TMDs and simulations for EIC

From: Nuclear physics with a medium-energy Electron-Ion Collider by A. Accardi et al.
Nucleon landscape

Nucleon is a many body dynamical system of quarks and gluons

Changing $x$ we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

See talks by Christian Weiss, Jianwei Qiu, Marc Schlegel, ...

These distributions are also referred to as 3D (three-dimensional) distributions

Plot courtesy of Christian Weiss

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Wigner distribution

Transverse Momentum Dependent distributions

$W(x, k_\perp, b_\perp)$

$\int d^2 b_\perp$ 

$\int d^2 k_\perp$

$f(x, k_\perp)$

Parton Distribution Functions

$\int d^2 k_\perp$

$f(x)$

Generalized Parton Distributions

$H(x, \xi, t)$

Form Factors

$\int dx$

$F(Q^2)$

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Transverse Momentum Dependent distributions

SIDIS

If produced hadron has low transverse momentum

\[ P_{hT} \sim \Lambda_{QCD} \ll Q \]

it will be sensitive to quark transverse $k_\perp$ momentum

TMD factorization proven in QCD

Ji, Ma, Yuan (2002)
Collins (2011)

\[ \Phi_{ij}(x, k_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} \ e^{ixP^+\xi^- - i\mathbf{k}_\perp \cdot \mathbf{\xi}_\perp} \ \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(0, \xi) \psi_i(\xi) | P, S_P \rangle \]

Gauge Invariant

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Transverse Momentum Dependent distributions

\[ \Phi_{ij}(x, k_\perp) = \int \frac{d\xi^-}{(2\pi)} \frac{d^2\xi_\perp}{(2\pi)^2} e^{ixP^+\xi^- - ik_\perp\xi_\perp} \langle P, S_P | \bar{\psi}_j(0) \mathcal{U}(0, \xi) \psi_i(\xi) | P, S_P \rangle |_{\xi^+ = 0} \]

SIDIS in IMF:

Gauge link

\[ \mathcal{U}(a, b; n) = e^{-ig \int_a^b d\lambda n \cdot A_\alpha(\lambda n) t_\alpha} \]

Ensures gauge invariance of the distribution, cannot be canceled by gauge choice

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TMDs

<table>
<thead>
<tr>
<th>N</th>
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<td>$g_{1T}$</td>
<td>$h_1$</td>
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8 functions in total (at leading Twist)

Each represents different aspects of partonic structure

Each function is to be studied

Correlation of transverse quark motion and the nucleon spin – Sivers function

\[ f(x, k_T, S_T) = f_1(x, k_T^2) - f_{1T}(x, k_T^2) \frac{k_x}{M} \]

This function gives access to 3D imaging

Spin-orbit correlation

Physics of gauge links is represented

Requires Orbital Angular Momentum

No polarisation:   

Polarisation:

EIC report, Boer, Diehl, Milner, Venugopalan, Vogelsang et al., 2011;  
EIC Whitepaper

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What do we know about TMDs

Sivers function
HERMES, COMPASS, RHIC, BELLE, BABAR and JLab are sources of experimental information for TMD physics

Hundreds of experimental points on Sivers function

There exists a number of extractions

HERMES 02 - COMPASS 04 - JLAB 11 -

Vogelsang, Yuan 05
Collins et al 06
Anselmino et al 06-09
Bacchetta, Radici 11
Extractions compare well with each other

\[ x f_{^1T}^{1q}(x) \]

Anselmino et al 2006
Collins et al 2006

d-quark
u-quark

HERMES x-range
Extractions compare well with models

- MIT bag model
- Quark-diquark model
- LCWF
- Quark-diquark model
- MIT bag model

Yuan 03
Gamberg, Goldstein, Schlegel 08
Yuan, Pasquini 10
Bacchetta et al 10
Avakian et al 10 etc

\[ x f_{1T}^{(1)u}(x) \]
Steady growth of interest

Title “Sivers” in the literature
Estimate of current uncertainty

Based on Anselmino et al 09

Up quark

Down quark

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Estimate of current uncertainty

Based on Anselmino et al 09

Current uncertainty in low x region
Estimate of current uncertainty

Based on Anselmino et al 09

Region of $x$ covered by existing data
Estimate of current uncertainty

Based on Anselmino et al 09

EIC can reach here
Estimate of current uncertainty

Based on Anselmino et al 09

Estimate of uncertainty is based on Monte Carlo method

\[ \chi^2 = \sum_{n=1}^{N_{tot}} \left( \frac{\sigma_n^{theor} - \sigma_n^{exp}}{\Delta \sigma_n^{exp}} \right)^2 \]
Estimate of current uncertainty

Based on Anselmino et al. 09

Estimate of uncertainty is based on Monte Carlo method

Value $A_0$ of parameter corresponds to the minimum of $\chi^2$
Estimate of current uncertainty

Based on Anselmino et al 09

Estimate of uncertainty is based on Monte Carlo method

\[ \chi^2 \]

\[ \Delta \chi^2 \]

\[ \chi^2_{min} \]

\[ 2\Delta A_0 \]

\[ V_{alue} \]

Error \( \Delta A_0 \) of parameter corresponds to

\[ \chi^2 = \chi^2_{min} + \Delta \chi^2 \]
Estimate of current uncertainty

Based on Anselmino et al. 09

Estimate of uncertainty is based on Monte Carlo method

\[ \chi^2 = \chi_{min}^2 + \Delta \chi^2 \]

We use Monte Carlo to populate

Robust in case of non symmetric errors

\[ 2\Delta A_0 \]

\[ Value \]
Estimate of current uncertainty

Based on Anselmino et al. 09

Estimate of uncertainty is based on Monte Carlo method

\[ \chi^2 \]

\[ \Delta \chi^2 \]

\[ \chi^2_{min} \]

Choice of \( \Delta \chi^2 \) is important

\[ \Delta \chi^2 = 1 \]

\[ \Delta \chi^2 = 3\% \chi^2 \]

\[ \Delta \chi^2 = f(N_{par}) \]
Estimate of current uncertainty

Based on Anselmino et al 09

Estimate of uncertainty is based on Monte Carlo method

\[ \chi^2 \]

\[ \Delta \chi^2 \]

\[ \chi^2_{min} \]

\[ 2\Delta A_0 \]

\[ V_{value} \]

Choice of \( \Delta \chi^2 \) is important

\[ \Delta \chi^2 = 1 \]

\[ \Delta \chi^2 = 3\% \chi^2 \]

\[ \Delta \chi^2 = f(N_{par}) \]

Anselmino et al 09

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What do we expect from EIC?
Kinematics

\[ Q^2 \approx sxy \]

JLab 12 and future Electron Ion Collider are complimentary.
Kinematics and existing data

Current data for Sivers asymmetry:
- COMPASS: $h^\pm$: $P_{hT} < 1.6$ GeV, $z > 0.1$
- HERMES: $\pi^{0,x}, K^\pm$: $P_{hT} < 1$ GeV, $0.2 < z < 0.7$
- JLab Hall-A: $\pi^\pm$: $P_{hT} < 0.45$ GeV, $0.4 < z < 0.6$

Planned:
- JLab 12

EIC $\sqrt{s} = 45$ GeV, $0.01 \leq y \leq 0.95$

$EIC \sqrt{s} = 140$ GeV, $0.01 \leq y \leq 0.95$
Kinematics and existing data

Current data for Sivers asymmetry:
- **COMPASS** $h^\pm$: $P_{hT} < 1.6 \text{ GeV}$, $z > 0.1$
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EIC $\sqrt{s} = 140 \text{ GeV}$, $0.01 \leq y \leq 0.95$
EIC $\sqrt{s} = 45 \text{ GeV}$, $0.01 \leq y \leq 0.95$
```
What do we expect at EIC?

Prediction for EIC 5x100 kinematics based on Anselmino et al 09

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What do we expect at EIC?

Estimates of experimental error for EIC at 10/fb

Part of the data set, demonstration only

HERMES data

EIC

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What do we expect at EIC?

Estimates of experimental error for EIC at 10/fb

Very small error in low-x region

Part of the data set, demonstration only

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Generate pseudo-data

Estimates of experimental error for EIC at 10/fb

Generate pseudodata around our predictions

Part of the data set, demonstration only
Generate pseudo-data

Based on Anselmino et al 09

Not actual data, demonstration only

Generate pseudodata $\pm \sigma$ around our predictions
Fit the pseudo-data

Based on Anselmino et al 09

Not actual data, demonstration only
Generate pseudo-data

$\sin (\Phi_s - \Phi_h)$

Based on Anselmino et al 09

Not actual data, demonstration only

Compare the error to the existing one
Tomographic scan of the nucleon

$\mathbf{x} f_1(x, k_T, S_T)$

$u$ quark

$d$ quark

$k_x$ (GeV)

$k_y$

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Results
Current knowledge

Expected at EIC

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Results

- $f_{1T}^{u,v}(x, k_T)$

$Q^2 = 2.4 \text{ GeV}^2$
Results

$-f_{1T}^{u} (x, k_T)$

$Q^2 = 2.4 \text{ GeV}^2$

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Drawbacks

• The estimate is pure \textit{statistical}

• No control over \textit{"systematical"} theoretical error, such as dependence on functional form, higher twist contributions, evolution etc

• Note that generated pseudodata does not have estimate on \textit{systematical} errors either

• No error propagation due to the use of existing PDFs and FFs (those have substantial uncertainty)

• Given all this one should comment results carefully
Advantages

• The estimate of the error uses the same method as the estimate of current error

• Thus the comparison of existing uncertainty and the projected one at EIC is trustworthy

• Some of theoretical uncertainties such as evolution can be controlled

• There is no doubt that EIC precision in TMD measurements is going to be great!
Thanks to Xin Qian (CALTECH), Tom Burton (BNL), Min Huang (JLab) for their contribution to the analysis.