Vacuum Cherenkov radiation

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Outline

- Cherenkov radiation in media
- Properties of vacuum Cherenkov radiation
- Research results: 1997 – 2018
Cherenkov radiation in media

Idaho National Laboratory’s Advanced Test Reactor core
(Argonne National Laboratory; Wikipedia)
Cherenkov radiation in media

- Experimental discovery: Cherenkov, Vavilov (1934)
- Theoretical explanation: Frank, Tamm (1937)
Cherenkov cone and Cherenkov angle
Cherenkov-type process in vacuo
Refractive index of vacuum (standard)
Refractive index of vacuum (isotropic)
Refractive index of vacuum (anisotropic)
Refractive index of vacuum (parity-odd)
Refractive index of vacuum (birefringent, 1 spacelike direction)
Refractive index of vacuum (birefringent, 2 spacelike directions)
Refractive index of vacuum (dispersive)
Different kinds of vacuum Cherenkov radiation
Modified photons: dispersive

\[ \tilde{\kappa}_{tr} = \frac{3}{5} \]
Modified photons: dispersive

\[ k_{AF}^3 = 5m_\psi \]
Modified fermions: spin-degenerate

\[ c^{(4)00} = -1/2 \]
Modified fermions: spin-degenerate

\[ b^{(3)0} = m_\psi \]
Research on vacuum Cherenkov radiation

Based on Standard-Model Extension:

- B. Altschul, Phys. Rev. Lett. 98, 041603 (2007): modified e.m., \textit{CPT}-even
- B. Altschul, Phys. Rev. D 75, 105003 (2007): modified e.m., \textit{CPT}-even
- B. Altschul, Phys. Rev. D 90, 021701 (2014): modified e.m., \textit{CPT}-odd
- MS, Phys. Rev. D 96, 095026 (2017): Dirac fermions

Beyond Standard-Model Extension (not expected to be complete):

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1. Cherenkov radiation in media
2. Lorentz-violating vacuum
3. Vacuum Cherenkov radiation
4. What we know
   - Isotropic modified speed of light
   - Electrodynamics of Vacuum Cherenkov radiation
   - Quantum field theory treatment
   - Birefringent modified Maxwell theory
   - Vacuum Cherenkov radiation at LEP
   - Timelike MCS theory
   - Gravitational Cherenkov radiation
   - Partonic description
   - Vacuum Cherenkov radiation for pions
   - Quantization of timelike MCS
   - Lorentz-violating fermions
   - Radiation of W bosons
5. Conclusion
Isotropic modified speed of light

Coleman, Glashow (1997)

- Isotropic modification of speed of light: Photons traveling with $c < 1$
- Threshold energy for $p \rightarrow p + \gamma$:

$$E_{th} = \frac{m}{\sqrt{1 - c^2}}.$$

- Modified action not available: scalar QED(?) Feynman rules and modified phase space
- Rate of energy loss at first order in $1 - c$:

$$\frac{dE}{dx} \sim -\frac{\alpha}{3} E^2 (1 - c) \left\{ 1 - \left( \frac{E_{th}}{E} \right)^2 \right\} \left\{ 1 - \frac{3}{8} \left( \frac{E_{th}}{E} \right)^2 \right\}$$

$$\sim -\frac{\alpha}{3} (1 - c) E^2,$$

- Observation of protons with $E = 10^{11}$ GeV on Earth:

$$1 - c < 5 \times 10^{-23}$$
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Classical description within *CPT*-odd modification of electromagnetism:

\[ \mathcal{L}_{\text{MCS}} = -\frac{1}{4} F^2 - m_{cs} \zeta^\mu A^\nu \tilde{F}_{\mu\nu} - A \cdot j, \]

Modified modes \( k_0 = \omega_k(\vec{k}) \) obtained from equation \( D(k_0, \vec{k}) = 0 \):

What are the conditions for vacuum Cherenkov radiation?

Observer Lorentz invariance: analysis in rest frame of charged particle

\( \implies \) Four-current must be stationary

\[ \tilde{j}_\mu(p_0, \vec{p}) = 2\pi \delta(p_0) \tilde{j}_\mu(\vec{p}) \]
General (time-independent) solution to field equations

\[ A^{\mu}(\vec{r}) = \int_{C_\omega} \frac{d^3k}{(2\pi)^3} \frac{N^{\mu\nu}(\vec{k})\tilde{j}_{\nu}(\vec{k})\exp(i\vec{k} \cdot \vec{r})}{D(0, \vec{k})} \]

⇒ Real, spacelike wave vectors must exist obeying dispersion equation

Wave pattern similar to water surface of moving boat

No sharply defined Cherenkov cone (dispersion)

Consistent with vacuum refractive index \( n > 1 \):

\[ 0 > k^2 = \omega^2 - \vec{k}^2 = \vec{k}^2 \left( \frac{\omega^2}{k^2} - 1 \right) = \vec{k}^2(v_{ph}^2 - 1), \quad v_{ph} \equiv \frac{\omega}{|\vec{k}|} = \frac{1}{n} < 1 \]

Boosting provides condition on Cherenkov angle:

\[ (k'^{\mu}) = \left( \frac{\vec{\beta} \cdot \vec{k}'}{k'} \right), \quad \frac{1}{n} = v_{ph} = \frac{\omega'}{|k'|} = \frac{\vec{\beta} \cdot \vec{k}'}{|\vec{k}'|} = |\vec{\beta}| \cos \theta \]
Electrodynamics of Vacuum Cherenkov radiation


- No net radiated energy in rest frame of localized source

$$\partial_\mu \theta^{\mu \nu} = j_\mu F^{\mu \nu} \Rightarrow \int_\sigma d\vec{\sigma} \cdot \vec{S} = \vec{0}$$

- Look at radiated three-momentum

$$\dot{\vec{P}} = i \int_{C_\omega} \frac{d^3k}{(2\pi)^3} \frac{N(\vec{k})}{D(0, \vec{k})} \vec{k}, \quad \frac{N(-\vec{k})}{D(0, -\vec{k})} = \frac{N(\vec{k})}{D(0, \vec{k})}$$

$$\Rightarrow$$ Vanishes for nonsingular integrand

$$\Rightarrow$$ Momentum is radiated for spacelike solutions of $D(0, \vec{k}) = 0$

- Circular polarization of radiation found

- Backreaction on accelerating particle:

$$\dot{p}^\mu = -\dot{\vec{P}}^\mu$$

$$\Rightarrow$$ Particle not moving on geodesic

- Estimate of radiation rate in quantum physics:

$$\dot{\vec{P}} \sim -q^2 m_{cs}^2 \text{sgn}(\zeta^0) |\mathcal{M}|^2 \zeta^2 \vec{e}_\zeta$$
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     - Vacuum Cherenkov radiation for pions
     - Quantization of timelike MCS
     - Lorentz-violating fermions
     - Radiation of W bosons
5. Conclusion
Consider QFT based on modified photons and standard fermions:

\[ \mathcal{L} = \mathcal{L}_\gamma + \mathcal{L}_{\psi \gamma}, \]
\[ \mathcal{L}_\gamma = -\frac{1}{4} F^2 + \frac{m_{cs}}{2} \zeta^\mu A_\nu \tilde{F}^{\mu\nu}, \quad \mathcal{L}_{\psi \gamma} = \bar{\psi}(i\gamma - m)\psi \]

Frame with purely spacelike preferred direction \((\zeta^\mu) = (0, \vec{\zeta})\)

\[ k_\parallel = \vec{k} \cdot \vec{\zeta}, \quad k_\perp = |\vec{k} - k_\parallel \vec{\zeta}| \]

Vacuum Cherenkov process \(e^- \rightarrow \Theta + e^-\) allowed for \(q_\parallel \neq 0\) without threshold

Tree-level Feynman diagram:
Quantum field theory treatment

Kaufhold, Klinkhamer (2006)

- Exact decay rate:

- Asymptotic behaviors:

\[ \Gamma_{|q_\parallel| \ll m} = \frac{2}{3} \alpha m_{cs} \left( \frac{|q_\parallel|}{m} \right)^3 \]

\[ \Gamma_{|q_\parallel| \gg m} = \frac{\alpha}{2} m_{cs} \left[ \ln \left( \frac{|q_\parallel|}{m_{cs}} \right) + C \right] \]

- Photon circularly polarized for \(|k_\parallel| \gg m\), linearly polarized for \(|k_\parallel|/m \mapsto 0\)

- Electron helicity shown to be preserved for large electron momentum

\[ \Rightarrow \text{Angular momentum violation} \]
Quantum field theory treatment

Kaufhold, Klinkhamer (2007)

- Consider MCS theory with purely spacelike $k^\mu_{AF}$
- Realistic matter sector: standard Dirac particles (see before)
- Toy model matter sector: scalar particles

\[ \mathcal{L} = \mathcal{L}_\gamma + \mathcal{L}_{\phi \gamma} \]
\[ \mathcal{L}_\gamma = -\frac{1}{4} F^2 + \frac{m_{cs}}{2} \zeta_{\mu \nu} A_{\nu} \tilde{F}^{\mu \nu}, \quad \mathcal{L}_{\phi \gamma} = (D_{\mu} \phi)^* (D^{\mu} \phi) - m^2 \phi^* \phi \]

- Calculation of decay rate for scalars and spinors

- Direction-dependent lifetime of high-energy protons: $\Gamma_p \sim (m_{cs}/2) |\cos \theta_p|$

\[ m_{cs} \leq 2 \times 10^{-33} \text{ eV} \sim 1/L_{universe} \]
Radiated-energy rate for scalars and spinors

Leading-order terms for low and intermediate energies independent of spin:

\[
\left. \frac{dW_{\text{scalar}}}{dt} \right|_{q \gg m} = \frac{\alpha}{4} m_{cs} q_{\parallel} + \ldots,
\left. \frac{dW_{\text{spinor}}}{dt} \right|_{q \gg m} = \frac{\alpha}{3} m_{cs} q_{\parallel} + \ldots
\]

Classical description based on Frank-Tamm formula:

\[
\left. \frac{d^2 W}{dt d\omega} \right|_{\text{Frank-Tamm}} = \frac{\beta Q^2}{4\pi} \sin^2 \theta_c(\omega) \omega
\]

\[\Rightarrow\] Cut-off frequency \(\omega_{\text{max}}\) must exist to render \(dW/dt\) finite
Take photon momentum (quantum effect!) into account:

\[
\cos \theta_c = \frac{1}{\beta n(\omega)} \left\{ 1 + \frac{\omega}{2E} [n(\omega)^2 - 1] \right\}
\]

\[\Rightarrow\] Even for \( n(\omega) = n \), natural cutoff frequency introduced

- Refractive index for \( \ominus \) mode of MCS theory:

\[
n(\omega) = 1 + \frac{m_{cs} | \cos \theta |}{2\omega} + \ldots, \quad \omega_{\text{max}} = E
\]

- Confirmation of leading-order radiated-energy rate for scalars:

\[
\left. \frac{dW}{dt} \right|_{\text{quantum}} = \frac{\alpha}{4} m_{cs} | q_\parallel | + \ldots
\]

- For particles with spin: extra contribution (e.g., magnetic dipole moment):

\[
\Delta \left. \frac{dW}{dt} \right|_{\text{spin}} = \frac{\alpha}{\beta} \int_0^{\omega_{\text{max}}} d\omega \omega \left\{ \frac{\omega^2}{4E^2} [n(\omega)^2 - 1] \right\} = \frac{\alpha}{12} m_{cs} | q_\parallel | + \ldots
\]
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5. Conclusion
Consider $CPT$-even extension of photon sector: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_F$

$$\mathcal{L}_0 = -\frac{1}{4} F^2 + \bar{\psi} [\gamma^\mu (i\partial_\mu - eA_\mu) - m] \psi, \quad \mathcal{L}_F = -\frac{1}{4} k^{\mu\nu\rho\sigma}_F F_{\mu\nu} F_{\rho\sigma}$$

- Weakly constrained 9 nonbirefringent coefficients $\tilde{k}^{\mu\nu}$
- Coordinate transformation between photon and matter sector:

$$\mathcal{L} \rightarrow \mathcal{L}' = \mathcal{L}_0 - \frac{1}{2} \tilde{k}^{\mu\nu} \bar{\psi} \gamma_\nu (i\partial_\mu - eA_\mu) \psi$$

- LV in photon sector: Cherenkov cone distorted and direction-dependent
- LV in fermion sector: Cherenkov cone standard
- Threshold energy:

$$E_{th} = \frac{m}{\sqrt{\tilde{k}_{jk} \hat{\nu}_j \hat{\nu}_k - (\tilde{k}_{0j} + \tilde{k}_{j0}) \hat{\nu}_j + \tilde{k}_{00}}}$$

- Problem: Cutoff for radiated-energy $P(\omega)$?
- Causality problems at $\omega \sim \Lambda_{\tilde{k}} = m/\sqrt{\tilde{k}}$

$$\Rightarrow \Lambda_{\tilde{k}} \text{ acts as physical cutoff}$$

$$P = \frac{e^2}{8\pi} \theta_c^2 \Lambda_{\tilde{k}}^2$$
MCS-theory and $CPT$-even modified photons:

$$
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{AF} + \mathcal{L}_F, \quad \mathcal{L}_0 = -\frac{1}{4} F^2 - j \cdot A,
$$

$$
\mathcal{L}_{AF} = \frac{1}{2} k_{AF}^\mu \tilde{F}_{\mu\nu} A^\nu, \quad \mathcal{L}_F = -\frac{1}{4} k_{F}^{\mu\nu\varrho\sigma} F_{\mu\nu} F_{\varrho\sigma}
$$

Cherenkov angle

$$
\theta_c^2 \approx 2 \left[ 1 - \frac{\omega(k\hat{v})}{kv} \right]
$$

$\Rightarrow$ Right-angled and circular, but varies with propagation direction

Single subluminal mode for MCS theory:

$$
\theta_c^2 \approx 2 \left( 1 - \frac{1}{v} + \frac{|k_{AF}^0 - \tilde{k}_{AF} \cdot \hat{v}|}{kv} \right)
$$

Two modes possible for modified Maxwell theory:

$$
\theta_c^2 \approx 2 \left( 1 - \frac{1}{v} - \frac{\rho(\hat{v}) \pm \sigma(\hat{v})}{v} \right)
$$

Power radiated (mode-by-mode analysis: $\tilde{n} = n(k, \hat{\epsilon}(i))$):

$$
P_{(i)}^{\text{classical}}(\omega, \phi) \sim |\hat{\epsilon}(i) \cdot \hat{\epsilon}(0)|^2 (\theta_c(i))^2 \omega(i)(k\hat{v})$$
Consider isotropic modified Maxwell theory:

\[
\mathcal{L} = \mathcal{L}_\gamma + \mathcal{L}_{\psi \gamma}, \quad \mathcal{L}_\gamma = \frac{1}{2} \left[ (1 + \tilde{\kappa}_{\text{tr}}) \vec{E}^2 - (1 - \tilde{\kappa}_{\text{tr}}) \vec{B}^2 \right]
\]

\[
\mathcal{L}_{\psi \gamma} = \frac{i}{2} \overline{\psi} \phi \psi - m \overline{\psi} \psi,
\]

Obtain radiated-energy rate in quantum field theory:

\[
\frac{1}{\alpha} \frac{dW}{dt} \left|_{\text{K-S}} \right. = \frac{7}{12} \alpha \tilde{\kappa}_{\text{tr}} q^2 + \ldots
\]

Threshold energy and large-energy behavior:

\[
E_{\text{th}} = \frac{m}{\sqrt{2\tilde{\kappa}_{\text{tr}}}}, \quad \left. \frac{dW}{dt} \right|_{\text{K-S}} \bigg|_{q \gg m} = \frac{7}{12} \alpha \tilde{\kappa}_{\text{tr}} q^2 + \ldots
\]
Birefringent modified Maxwell theory

Klinkhamer, MS (2008)

- Compare to semi-classical description: \( n \approx 1 + \tilde{\kappa}_{tr} \)

\[
\frac{dW}{dt}\bigg|_{\text{quantum}} = \frac{\alpha}{3} \tilde{\kappa}_{tr} q^2 + \cdots = \frac{dW}{dt}\bigg|_{\text{C-G}}
\]

\[
\Delta \frac{dW}{dt}\bigg|_{\text{spin}} = \frac{\alpha}{4} \tilde{\kappa}_{tr} q^2 + \cdots = \frac{dW}{dt}\bigg|_{\text{K-S}} - \frac{dW}{dt}\bigg|_{\text{C-G}}
\]

- Investigate ultra-high-energy cosmic rays

- Pierre-Auger event used of \( E_{\text{prim}} = 212 \text{ EeV} \)

\[
\tilde{\kappa}_{tr} < 6 \times 10^{-20}
\]
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5. Conclusion
Consider CPT-even isotropic modifications for photons:

\[ \mathcal{L} = \mathcal{L}_\gamma + \mathcal{L}_{e\gamma}, \quad \mathcal{L}_\gamma = \frac{1}{2} \left[ (1 + \tilde{\kappa}_{tr}) \vec{E}^2 - (1 - \tilde{\kappa}_{tr}) \vec{B}^2 \right] \]

\[ \mathcal{L}_{e\gamma} = \frac{1}{2} \overline{\psi} (\not{\partial} + c_{\mu}^{\mu} \gamma_\mu D_\nu) \psi - m_e \overline{\psi} \psi, \quad (c_{\mu}^{\mu}) = c_e^{00} \times \text{diag} \left( 1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \]

Goal: Improve existing constraints on \( \tilde{\kappa}_{tr} \)

Threshold energy and decay rate close to threshold:

\[ E_{th} = \frac{m_e}{\sqrt{2 \tilde{\kappa}_{tr}}}, \quad \Gamma = \alpha m_e^2 \frac{(E - E_{th})^2}{2E^3} \]

Consider LEP collider (\( e^+e^- \)): \( E_{\text{LEP}} = 104.5 \text{ GeV} \)

Potential Cherenkov losses must be within uncertainty of synchrotron losses

\[ \frac{dE_{vcr}}{dL} \leq 10^{-4} \frac{dE_{\text{syn}}}{dL}, \quad \frac{dE_{\text{syn}}}{dL} = 2.580 \times 10^{-20} \text{ GeV} \]

Near threshold: dominant process is single-photon emission of \( E_\gamma \approx E - E_{th} \)

\[ \frac{dE_{vcr}}{dL} \geq \frac{E - E_{th}}{1/\Gamma} \implies E_{th} \lesssim 1.5 \text{ MeV} \]

Constraint on total isotropic combination:

\[ 0 \leq \tilde{\kappa}_{tr} - \frac{4}{3} c_{e}^{00} \leq 1.2 \times 10^{-11} \]
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Consider MCS theory:
\[ \mathcal{L} = -\frac{1}{4} F^2 + \zeta^{\mu} A^\nu \tilde{F}_{\mu\nu} - j \cdot A, \quad \zeta^{\mu} \equiv m_c s \zeta^{\mu} \]

Timelike sector: energy density not bounded from below
\[ \mathcal{E} = \frac{1}{2} \left[ \vec{E}^2 + \vec{B}^2 \right] - \zeta_0 \vec{B} \cdot \vec{A} \]

\[ \implies \text{Existence of exponentially increasing solutions} \]

Phase space estimate gives total power emitted into positive-frequency modes:
\[ P = \frac{\zeta_0^2 q^2 v^3}{30\pi} + \ldots \]

Obtain power emitted by long-wavelength modes: contribution of \( O(v^2) \)

Standard magnetic field of moving charge
\[ \vec{B}_0 = \frac{qv \sin \theta}{4\pi} \frac{r^2}{r^2 - \hat{e}_\phi} \]

Additional term in modified Ampère’s law
\[ \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J}_{\text{eff}} + \vec{J}, \quad \vec{J}_{\text{eff}} = 2\zeta_0 \vec{B} \]

Finding \( \vec{B} \) associated to \( \vec{J}_{\text{eff,0}} = 2\zeta_0 \vec{B}_0 \) is standard problem in electrodynamics
Magnetic field at first order in $\zeta_0$:
\[ \vec{B} = \vec{B}_0(\vec{r}) + \frac{\zeta_0 q v}{4\pi r} (2 \cos \theta \hat{e}_r - \sin \theta \hat{e}_{\theta}) \]

Consider outgoing energy flux $\vec{S} \cdot \hat{e}_r$ with
\[ \vec{S} = \vec{E} \times \vec{B} + \zeta_0 \vec{A} \times \vec{E} - \zeta_0 A_0 \vec{B} \]

No contribution from standard $\vec{E}_0 \sim \hat{e}_r$

$\vec{A}, \vec{B} \sim v$, inductive part of $\vec{E}$ coming from $\vec{B}$ is $\sim v^2$\[ \implies \text{no contribution from first two terms at } O(v^2) \]

$\zeta_0$-dependent contribution to $A_0$ of $O(v^2)$
\[ \vec{S} \cdot \hat{e}_r = -\frac{\zeta_0^2 q^2 v}{8\pi^2 r^2} \cos \theta \]

Radial flux is odd function in $\cos \theta$
\[ \lim_{r \to \infty} \int_0^{2\pi} d\phi \int_{-1}^1 d(\cos \theta) \vec{S} \cdot \hat{e}_r = 0 \]
Cherenkov radiation in media Lorentz-violating vacuum Vacuum Cherenkov radiation

What we know

Timelike MCS theory

Altschul, Schober (2015)

- Generalization of results from 2014
- Simplification: Consider steady motion (neglect recoil)

\[ \vec{r}(t) = vt\hat{z} \]

\[ \implies \text{Emitted fields } \vec{W} \in \{\vec{E}, \vec{B}\} \text{ move along with particle} \]

\[ \vec{W}(\vec{r}, t) = \vec{W}(\vec{r} - vt\hat{z}, 0) \]

\[ \vec{W} = \sum_{n, m=0}^{\infty} \zeta_m^0 v^n \vec{W}(m, n) \]

- Definite forms \( \vec{W}^{(m>0, n)} \)

  - \( m + n \) even: \( \vec{W}^{(m, n)} = W_r(r, \theta)\hat{r} + W_\theta(r, \theta)\hat{\theta} \) (toroidal)

  - \( m + n \) odd: \( \vec{W}^{(m, n)} = W_\phi(r, \theta)\hat{\phi} \) (azimuthal)

- Figures of fields
Timelike MCS theory

Altschul, Schober (2015)

- Dependence on \( \cos \theta \): even function (+), odd function (−)

<table>
<thead>
<tr>
<th>Field</th>
<th>( \hat{e}_r )</th>
<th>( \hat{e}_\theta )</th>
<th>( \hat{e}_\phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toroidal ( \vec{E} )</td>
<td>+</td>
<td>−</td>
<td>0</td>
</tr>
<tr>
<td>Azimuthal ( \vec{E} )</td>
<td>0</td>
<td>0</td>
<td>−</td>
</tr>
<tr>
<td>Toroidal ( \vec{B}, \vec{A} )</td>
<td>−</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Azimuthal ( \vec{B}, \vec{A} )</td>
<td>0</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

- \( A_0^{(0,n)} \) is even in \( \cos \theta \), \( A_0^{(m>0,n)} = 0 \)
- Total outgoing energy flux:

\[
\lim_{r \to \infty} \int_0^{2 \pi} \int_{-1}^{1} d\phi \, d(\cos \theta) \vec{S} \cdot \hat{e}_r \\
\vec{S} = \vec{E} \times \vec{B} + \zeta_0 \vec{A} \times \vec{E} - \zeta_0 A_0 \vec{B}
\]

- Contributing combinations: \( E_\theta B_\phi, E_\phi B_\theta, A_\theta E_\phi, A_\phi E_\theta, A_0 B_r \)
- Each combination is odd in \( \cos \theta \)
  \( \implies \) Total outgoing energy flux vanishes for all orders of \( \nu \)
- Recall energy density

\[
\mathcal{E} = \frac{1}{2} \left[ \vec{E}^2 + \vec{B}^2 \right] - \zeta_0 \vec{B} \cdot \vec{A}
\]

\( \implies \) Negative energy carried away from long-wavelength modes cancels with positive energy from short-wavelength modes
Gravitational Cherenkov radiation

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5. Conclusion
Consider modified linearized Einstein equations:

\[ 0 = (\eta^{\alpha\beta} + \hat{s}^{\alpha\beta})R_{\alpha\mu\beta\nu} \]

\[ \hat{s}_{\mu\nu} \equiv (\hat{s}^{(4)})_{\mu\nu} + (\hat{s}^{(6)})_{\mu\nu}q^\rho\partial_\rho\partial_\sigma + \ldots \]

Fix gauge and look for modified gravitational-wave solutions

\[ h_{\mu\nu}(x) = A_{\mu\nu}(q) \exp(iq \cdot x), \quad q_0^2 = \vec{q}^2 + \hat{s}^{\alpha\beta}q_\alpha q_\beta \]

Phase velocity of gravity modified by refractive index

\[ n \approx 1 - \frac{1}{2q_0^2} \left[ (\hat{s}^{(4)})_{\mu\nu}q\mu q\nu - (\hat{s}^{(6)})_{\mu\nu}q^\rho q^\sigma q\mu q\nu q_\rho q_\sigma + \ldots \right] \]

\[ \implies \text{Gravitational Cherenkov radiation can be produced} \]

Derive Feynman rules for scalar-graviton, photon-graviton, spinor-graviton vertex

Obtain matrix element squares for following processes:
Treatment of mixing of different $d$ technically challenging

$\implies$ Consider $d$ fixed at a time

Compute energy loss by integrating over final-particle phase space:

$$\frac{dE}{dt} = -F^w(d)G_N(s^{(d)})^2 |\vec{p}|^{2d-4}$$

Observation of cosmic ray on Earth with energy $E_f$ after distance $L$ leads to constraints:

$$s^{(d)}(\hat{\rho}) < \sqrt{\frac{2d - 5}{F^w(d)G_NE_f^{2d-5}L}}$$

Cosmic rays come from different directions: introduce spherical coefficients

$$s^{(d)}(\hat{\rho}) = \sum_{j,m} Y_{jm}(\hat{\rho})s^{(d)}_{jm}$$

Consider big compilation ($\approx 300$) of cosmic-ray events with $E > 60 \times 10^9$ GeV

One-sided bounds for isotropic and two-sided ones for anisotropic coefficients

$$-10^{-13} \lesssim s^{(4)}_{ij} \lesssim 10^{-13}$$

$$-10^{-29} \text{ GeV}^{-2} \lesssim s^{(6)}_{ij} \lesssim 10^{-29} \text{ GeV}^{-2}$$

$$-10^{-45} \text{ GeV}^{-4} \lesssim s^{(8)}_{ij} \lesssim 10^{-45} \text{ GeV}^{-4}$$
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5. Conclusion
Partonic description

Díaz, Klinkhamer (2015)

- Isotropic $CPT$-even photon framework
- Generalization to fermions with internal structure using parton model
- Emission rate by charged parton carrying momentum fraction $x$:
  \[
  \frac{d^2 \hat{\Gamma}_{\text{parton}}}{dx d\omega}
  \]
- Energy-momentum consideration: radiated photon energy has cutoff
  \[
  \omega_{\text{max}} = x f(\tilde{\kappa}_\text{tr}, E, M_p)
  \]
  $\implies$ Cherenkov emission suppressed for $x \ll 1$ (quark see unimportant)
- Momentum transfer suppressed by Lorentz violation:
  \[
  Q^2 \leq \frac{2\tilde{\kappa}_\text{tr}}{1 - \tilde{\kappa}_\text{tr}} \omega_{\text{max}}^2
  \]
- For $E \gg E^\text{th}$: $Q^2 > M_p^2$
  $\implies$ Photon emission can produce additional hadrons

\[\text{CT14 NNLO}\]
Partonic description

Díaz, Klinkhamer (2015)

- Total power radiated by composite proton:
  \[
  P(E) = \sum_{\text{parton}} \int_0^1 dx \int_0^{\omega_{\text{max}}} d\omega f_{\text{parton}}(x, Q^2) \omega \frac{d^2\hat{\Gamma}_{\text{parton}}}{dx d\omega}
  \]

- Radiation length \( l(E) \equiv cE/P(E) \)

- Threshold energy and constraint on LV coefficient (2008) unaffected
Vacuum Cherenkov radiation for pions

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Vacuum Cherenkov radiation for pions

Altschul (2016)

Consider Cherenkov-like process for photons radiating pions: $\gamma \rightarrow \gamma + \pi^0$

Lagrange density

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_\gamma + \mathcal{L}_{\pi\gamma}$$

$$\mathcal{L}_\pi = \frac{1}{2} (\partial^\mu \pi^0)(\partial_\mu \pi^0) + \frac{1}{2} k^{\mu\nu} (\partial_\mu \pi^0)(\partial_\nu \pi^0) - \frac{m_{\pi}^2}{2} (\pi^0)^2$$

$$\mathcal{L}_\gamma = -\frac{1}{4} F^2, \quad \mathcal{L}_{\pi\gamma} = -2g_{\pi} F_\mu \tilde{F}^{\mu\nu}$$

Modified dispersion law

$$E_{\pi} = \sqrt{[1 + 2\delta(\hat{p})]\hat{p}^2 + m_{\pi}^2}, \quad \delta(\hat{p}) = -\frac{1}{2}[k_{00} + (k_{0j} + k_{j0})\hat{p}_j + k_{ij}\hat{p}_i\hat{p}_j]$$

Pion emission above threshold energy

$$E_{\text{th}} = \frac{m_{\pi}}{\sqrt{-2\delta}}$$

Estimate decay rate from $\Gamma(\pi^0 \rightarrow 2\gamma)$ (in rest frame)

Daughter particles predominantly radiated into angular range $\sim m_{\pi}/E_{\text{th}}$:

$$\Gamma \sim \frac{m_{\pi}}{E_{\text{th}}\tau_{\pi}}, \quad \tau_{\pi} \approx 8.4 \times 10^{-17} \text{ s}$$

Observation of $\gamma$ from sufficiently distant source: $E_{\gamma} < \alpha E_{\text{th}}, \alpha = 1.01$

$$\delta > -7 \times 10^{-13}, \quad E_{\gamma} = 80 \text{ TeV}$$
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Quantization of timelike MCS

Colladay, McDonald, Potting (2016)

- Include gauge fixing and photon mass term into MCS Lagrangian:
  \[
  \mathcal{L}_A = -\frac{1}{4} F^2 + \zeta^\mu A^\nu \tilde{F}_{\mu\nu} + \frac{m_\gamma^2}{2} A^2 - \frac{1}{2\xi} (\partial \cdot A)^2, \quad \zeta^\mu \equiv m_{cs} \zeta^\mu
  \]

- Timelike MCS: no complex energies for \( m_\gamma \geq \zeta^0 \)
- Compare to current limits:
  \[
  m_\gamma < 1 \times 10^{-27} \text{ GeV}, \quad \zeta^0 \lesssim 10^{-43} \text{ GeV}
  \]

- 1 unphysical (gauge) polarization mode and 3 physical modes
- Standard equal-time commutation relations can be imposed
  \( \implies \) Consistent quantization procedure possible
- Obtain vacuum-Cherenkov decay rate based on these results

\[
\begin{align*}
W |q| \gg q_{th} & \sim m_{cs} |q| \\
W |q| \gg q_{th} & \sim \frac{m_{cs}^2}{m^2} q^2
\end{align*}
\]
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Lorentz-violating fermions

MS, Reis (2017)

- Lorentz violation in fermion sector

\[ \mathcal{L}_{\text{Dirac}} = \bar{\psi} \left( \frac{i}{2} \Gamma^\nu \partial_\nu - M \right) \psi \]

\[ \Gamma^\nu \equiv \gamma^\nu + c^{(4)\mu\nu} \gamma_\mu + \left[ \begin{array}{c} d^{(4)\mu\nu} \gamma^5 \gamma_\mu + e^{(4)\nu} \mathbb{1}_4 + i f^{(4)} \gamma^5 + (1/2) g^{(4)\mu\nu} \sigma_{\mu\nu} \end{array} \right] \]

\[ M \equiv m_\psi + a^{(3)\mu} \gamma_\mu + b^{(3)\mu} \gamma^5 \gamma_\mu + (1/2) H^{(3)\mu} \sigma_{\mu\nu} \]


- Spin-conserving process forbidden for \( a, b, \) and \( H \)
Lorentz-violating fermions

MS, Reis (2017)

- Threshold for $c, d$ coefficients:

  \[
  q^\text{th}_c = \frac{1}{2} \sqrt{\frac{3}{2}} \frac{m_\psi}{\sqrt{-\hat{c}}}, \quad q^\text{th}_d = \frac{1}{2} \sqrt{\frac{3}{2}} \frac{m_\psi}{\sqrt{\pm \hat{d}}}
  \]

- Threshold for $e, g$ coefficients:

  \[
  q^\text{th}_e = \frac{m_\psi}{\pm \hat{e}}, \quad q^\text{th}_g = \frac{m_\psi}{\pm \hat{g}}
  \]

- Pierre-Auger event of $E = 212$ EeV

- $u$ and $d$ quarks of proton carrying 10% of $E$

  \[
  -3 \times 10^{-23} < c^{(4)u}_{00} - (3/8)(f^{(4)u}_0)^2, \\
  -3 \times 10^{-23} < d^{(4)u}_{00} < 3 \times 10^{-23} \\
  -9 \times 10^{-12} < e^{(4)u}_0 < 9 \times 10^{-12} \\
  -9 \times 10^{-12} < \hat{g}^{u} < 9 \times 10^{-12}
  \]
Characteristics of helicity decay for $\bar{b}$

**Asymptotic behaviors of decay rate:**

\[
\Gamma _{<} \sim \frac{32\alpha }{3} \left( \frac{q}{m_{\psi}^2} \right) ^2 \bar{b} ^3 , \quad q_{\text{max}} \approx \frac{m_{\psi}^2}{\bar{b}} \\
\Gamma _{>} \sim \frac{\alpha }{2} \left[ \ln \left( \frac{4q\bar{b}}{m_{\psi}^2} \right) - \frac{3}{2} \right] \frac{m_{\psi}^2}{q}
\]
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Consider vacuum Cherenkov radiation with LV photons replaced by W bosons

Apply methodology developed with the difference that $m_W \gg m_\gamma$

$$\mathcal{L}_W = \frac{1}{2} k_2^{\mu} W^{+,\nu} \mathcal{W}^{\mu\nu} + \text{H.c.}$$

Energy-momentum conservation fulfilled for W boson mode $\oplus$ with $\Lambda_+(p) = 0$

Rest frame of incoming particle useful for illustration purposes:

- Purely timelike, purely spacelike, and lightlike $k_2^\mu$ considered
Colladay, Nordmaans, Potting (2017)

- Threshold momentum of incoming fermion:
  \[ |\vec{q}|_{\text{th}} = \frac{m_W(m_W + 2m_2)}{2|\vec{\kappa}|} + \ldots \]

- Approximated decay rate \( \Gamma(q) \) obtained at tree-level

- Consider elementary process to take place inside proton (breakup possible)

- Generalize decay rate within parton model:
  \[ \Gamma_{\text{parton model}} = \sum_{\text{quark}} \int_0^1 dx \left[ f_{\text{quark}}(x) + \bar{f}_{\text{quark}}(x) \right] \tilde{\Gamma}(xq) \]

  \( \implies \) Decay rate at threshold region considerably suppressed

- Estimate of average traveling length before decay:
  \( L \approx 10^3 \) km

  \[ |k_2^\mu| < 1 \times 10^{-7} \text{ GeV} \]
Summary and outlook

- Cherenkov-type processes in a Lorentz-violating vacuum
- Energy loss of massive particles by radiating photons, W bosons, gravitons, etc.
- Can but need not necessarily be threshold effect
- Occurs by either modifying matter sector or photon, W boson, graviton etc. sectors
- Absence strongly constrains Lorentz violation
- Vacuum Cherenkov radiation well understood for minimal SME photon sector
- Some studies on fermion sector, photons radiating pions, W bosons, and gravitational Cherenkov radiation
  - Other sectors
  - Nonminimal SME
  - Microscopic understanding (beyond SME)
  - Radiative corrections(?)