Babar anomaly and the pion form factors

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high $Q^2$ :: LO pQCD (twist/$\alpha_s$ expansion)
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Questions:

1. Role of hadronic vs partonic d.o.f
2. Is there indication that all-orders re-summation is needed

with M.Gorshtein, P.Guo, J.L.Londergan, F.L.Estrada
BaBar anomaly \[ e^+e^- \rightarrow \pi^0e^+e^- \]


\[ F_{\gamma^*\gamma\pi}(Q^2) \]

\[ Q^2 \sim 1/r^2 \]

theory: G.P. Lepage, S. Brodsky
BaBar anomaly $e^+e^- \rightarrow \pi^0 e^+e^-$


$e^-$

$\gamma^*(Q^2) \\ F_{\gamma^*\gamma\pi}(Q^2) \\ \pi^0$

$e^+$

$\gamma$

$Q^2 \sim 1/r^2$

theory: G.P.Lepage, S.Brodsky

"pointlike"

"harder"

"hard"

"soft"
problems with LO pQCD in exclusive reactions


\[ \frac{1}{\Delta t} \sim \Delta E = \sum_i \frac{\mu_{i,\perp}^2}{z_i P_z} \]

valid for \( z_i P_z \) large i.e. NOT in the end-point region
similar final states but different asymptotic predictions

\[ s = q^2 \]

\[ \Delta t \sim \Lambda_{QCD}^{-1} \]

\[ s F_{\gamma\pi}(s) \sim O(1) \]

\[ s F_{2\pi}(s) \sim O(\alpha_s(Q^2)) \]

... but it does look different on the Light Front

S. Brodsky, P. Lepage, A. Radyushkin, A. Efremov
Pion form factors: still a mystery

\[ Q^2 \frac{F_{\gamma\pi}(Q^2)}{F_{\gamma\pi}(0)} \rightarrow 8\pi^2 f_\pi^2 \]

\[ Q^2 \frac{F_{2\pi}(Q^2)}{F_{2\pi}(0)} \rightarrow 8\pi f_\pi^2 \left( \frac{\alpha_s(Q^2)}{0.5} \right) \]

\( (\alpha_s = 0.5) \)
\[
\left| \frac{F_{\gamma\pi}(s)}{F_{\gamma\pi}(0)} \right|^2
\]

- e.m.
- transition BaBar
- transition CLEO
- transition Belle

\(\rho\) meson

\(\rho'\) meson

resonance tails or QCD (duality)
Dispersive analysis

\[ F(s) = F(0) + \frac{s}{\pi} \int_{s_{th}} ds' \frac{\text{Im } F(s')}{s'(s' - s)} \]

\[ F = F_{2\pi}, F_{\gamma \pi} \]
**Dispersive analysis**

\[ F(s) = F(0) + \frac{s}{\pi} \int_{s_{th}} ds' \frac{\text{Im} \, F(s')}{s'(s' - s)} \]

\[ F = F_{2\pi}, F_{\gamma\pi} \]

**EM F.Factor**

\[ \text{Im} \, F_{2\pi}(s) = t_{2\pi,2\pi}^* \rho_{2\pi} F_{2\pi} + t_{2\pi,K\bar{K}}^* \rho_{2K} F_{2K} + \sum_X t_{2\pi,X}^* \rho_X F_X \]

\[ = t^* \rho F_{2\pi} + R \]

\[ t(s) = \int dz_s t_I^{I=1}(s, t(z_s)) P_1(z_s) \quad \rho(s) = (1 - s/s_{th})^{1/2} \]
\(F(s) = F(0) + \frac{s}{\pi} \int_{s_{th}} ds' \frac{\text{Im } F(s')}{s'(s' - s)}\)

\[F = F_{2\pi}, F_{\gamma \pi}\]

\(\text{EM F.Factor}\)

\(\text{Im } F_{2\pi}(s) = t_{2\pi,2\pi}^* \rho_{2\pi} F_{2\pi} + t_{2\pi,KK}^* \rho_{KK} F_{KK} + \sum_{X} t_{2\pi,X}^* \rho_X F_X\)

\[= t^* \rho F_{2\pi} + R\]

\(t(s) = \int dz_s t^I = 1(s, t(z_s)) P_1(z_s)\)

\(\rho(s) = (1 - s/s_{th})^{1/2}\)

\(F_{2\pi}(s) = 1 + \frac{1}{\pi} \int_{s_{th}} ds' \frac{t^*(s') \rho(s') F_{2\pi}(s') + R(s')}{s'(s' - s)}\)

(so far) Exact representation of the electromagnetic form factor
\*\* Dispersive analysis cont.

\[ F_{2\pi}(s) = 1 + \frac{1}{\pi} \int_{s_{th}} ds' t'(s') \rho(s') F_{2\pi}\left(s'\right) + R(s') \]

\[ R(s') \propto \theta(s' - (4m_\pi)^2) \]

solution
Dispersive analysis cont.

\[ F_{2\pi}(s) = 1 + \frac{1}{\pi} \int_{s_{th}} ds' \frac{t^*(s') \rho(s') F_{2\pi}(s') + R(s')}{s'(s' - s)} \]

Solution

\[ F_{2\pi}(s) = \frac{N(s)}{D(s)} \]

---

elastic \hspace{1cm} inelastic \hspace{1cm} \[ R(s') \propto \theta(s' - (4m_{\pi})^2) \]
\* Dispersive analysis cont.

\[
F_{2\pi}(s) = 1 + \frac{1}{\pi} \int_{s_{th}} ds' \frac{t^*(s')\rho(s')F_{2\pi}(s') + R(s')}{s'(s' - s)}
\]

\[
F_{2\pi}(s) = \frac{N(s)}{D(s)}
\]

\[
N(s) = 1 + \frac{s}{\pi} \int_{s_i} ds' \frac{D(s')Re R(s')}{[1 - it^*(s')\rho(s')]s'(s' - s)}
\]

\[
D(s) = \exp \left( -\frac{s}{\pi} \int_{s_{th}} ds' \frac{\phi(s')}{s'(s' - s)} \right)
\]

\[
\phi = \arctan Re \frac{t}{1 - Im \, t\rho}
\]
Dispersive analysis cont.

\[ F_{2\pi}(s) = 1 + \frac{1}{\pi} \int_{s_{th}} ds' \frac{t^*(s') \rho(s') F_{2\pi}(s') + R(s')}{s'(s' - s)} \]

solution

\[ F_{2\pi}(s) = \frac{N(s)}{D(s)} \]

\[ N(s) = 1 + \frac{s}{\pi} \int_{s_{i}} ds' \frac{D(s') \Re R(s')}{[1 - it^*(s') \rho(s')] s'(s' - s)} \]

\[ D(s) = \exp \left( -\frac{s}{\pi} \int_{s_{th}} ds' \frac{\phi(s')}{s'(s' - s)} \right) \]

\[ \phi = \arctan \Re t / (1 - \Im t \rho) \]

input: \( t(s), R(s) \)

on shell, exclusive \( \pi\pi \rightarrow X \) amplitudes + associated form factors

output: \( F_{2\pi}(s) \)

\( R(s') \propto \theta(s' - (4m_{\pi})^2) \)
\[ t = \frac{\eta e^{2i\delta} - 1}{2i\rho} \]
\[ t = \frac{\eta e^{2i\delta} - 1}{2i\rho} \]

**ππ P-wave amplitude**

\[ \text{Re } t(s) \text{ and } \text{Im } t(s) \]

\( \rho' \) is inelastic

**Fit to phase shift data below** \( s^{1/2} = 1.9 \text{ GeV} \)
\[ t = \frac{\eta e^{2i\delta} - 1}{2i\rho} \]

\( \pi\pi \) P-wave amplitude

Im \( t \)

Re \( t \)

\( \rho' \) is inelastic

fit to phase shift data below \( s^{1/2} = 1.9 \) GeV

Regge fit

fit to phase shift data below \( s^{1/2} = 1.9 \) GeV
$\pi\pi$ P-wave amplitude

$$t = \frac{\eta e^{2i\delta} - 1}{2i\rho}$$

$\rho'$ is inelastic

Fit to phase shift data below $s^{1/2} = 1.9$ GeV

Regge fit

Regge fit
\( \pi \pi \) P-wave amplitude

\[ t = \begin{cases} \pi^+ \pi^- & \text{Im } t \\ F_{2\pi} & \text{Re } t \\ F_{2K} & + \sum_x \\ \kappa & \text{Regge fit} \end{cases} \]

\( \rho' \) is inelastic

Fit to phase shift data below \( s^{1/2} = 1.9 \text{ GeV} \)

Regge fit

\[ s [\text{GeV}^2] \]
Inelastic contribution (I)

\[ R = t_{2\pi, K\bar{K}}^* \rho_{2K} F_{2K} + \sum_X t_{2\pi, X}^* \rho_X F_X \]
\[ R = \sum_{\pi, K} t_{2\pi, K}^* \rho_{2K} F_{2K} + \sum_{X} t_{2\pi, X}^* \rho_{X} F_{X} \]

\[ \rho' \text{ is inelastic} \]
\[ R = t_{2\pi,K}^* \rho_{2K} F_{2K} + \sum_X t_{2\pi,X}^* \rho_X F_X \]

fit to phase shift data below \( s^{1/2} = 1.9 \text{ GeV} \) and Regge above

\[ |F_K(w)|^2 \]

\[ w = \sqrt{s} \text{[GeV]} \]

FIG. 8: Real (top) and imaginary (bottom) parts of the isovector, \( P \)-wave amplitude, \( t_{\pi K}(s)/(q_{\pi q K}) \) (solid lines). The dashed line is the result of the \( K \)-matrix parameterization.
Inelastic contribution (II)

\[ R = t^{*}_{2\pi, K\bar{K}} \rho_{2K} F_{2K} + \sum_{X} t^{*}_{2\pi, X} \rho_{X} F_{X} \]
Inelastic contribution (II)

\[ R = t_{2\pi,K\bar{K}}^* \rho_{2K} F_{2K} + \sum_X t_{2\pi,X}^* \rho_X F_X \]

\[ \text{Im } t_{q\bar{q},\pi\pi} = \beta_\pi(t) s^\alpha_q(t) \]

\[ t_{q\bar{q},\pi\pi} = \int dz t_{q\bar{q},\pi\pi}(s, t) P_1(z) \]

\[ s = q^2 \]
\[ R = t^{*}_{2\pi, K K} \rho_{2K} F_{2K} + \sum_{X} t^{*}_{2\pi, X} \rho_{X} F_{X} \]

**Mandelstam branchings**

\[ \alpha_{q} \left( \frac{t}{4} \right) \sim \frac{\alpha_{\rho}(t) + 1}{2} \]

\[ \alpha_{q}(t \sim 0) \sim 0.75 \]

\[ s \sum_{q\bar{q}} t^{*}_{2\pi, q\bar{q}} \rho_{q\bar{q}} F_{q\bar{q}} \sim s^{\alpha_{q}(0) - 1/2} \]

**Im** \( t_{q\bar{q}, \pi\pi} = \beta_{\pi}(t)s^{\alpha_{q}(t)} \)

\[ t_{q\bar{q}, \pi\pi} = \int dz_{t} t_{q\bar{q}, \pi\pi}(s, t) P_{1}(z_{s}) \]
why reggezation enhances amplitudes

multi-particle production in $e^+e^-$

“leading Fock components”

no central plateau

Reggized quark
why reggezation enhances amplitudes

multi-particle production in $e^+e^-$

“leading Fock components”

no central plateau
pion e.m. form factor (summary)

\[ \text{Im} F_{2\pi} = t^{*}_{2\pi,2\pi} \rho_{2\pi} F_{2\pi} + t^{*}_{K\bar{K},2\pi} \rho_{2K} F_K + \sum_{X} t^{*}_{X,2\pi} \rho_X F_X \]

\( F_{2\pi} \)
\( t^{*}_{2\pi,2\pi} \rho_{2\pi} F_{2\pi} \)
\( t^{*}_{K\bar{K},2\pi} \rho_{2K} F_K \)
\( \sum_{X} t^{*}_{X,2\pi} \rho_X F_X \)

\( F_{X=q\bar{q}} = 1 \)
\( t_{q\bar{q},\pi} = \beta s^\alpha q \)

\( \rho \) meson
\( \rho' \) meson

curves: dispersion relation solution with reggized quarks to describe large-s region

possible \( (Q^2)^a \) enhancement (from multi-particle production -- Reggized quark)
\[ \text{Im} F_{\pi\gamma} = t^*_{2\pi,\pi\gamma}\rho_{2\pi}F_{2\pi} + t^*_{3\pi,\pi\gamma}\rho_{3\pi}F_{3\pi} + \sum_{X} t^*_{X,\pi\gamma}\rho_{X}F_{X} \]

Reggized quark

Electromagnetic transition

\( \mu_2 = 1 \text{ GeV}^2 \)

\( \mu_2 = 10 \text{ GeV}^2 \)
From the s-channel:

\[ \text{Im} F(s) = \sum_X t_X^*(s) \rho_X(s) F_X(s) \]

\[ \text{Im} F_{\pi\gamma} = t_{2\pi,\pi\gamma}^* \rho_{2\pi} F_{2\pi} + t_{3\pi,\pi\gamma}^* \rho_{3\pi} F_{3\pi} + \sum_X t_{X,\pi\gamma}^* \rho_X F_X \]

\( (Q^2)^a \) enhancement from multi-particle production

Curves: dispersion relation solution with reggized quarks to describe large-s region

Multi-particle ladder -- Reggized quark (aka diffractive dissociation)
Figure 3: Left panel: experimental data for $|Q^2 F_{\eta\gamma}(Q^2)|$ in the space-like region from Refs. [23, 22] and high-$Q^2$ timelike data from Refs. [24] in comparison with unitarized VDM (solid line) and our full model (VDM + Regge) with $\mu^2 = 5$ GeV$^2$ (dashed line). Right panel: the same for $|Q^2 F_{\eta'\gamma}(Q^2)|$. 
Summary

* In the available energy range factors dominated by resonances

* Complete analysis requires self consistency: (e.g. kaon form factor, Im part of inelasticity)

* Importance of Regge trajectories and not elementary particles