matrix elements (E1, E2, E3, M1)
- Static and dynamic polarizabilities & applications
  - Dipole (scalar and tensor)
  - Quadrupole, Octupole
  - Light shifts
  - Black-body radiation shifts
- Magic wavelengths
- Hyperfine constants
- $C_3$ and $C_6$ coefficients
- Parity-nonconserving amplitudes (derived weak charge and anapole moment)
- EDM enhancement factors
- Isotope shifts (field shift and one-body part of specific mass shift)
- Atomic quadrupole moments
- Nuclear magnetic moment (Fr), from hyperfine data
- Lorentz-violating matrix elements

http://www.physics.udel.edu/~msafrono

Systems

- Li, Na, Mg II, Al III, Si IV, P V, S VI, K, Ca II, In, In-like ions, Ga, Ga-like ions, Rb, Cs, Ba II, Tl, Fr, Th IV, U V, other Fr-like ions, Ra II
More than one valence electron
Electron-electron correlation separates into two problems

Example: Thorium $Z = 90$

Four valence electrons outside of a closed core

$$[1s^2 \ldots 4f^{14}5d^{10}6s^26p^6] \ 6d^2\ 7s^2\ \ ^3F_2\ \text{ground state}$$

Problem 1: core-core and core-valence correlations
Problem 2: valence-valence correlations
Main idea: solve two problems by different methods

Configuration interaction works well for systems with a few valence electrons but can not accurately account for core-valence and core-core correlations.

Coupled-cluster method accounts well for core-core and core-valence correlations (as demonstrated by work on alkali-metal atoms).

Therefore, two methods are combined to acquire benefits from both approaches.
Configuration interaction method

\[ \Psi = \sum_i c_i \Phi_i \leftarrow \text{Single-electron valence basis states} \]

\[ (H - E) \Psi = 0 \]

Example: two particle system:

\[ H = h_1(r_1) + h_1(r_2) + h_2(r_1, r_2) \]

\[ \frac{1}{|r_1 - r_2|} \]
Electron-electron correlation separates into two problems

Thorium Z = 90

Core

\[ 1s^2 \ldots 6p^6 \]

Valence electrons

\[ 6d^2 \, 7s^2 \]

Use all-order (coupled-cluster) method to treat core and core-valence correlations

Use configuration interaction (CI) method to treat valence correlations

\[ \Psi = \sum_i c_i \Phi_i \]

\[ (H_{\text{eff}} - E) \Psi = 0 \]
Use **configuration interaction** (CI) method to treat valence correlations

\[ \Psi = \sum_i c_i \Phi_i \]

Number of configurations blows up exponentially with the number of valence electrons!

\[ 150^2 = 22500 \]
\[ 150^4 = 506250000 \]
\[ 150^6 = 11390625000000 \]
Computational and other challenges

1. Computer calculations should finish within “reasonable time”

2. Evaluation and reduction of numerical uncertainties

3. Estimation of “missing physics” uncertainties
Calculations should finish in “reasonable time” less than the lifetime of the Universe.
Calculations should finish in “reasonable time”

Before your current grant proposal runs out
Calculations should finish in “reasonable time”

Before your patience runs out
How to estimate what we do not know?

1. Calculate properties of similar “reference” systems where experimental data exist.

2. Use several different methods of increasing precision and compare results.

3. Calculate all major corrections separately, check for possible cancelations – use to estimate uncertainty.

4. Test the methods of evaluating uncertainties on “reference” systems.

Need experimental benchmarks!
Present CI + all-order code capabilities

Properties:

- Energy levels, ionization potentials, and affinities
- E1, E2, E3, M1, M2, M3 transition matrix elements
- A and B hyperfine constants
- E1 and E2 static and dynamic polarizabilities
- Van der Waals C\(_6\) and C\(_8\) coefficients
- Sensitivity coefficients to variation of fine-structure constant
- Parity-violating, T-odd, Lorentz-violating matrix elements
- Isotope shifts

Systems studied: Mg, Al II, Si III, Ca, Cd, Zn, Sr, In, Ba, La, La II, Ce, Ce II, Ce III, Yb, Lu, Lu II, Hf, Hg, Tl, Tl II, Pb, Pb III, Ra, Th, Th II, Th III, Th V, UIII, UVI, U, superheavy atoms No (Z=102), Lr (Z=103) and Rf (Z=104), and highly-charged Cd-like, In-like, Sn-like, Hg-like, Tl-like ions, negative La-ion
### Example: Cl + all-order results

Two-electron binding energies, differences with experiment

<table>
<thead>
<tr>
<th>Atom</th>
<th>CI</th>
<th>CI + MBPT</th>
<th>CI + All-order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg</td>
<td>1.9%</td>
<td>0.11%</td>
<td>0.03%</td>
</tr>
<tr>
<td>Ca</td>
<td>4.1%</td>
<td>0.7%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Zn</td>
<td>8.0%</td>
<td>0.7%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Sr</td>
<td>5.2%</td>
<td>1.0%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Cd</td>
<td>9.6%</td>
<td>1.4%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Ba</td>
<td>6.4%</td>
<td>1.9%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Hg</td>
<td>11.8%</td>
<td>2.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Ra</td>
<td>7.3%</td>
<td>2.3%</td>
<td>0.67%</td>
</tr>
</tbody>
</table>

CI-MBPT: A package of programs for relativistic atomic calculations based on a method combining configuration interaction and many-body perturbation theory

M.G. Kozlov\textsuperscript{a,b,*}, S.G. Porsev\textsuperscript{a,c,**}, M.S. Safronova\textsuperscript{c,d}, I.I. Tupitsyn\textsuperscript{e}
A Test of Lorentz Symmetry with $^{40}\text{Ca}^+$ ions
Electron-Photon sector

LETTER

Michelson–Morley analogue for electrons using trapped ions to test Lorentz symmetry

T. Pruttivarasin$^{1,2}$, M. Ramm$^1$, S. G. Porsev$^{3,4}$, I. I. Tupitsyn$^5$, M. S. Safronova$^{3,6}$, M. A. Hohensee$^{1,7}$ & H. Häffner$^1$
Electron sector

Coulomb force is symmetric

any LV-signal is attributed to

the electron

Photon sector

Interpretation of LV tests for atoms with zero nuclear spin

electron obeys Lorentz symmetry

any LV-signal is attributed to the photon

Lorentz-violation parameters

\[ c_{\mu\nu}' = c_{\mu\nu} + k_{\mu\nu} / 2 \]

the electron (\( c_{\mu\nu} \)) sector

the photon (\( k_{\mu\nu} \)) sector
Ca\(^+\) ion: building quantum computers

Long-lived state 1.2 second lifetime

Quantum Bit

\(\psi = a\ket{0} + b\ket{1}\)

http://www.uibk.ac.at/th-physik/qo/research/trappedions.html
### LV-violating matrix elements for the 3d states of Ca$^+$

TABLE VI: Lowest-order DF, DF+RPA, second-order MBPT, third-order MBPT, CI+single-double excitations, and all-order results for the $\langle 3d_{3/2}|p^2|3d_{3/2}\rangle$ and $\langle 3d_j||T_0^{(2)}||3d_j\rangle$ reduced matrix elements in Ca$^+$ in atomic units. The values in the DF(FC) and DF columns are lowest-order DF values calculated with and without the frozen core approximation. Final data are given in bold font.

<table>
<thead>
<tr>
<th>Matrix element</th>
<th>DF(FC)</th>
<th>DF</th>
<th>RPA</th>
<th>MBPT2</th>
<th>MBPT3</th>
<th>CI+SD</th>
<th>All-order</th>
<th>VT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 3d_{3/2}</td>
<td>p^2</td>
<td>3d_{3/2}\rangle$</td>
<td>3.05</td>
<td>0.67</td>
<td>0.66</td>
<td>0.79</td>
<td>1.37</td>
<td>0.73</td>
</tr>
<tr>
<td>$\langle 3d_{5/2}</td>
<td>p^2</td>
<td>3d_{5/2}\rangle$</td>
<td>3.04</td>
<td>0.66</td>
<td>0.66</td>
<td>0.79</td>
<td>1.36</td>
<td>0.73</td>
</tr>
<tr>
<td>$\langle 3d_{3/2}</td>
<td></td>
<td>T^{(2)}</td>
<td></td>
<td>3d_{3/2}\rangle$</td>
<td>5.45</td>
<td>6.22</td>
<td>5.72</td>
<td>6.66</td>
</tr>
<tr>
<td>$\langle 3d_{5/2}</td>
<td></td>
<td>T^{(2)}</td>
<td></td>
<td>3d_{5/2}\rangle$</td>
<td>7.12</td>
<td>8.11</td>
<td>7.47</td>
<td>8.67</td>
</tr>
</tbody>
</table>
LV-violating matrix elements for the 3d states of Ca$^+$

TABLE V: Lowest-order DF, DF+RPA, third-order MBPT (column 3$^{rd}$), and all-order results for $\langle 3d_j||T_0^{(2)}||3d_j \rangle$ reduced matrix elements in Ca$^+$ in atomic units. The all-order ab initio results are listed in columns SD and SDpT; scaled all-order data are listed in columns SD$_{sc}$ and SDpT$_{sc}$.

<table>
<thead>
<tr>
<th>Matrix element</th>
<th>DF</th>
<th>+RPA</th>
<th>3$^{rd}$</th>
<th>SD</th>
<th>SDpT</th>
<th>SD$_{sc}$</th>
<th>SDpT$_{sc}$</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 3d_{3/2}</td>
<td></td>
<td>c\gamma_0 (\gamma^j p_j - 3\gamma^3 p_3)</td>
<td></td>
<td>3d_{3/2} \rangle$</td>
<td>5.454</td>
<td>5.723</td>
<td>7.239</td>
<td>7.204</td>
</tr>
<tr>
<td>$\langle 3d_{5/2}</td>
<td></td>
<td>c\gamma_0 (\gamma^j p_j - 3\gamma^3 p_3)</td>
<td></td>
<td>3d_{5/2} \rangle$</td>
<td>7.116</td>
<td>7.468</td>
<td>9.439</td>
<td>9.397</td>
</tr>
</tbody>
</table>
A Test of Lorentz Symmetry with electrons in Ca$^+$

Scalar shift cancels out for different $m_J$ states:

$$\delta \mathcal{H} = - C_0^{(2)} \frac{(p^2 - 3p_z^2)}{6m_e}$$

$$\Delta E \frac{\hbar}{h} = \left[ (2.16 \times 10^{15}) - (7.42 \times 10^{14}) m_j^2 \right] C_0^{(2)}$$

We calculate the expectation value of the above Hamiltonian for the 3d$_{5/2}$ state of Ca$^+$

Lorentz violating energy shift depends on magnetic quantum number $m_j$!
Lorentz violating energy shift depends on magnetic quantum number $m_j$

Use the $D_{5/2}$-manifold of $^{40}\text{Ca}^+$

Slide courtesy of Hartmut Häffner

DO NOT NEED TO ROTATE THE EXPERIMENT!
The frequency difference (in Hz) for a pair of Ca+ ions used is given by

\[
\frac{2}{\hbar} \left( E_{m_j=5/2} - E_{m_j=1/2} \right) = (-1.484 \times 10^{15} \text{ Hz}) \times \left( (5/2)^2 - (1/2)^2 \right) C_0^{(2)} \\
= (-8.9(2) \times 10^{15} \text{ Hz}) \times C_0^{(2)}
\]
Test of Lorentz invariance with Ca$^+$

Measure the energy difference between atomic levels with $m_j=1/2$ and $m_j=5/2$ for 24 hours as Earth rotates.

**Change in energy will indicate Lorentz violation.**

No change will set limit on Lorentz-violating tensor $c_{\mu\nu}$.

\[
\vec{B} \quad m_j=1/2 \quad m_j=5/2
\]

Superposition of two states

$|\Psi\rangle = |\text{state 1}\rangle + e^{i\omega t}|\text{state 2}\rangle$

Lorentz violation will modulate energy with the Earth’s rotation.
Problem: Systematic errors due to magnetic Field fluctuation

Energy levels fluctuate in varying magnetic field

Quantum Information solution: create superposition of two ions which is protected from magnetic field fluctuations.

\[ |\Psi^R\rangle = \frac{1}{\sqrt{2}} \left( | -5/2, +5/2 \rangle + | -1/2, +1/2 \rangle \right) \]
Decoherence Free Subspace

\[ \Psi = |-1/2, +1/2\rangle + |-5/2, +5/2\rangle \]

Entanglement of two ions

\[ |\psi\rangle = \frac{00\rangle + 11\rangle}{\sqrt{2}} \]

<table>
<thead>
<tr>
<th>Results of the measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>First qubit</td>
</tr>
<tr>
<td>Second qubit</td>
</tr>
</tbody>
</table>

\[ |\psi\rangle \neq |\alpha\rangle \otimes |\beta\rangle \]

Entangled states
Decoherence-free states

Use superposition of two ions to reject magnetic field fluctuations common to both ions.

\[ m_J = -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \]

\[ D_{\frac{5}{2}} \]

\[ \left| \Psi^R \right\rangle = \frac{1}{\sqrt{2}} \left( \left| -\frac{5}{2}, \frac{5}{2} \right\rangle + \left| -\frac{1}{2}, \frac{1}{2} \right\rangle \right) \]
As the Earth rotates, the direction of the electron motion changes and any violation of local Lorentz invariance will give rise to variations in the electron’s energy.

A Test of Lorentz Symmetry with Ca+ ions

Results:

(1) Verified the isotropy of the electron’s dispersion relation at the level of one part in $10^{18}$, a 100-fold improvement on previous work [Dy, PRL 111, 050401 (2013)].

(2) Tested the rotational invariance of the Coulomb potential: fivefold-improved limit on anisotropies in the speed of light [comparing to PRL 103, 090401 (2009)].

Demonstration of the potential of quantum information techniques in the search for physics beyond the standard model.

Which is the best system for next generation test of Lorentz symmetry?
What is the best atomic system?

(1) Need the largest matrix element of of Lorentz violating Hamiltonian:

\[ \delta \mathcal{H} = -C_0^{(2)} \frac{(p^2 - 3p_z^2)}{6m_e} \]

Long-lived state 1.2 second lifetime NOT LONG ENOUGH!

(2) Need long lifetime of the excited atomic state for Lorentz violation probe or Find atoms with suitable ground state for LV probe in the Ca^+

[Diagram of Ca^+ with levels S_{1/2} and D_{5/2}]
<table>
<thead>
<tr>
<th>Ion</th>
<th>State</th>
<th>Method</th>
<th>$&lt;i|T^{(2)}|i&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca$^+$</td>
<td>$3d^2D_{3/2}$</td>
<td>LCCSD</td>
<td>7.09(12)</td>
</tr>
<tr>
<td></td>
<td>$3d^2D_{5/2}$</td>
<td>LCCSD</td>
<td>9.25(15)</td>
</tr>
<tr>
<td>Ba$^+$</td>
<td>$5d^2D_{3/2}$</td>
<td>LCCSD</td>
<td>6.83</td>
</tr>
<tr>
<td></td>
<td>$5d^2D_{5/2}$</td>
<td>LCCSD</td>
<td>8.65</td>
</tr>
<tr>
<td>Yb$^+$</td>
<td>$4f^{14}5d^2D_{3/2}$</td>
<td>LCCSD</td>
<td>9.96</td>
</tr>
<tr>
<td></td>
<td>$4f^{14}5d^2D_{5/2}$</td>
<td>DF</td>
<td>7.23</td>
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$T_0^{(2)} = p^2 - 3p_3^2$
Need the largest matrix element of Lorentz violating Hamiltonian:

\[ \delta \mathcal{H} = -C_0^{(2)} \frac{(p^2 - 3p_z^2)}{6m_e} \]

\[ \langle r \rangle = 2.64 \text{ a.u.} \]
\[ \langle \delta H \rangle = 9.25 \text{ a.u.} \]
Need the largest matrix element of Lorentz violating Hamiltonian:

\[ \delta \mathcal{H} = - C_0^{(2)} \left( \frac{p^2 - 3p_z^2}{6m_e} \right) \]

\[ \langle r \rangle = 2.64 \text{ a.u.} \]
\[ \langle \delta H \rangle = 9.25 \text{ a.u.} \]
Need the largest matrix element of Lorentz violating Hamiltonian:
\[ \delta \mathcal{H} = -C_0^{(2)} \frac{(p^2 - 3p_z^2)}{6m_e} \]

\[ \langle r \rangle = 0.76 \text{ a.u.} \]
\[ \langle \delta H \rangle = 135 \text{ a.u.} \]

\[ \langle r \rangle = 2.64 \text{ a.u.} \]
\[ \langle \delta H \rangle = 9.25 \text{ a.u.} \]
Even Yb isotopes or 171 No LV contribution from the nucleus

\[ ^{173}\text{Yb } I=3/2 \text{ deformed nucleus} \]
4f shell is also highly contracted in highly-charged ions: LLI matrix elements are also enhanced

\[ \langle i | T^{(2)} | i \rangle = 149 \text{ a.u.} \]

Highly-charged ions for \( \alpha \)-variation

HIGHLY-CHARGED IONS

3s^2 \, ^1S_0 \rightarrow 3s3p \, ^3P_0 \text{ transition in Mg-like ions}

Cl^{6+} 102 \text{ nm}
Si^{2+} 190 \text{ nm}
Al^{+} 267 \text{ nm}
Mg 458 \text{ nm}
How do you get optical transitions in highly-charged ions?

Accidental wonders of atomic structure!

<table>
<thead>
<tr>
<th>Period</th>
<th>Group</th>
<th>Element</th>
<th>Atomic Number</th>
<th>Electron Configuration</th>
<th>Ionization Energy (eV)</th>
<th>Spectral Lines</th>
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<td>289 nm, 163 nm</td>
</tr>
</tbody>
</table>

289 nm and 163 nm are emission lines in the spectra of Sn-like Ba<sup>6+</sup> and Sn-like Pr<sup>9+</sup> ions, respectively.
| Ion     | N  | Level         | $J$ | $|\langle J \parallel T^{(2)} \parallel J \rangle|$ | $|\Delta E/(\hbar C_0^{(2)})|$ |
|---------|----|---------------|-----|---------------------------------|-----------------------------------|
| Ca$^+$  | 19 | $3d$          | $5/2$ | 9.3                              | $4.5 \times 10^{15}$ [9]         |
| Yb$^+$  | 69 | $4f^{13}6s^2$ | $7/2$ | 135                             | $6.1 \times 10^{16}$ [10]        |
| Tm      | 69 | $4f^{13}6s^2$ | $7/2$ | 141                             | $6.4 \times 10^{16}$             |
| Yb      | 70 | $4f^{13}5d6s^2$ | 2   | 74                             | $3.9 \times 10^{16}$             |
| Th$^{3+}$ | 87 | $5f$         | $5/2$ | 47                              | $2.2 \times 10^{16}$             |
| Sm$^{15+}$ | 47 | $4f$         | $5/2$ | 128                             | $5.7 \times 10^{16}$             |
| Sm$^{14+}$ | 48 | $4f^2$      | 4    | 124                             | $5.5 \times 10^{16}$             |
| Sm$^{13+}$ | 49 | $5s^24f$     | $5/2$ | 120                             | $5.8 \times 10^{16}$             |
| Eu$^{14+}$ | 49 | $4f^25s$    | $7/2$ | 120                             | $5.4 \times 10^{16}$             |
| Nd$^{10+}$ | 50 | $4f^2$      | 4    | 96                              | $4.3 \times 10^{16}$             |
| Cf$^{15+}$ | 83 | $5f6p^2$  | $5/2$ | 112                             | $5.4 \times 10^{16}$             |
| Cf$^{17+}$ | 81 | $5f$        | $5/2$ | 144                             | $6.9 \times 10^{16}$             |
| Os$^{18+}$ | 58 | $4f^{12}$ | 6    | 367                             | $1.4 \times 10^{17}$             |
| Pt$^{20+}$ | 58 | $4f^{12}$ | 6    | 412                             | $1.6 \times 10^{17}$             |
| Hg$^{22+}$ | 58 | $4f^{12}$ | 6    | 459                             | $1.8 \times 10^{17}$             |
| Pb$^{24+}$ | 58 | $4f^{12}$ | 6    | 507                             | $2.0 \times 10^{17}$             |
| Bi$^{25+}$ | 58 | $4f^{12}$ | 6    | 532                             | $2.1 \times 10^{17}$             |
| U$^{34+}$ | 58 | $4f^{12}$ | 6    | 769                             | $3.0 \times 10^{17}$             |
Coulomb crystallization of highly charged ions

L. Schmöger,¹,² O. O. Versolato,¹,²* M. Schwarz,¹,² M. Kohnen,² A. Windberger,¹ B. Piest,¹ S. Feuchtenbeiner,¹ J. Pedregosa-Gutierrez,³ T. Leopold,² P. Micke,¹,² A. K. Hansen,⁴† T. M. Baumann,⁵ M. Drewsen,⁴ J. Ullrich,² P. O. Schmidt,²,⁶ J. R. Crespo López-Urrutia¹†

A

EBIT: HCl production at MK temperatures

Beamline: Deceleration and pre-cooling of HCl bunches

Paul trap: Crystallization and cooling of HCl to mK temperatures

loading HCls
Strongly enhanced effects of Lorentz symmetry violation in entangled Yb\(^+\) and highly-charged ions

Possible factor of \textbf{27000} improvement in Lorentz symmetry test comparing to Ca\(^+\)!

- Sensitivity (larger matrix element): \(x\ 15\) (Yb\(^+\)) – \(50\) (HCl)
- Longer probe time (to 60 seconds): \(x\ 45\)
- Year-long measurement: \(x\ 20\)
- Pure state preparation: \(x\ 2\)


Limits to Lorentz violation

Lorentz violating effects are suppressed by some power of $R$

$$R = \frac{\text{Electroweak scale}}{\text{Plank energy scale}} \sim 2 \times 10^{-17}$$

Present limits for electron sector

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>$c_{JK} &lt; 10^{-18}$</td>
</tr>
<tr>
<td>[2]</td>
<td>$c_{TJ} &lt; 10^{-13} - 10^{-15}$</td>
</tr>
</tbody>
</table>

Yb+ projected limit [3]

$\begin{align*}
  c_{JK} & < 10^{-23} \\
  c_{TJ} & < 10^{-19}
\end{align*}$

Yb+ experiment sensitivity will be significantly below O(1) suppression limit.

Problem: still need two ions for the scheme to work so need a new idea to directly use optical atomic clocks
Ingredients for an atomic clock

1. Atoms are all the same and will oscillate at exactly the same frequency (in the same environment): you now have a perfect oscillator!

2. Take a sample of atoms (or just one)

3. Build a device that produces oscillatory signal in resonance with atomic frequency

4. Count cycles of this signal

Ludlow et al., RMP 87, 637 (2015)
valentinagurarie.wordpress.com/tag/atom/
What do we need to build a clock?

Schematic view of an optical atomic clock: the local oscillator (laser) is resonant with the atomic transition. A correction signal is derived from atomic spectroscopy that is fed back to the laser. An optical frequency synthesizer (optical frequency comb) is used to divide the optical frequency down to countable microwave or radio frequency signals.

Requirements for atomic reference

(1) Metastable level
(2) Near optical transition

Strontium optical lattice neutral atom clock

Yb+ single trapped ion clock

http://www.nist.gov/pml/div689/20140122_strontium.cfm
How good is a clock: stability

**Stability** is a measure of the precision with which we can measure a quantity (think of how widely scattered a group of arrows at target might be), and is usually stated as a function of averaging time since for many noise processes the precision increases (i.e., the noise is reduced through averaging) with more measurements.

The stability is usually set by the combination of the inherent frequency purity of the physical system and the signal-to-noise ratio with which we can measure the system.

How good is a clock: uncertainty

In contrast, the (absolute) **uncertainty** for an atomic clock tells us how well we understand the physical processes that can shift the measured frequency from its unperturbed ("bare"), natural atomic frequency (think of how off-centre our group of arrows might be).

Small absolute uncertainty is clearly an essential part of a good primary frequency standard and requires extensive evaluation of all known physical shifts (usually called "systematic effects").

Clock instability

Let us first consider the formula for clock instability, $\sigma_y$, in the regime where it is limited by fundamental (as opposed to technical) noise sources, such as atomic statistics based on the number of atoms:

$$
\sigma_y(\tau) \approx \frac{\Delta \nu}{\nu_0} \sqrt{\frac{T_c}{\tau}} \sqrt{N}
$$

- spectroscopic linewidth of the clock system
- clock transition frequency
- the number of atoms or ions used in a single measurement
- the time required for a single measurement cycle
- the averaging period
Sr clock will lose 1 second in 15 billion years!

http://www.nist.gov/pml/div689/20140122_strontium.cfm

Sr: $2 \times 10^{-18}$
Optical vs. microwave clocks

PTB Yb$^+$

JILA Sr
Search for physics beyond the standard model with atomic clocks

Atomic clocks can measure and compare frequencies to exceptional precisions!

If fundamental constants change (now) due to various “new physics” effects atomic clock may be able to detect it.
New Methods for Testing Lorentz Invariance with Atomic Systems

R. Shaniv,¹ R. Ozeri,¹ M. S. Safronova,²,³ S. G. Porsev,²,⁴ V. A. Dzuba,⁵ V. V. Flambaum,⁵ and H. Häffner⁶

A broadly applicable experimental proposal to search for the violation of local Lorentz invariance (LLI) with atomic systems.

The new scheme uses dynamic decoupling and can be implemented in current atomic clocks experiments, both with single ions and arrays of neutral atoms.

Moreover, the scheme can be performed on systems with no optical transitions, and therefore it is also applicable to highly charged ions which exhibit particularly high sensitivity to Lorentz invariance violation.

The scheme is scalable for many atoms or ions.
Yb$^+$ clock comparison for hundredfold improved LV bounds

C. Sanner, N. Huntemann, R. Lange, Chr. Tamm, E. Peik, M. Safronova and S. Porsev
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Hartmut Häffner, UC Berkeley
Ravid Shaniv, Roee Ozeri, The Weizmann Institute of Science, Israel