Saturation Physics and Di-Hadron Correlations

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Outline

1. Introduction

2. A Tale of Two Gluon Distributions

3. Di-hadron productions
   - DIS dijet
   - Dijet (dihadrons) in $pA$

4. Conclusion
Phase diagram in QCD

Consider the evolution inside a hadron:

- Low $Q^2$ and low $x$ region ⇒ saturation region.
- Use BFKL equation and BK equation instead of DGLAP equation.
- BK equation is the non-linear small-$x$ evolution equation which describes the saturation physics.
$k_t$ dependent parton distributions

The unintegrated quark distribution

$$f_q(x, k_\perp) = \int \frac{d\xi^- d^2 \xi_\perp}{4\pi(2\pi)^2} e^{ix^+ \xi^- + i \xi_\perp \cdot k_\perp} \langle P | \bar{\psi}(0) \mathcal{L}^\dagger(0) \gamma^+ \mathcal{L}(\xi^-, \xi_\perp) \psi(\xi_\perp, \xi^-) | P \rangle$$

as compared to the integrated quark distribution

$$f_q(x) = \int \frac{d\xi^-}{4\pi} e^{ix^+ \xi^-} \langle P | \bar{\psi}(0) \gamma^+ \mathcal{L}(\xi^-) \psi(0, \xi^-) | P \rangle$$

- The dependence of $\xi_\perp$ in the definition.
- Gauge invariant definition.
- Light-cone gauge together with proper boundary condition $\Rightarrow$ parton density interpretation.
- The gauge links come from the resummation of multiple gluon interactions.
- Gauge links may vary among different processes.
Saturation physics describes the high density parton distributions in the high energy limit.

- Initial condition: McLerran-Venugopalan Model plus small-x evolution $\Rightarrow$ dense gluon distributions.
- In a physical process, in order to probe the dense nuclear matter precisely, the proper factorization is required.
- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (parton distributions and fragmentation functions). Hard factor should always be finite and free of divergence of any kind.
Dilute-Dense factorizations

The effective Dilute-Dense factorization

- Protons and virtual photons are dilute probes of the dense gluons inside target hadrons.
- For $pA$ (dilute-dense system) collisions, there is an effective $k_t$ factorization.

\[
\frac{d\sigma^{pA\rightarrow qX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p q(x_p, \mu^2) x_A f(x_A, q^2_\perp) \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}.
\]

- For dijet processes in $pp$, $AA$ collisions, there is no $k_t$ factorization[Collins, Qiu, 08],[Rogers, Mulders; 10].
- At forward rapidity $y$, $x_p \propto e^y$ is large, while $x_A \propto e^{-y}$ is small.
- Ideal opportunity to search gluon saturation.
- Systematic framework to test saturation physics predictions.
Factorization for single inclusive hadron productions

- [G. Chirilli, BX and F. Yuan, Phys. Rev. Lett. 108, 122301 (2012)] Obtain a systematic factorization for the $p + A \rightarrow H + X$ process by systematically remove all the divergences!
- Gluons in different kinematical region give different divergences. 1. soft, collinear to the target nucleus; 2. collinear to the initial quark; 3. collinear to the final quark.

\[ P_+^A \simeq 0 \]
\[ P_-^p \simeq 0 \]
\[ k^+ \simeq 0 \]

- All the rapidity divergence is absorbed into the UGD $\mathcal{F}(k_\perp)$ while collinear divergences are either factorized into collinear parton distributions or fragmentation functions.
- Large $N_c$ limit is vital for the factorization in terms of getting rid of higher point functions.
- **Consistent check**: take the dilute limit, $k_\perp^2 \gg Q_s^2$, the result is consistent with the leading order collinear factorization formula.
- In terms of resummation, we will be able to resum up to $\alpha_s(\alpha_s \ln k_\perp^2)^n$ and $\alpha_s(\alpha_s \ln 1/x)^n$ terms.
Using this factorization technique, we can imagine that a lot of other NLO calculations can be achieved in the near future.

- **NLO Drell-Yan lepton pair production** and **NLO dijet productions** in $pA$ collisions.
- **Single inclusive DIS** at NLO. (see similar work [Balitsky, Chirilli, 10], [Beuf, 11])
- **Direct photon production** in $pA$ collisions at NLO (straightforward) and NNLO (similar to the DY case at NLO). **Universality and large $N_c$**
- **The CSS resummation and Sudakov suppression factor** in small-$x$ physics. (work in progress with A. Mueller and F. Yuan)
Sudakov factor

[Mueller, Xiao, Yuan, work in progress] Two scale problem (CSS resummation)

\[ Q_1^2 \gg Q_2^2 \Rightarrow \frac{\alpha_s C_R}{2\pi} \ln^2 \frac{Q_1^2}{Q_2^2} \]

For \( pA \rightarrow H(M_H, k_\perp) + X: \)

\[ S_{\text{sud}}(M_H^2, r_\perp^2) = \frac{\alpha_s N_c}{2\pi} \ln^2 \frac{M_H^2 r_\perp^2}{C^2} + \cdots \]

with \( C = 2e^{-\gamma_E} \):

\[ d\sigma^{(\text{resum})} \bigg|_{k_\perp \ll M_H} = \sigma_0 \int \frac{d^2 x_\perp d^2 x'_\perp}{(2\pi)^2} e^{i k_\perp \cdot r_\perp} e^{-S_{\text{sud}}(M_H^2, r_\perp^2)} S_{YY}^{\text{WW}} \ln 1/x_g(x_\perp, x'_\perp) \]

\[ \times x g_p(x, \mu^2 = c_0^2/r_\perp^2) \left[ 1 + \frac{\alpha_s}{\pi} \frac{\pi^2}{2} N_c \right]. \]

- For dijet processes, replace \( M_H^2 \) by \( 4P_\perp^2 \).
- **Mismatch** between rapidity and collinear divergence between graphs \( \Rightarrow S_{\text{sud}}(M_H^2, r_\perp^2) \).
A Tale of Two Gluon Distributions

In small-x physics, two gluon distributions are widely used: [Kharzeev, Kovchegov, Tuchin; 03]

I. Weizsäcker Williams gluon distribution ([KM, 98] and MV model):

\[ xG^{(1)} = \frac{S_{\perp} N_c^2 - 1}{\pi^2 \alpha_s} \frac{N_c}{N_c} \]
\[ \times \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-ik_{\perp} \cdot r_{\perp}} \left( 1 - e^{-\frac{r_{\perp}^2 Q_{sg}^2}{2}} \right) \]

II. Color Dipole gluon distributions:

\[ xG^{(2)} = \frac{S_{\perp} N_c}{2\pi^2 \alpha_s} k_{\perp}^2 \]
\[ \times \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-ik_{\perp} \cdot r_{\perp}} e^{-\frac{r_{\perp}^2 Q_{sg}^2}{4}} \]

Remarks:

- The WW gluon distribution simply counts the number of gluons.
- The Color Dipole gluon distribution often appears in calculations.
- Does this mean that gluon distributions are non-universal? Answer: Yes and No!
A Tale of Two Gluon Distributions

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution

\[ xG^{(1)} = \frac{S_{\perp} N_c^2 - 1}{\pi^2 \alpha_s N_c} \left( 1 - e^{-\frac{r_{\perp}^2 Q_s^2}{2}} \right) \]

\[ \times \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-i k_{\perp} \cdot r_{\perp}} \left( 1 - e^{-\frac{r_{\perp}^2 Q_s^2}{2}} \right) \]

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A Tale of Two Gluon Distributions

In terms of operators (known from TMD factorization), we find these two gluon distributions can be defined as follows: [F. Dominguez, C. Marquet, BX and F. Yuan, 11]

I. Weizsäcker Williams gluon distribution:

\[ xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i} (\xi^- , \xi_\perp) U^{[+]\dagger} F^{+i} (0) U^{[+]} | P \rangle. \]

II. Color Dipole gluon distributions:

\[ xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+ \xi^- - ik_\perp \cdot \xi_\perp} \text{Tr} \langle P | F^{+i} (\xi^- , \xi_\perp) U^{[-]} U^{[+]\dagger} F^{+i} (0) U^{[+] | P \rangle. \]

Remarks:

- The WW gluon distribution is the conventional gluon distributions. In light-cone gauge, it is the gluon density. (Only final state interactions.)
- The dipole gluon distribution has no such interpretation. (Initial and final state interactions.)
- Both definitions are gauge invariant.
- Same after integrating over \( q_\perp \).
A Tale of Two Gluon Distributions

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Questions:

- Can we distinguish these two gluon distributions? Yes, We Can.
- How to measure \( xG^{(1)} \) directly? DIS dijet.
- How to measure \( xG^{(2)} \) directly? Direct \( \gamma \)+Jet in \( pA \) collisions.
- For single-inclusive particle production in \( pA \) up to all order.
- What happens in gluon+jet production in \( pA \) collisions? It’s complicated!
**DIS dijet**

[F. Dominguez, C. Marquet, BX and F. Yuan, 11]

\[
\frac{d\sigma}{d\mathcal{P}.S.} \propto N_c\alpha_{em}e_q^2 \int \frac{d^2x}{(2\pi)^2} \frac{d^2x'}{(2\pi)^2} \frac{d^2b}{(2\pi)^2} \frac{d^2b'}{(2\pi)^2} e^{-ik_{1\perp}(x-x')} \times e^{-ik_{2\perp}(b-b')} \sum \psi_T^*(x-b)\psi_T(x'-b') \\
\left[ 1 + S_{xg}^{(4)}(x, b; b', x') - S_{xg}^{(2)}(x, b) - S_{xg}^{(2)}(b', x') \right],
\]

- Eikonal approximation $\Rightarrow$ Wilson Line approach [Kovner, Wiedemann, 01].
- In the dijet correlation limit, where $u = x - b \ll v = zx + (1 - z)b$.
- $S_{xg}^{(4)}(x, b; b', x') = \frac{1}{N_c} \langle \text{Tr} U(x)U^+(x')U(b')U^+(b) \rangle_{xg} \neq S_{xg}^{(2)}(x, b)S_{xg}^{(2)}(b', x')$.
- Quadrupoles are generically **different** objects and only appear in dijet processes.
DIS dijet

The dijet production in DIS.

TMD factorization approach:

\[
d\sigma_{\gamma^* T A \rightarrow q\bar{q} + X} = \delta(x_{\gamma^*} - 1) x_g G^{(1)}(x_g, q_\perp) H_{\gamma^* T g \rightarrow q\bar{q}},
\]

Remarks:

- Dijet in DIS is the only physical process which can measure Weizsäcker Williams gluon distributions.
- Golden measurement for the Weizsäcker Williams gluon distributions of nuclei at small-x. The cross section is directly related to the WW gluon distribution.
- EIC and LHeC will provide us perfect machines to study the strong gluon fields in nuclei. Important part in EIC and LHeC physics.
Di-Hadron correlations in DIS

Di-pion correlations at EIC\cite{J. H. Lee, BX, L. Zheng}

\[ C(\Delta \phi) = \frac{\int |p_1\perp|, |p_2\perp| \frac{d\sigma^{eA \rightarrow h_1h_2}}{dy_1 dy_2 d^2 p_1\perp d^2 p_2\perp}}{\int |p_1\perp| \frac{d\sigma^{eA \rightarrow h_1}}{dy_1 d^2 p_1\perp}} \]

- EIC stage II energy $30 \times 100 \text{GeV}$.
- Using: $Q_{sA}^2 = c(b) A^{1/3} Q_s^2(x)$.
- **Physical picture**: Dense gluonic matter suppresses the away side peak.
Di-Hadron correlations in DIS


\[ J_{eA} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{\sigma_{eA}^{\text{pair}}}{\sigma_{eA}} / \frac{\sigma_{ep}^{\text{pair}}}{\sigma_{ep}} \]

\( J_{eA} \) is normalized to unity in the dilute regime.

**Physical picture:** The cross sections saturates at low-\( \chi^A \).

\( J_{eA} \) and \( J_{eAu} \) nicely demonstrates on the one hand the correspondence between the physics in calculations from [165, 166] and appear to describe the data rather well. This example the full parton energy. The two curves in the right panel of Fig. 3.22 represent the CGC in the Au nucleus, derived from the kinematics of the measured hadrons assuming they carry measured in hadron-hadron collisions. Instead by analogy to Eq. (3.15) with (away side) the measurement shown in the right panel of Fig. 3.21 this study requires the additional a function of the the longitudinal momentum fraction of the probed gluon signifies suppression of di-hadron correlations. In the left panel of Fig. 3.22, \( J_{eAu}\) = 1 in absence of correlations.
\( \gamma + \text{Jet in } pA \text{ collisions} \)

The direct photon + jet production in \( pA \) collisions. (Drell-Yan follows the same factorization.)

TMD factorization approach:

\[
\frac{d\sigma}{dP \cdot S} \bigg\vert_{(pA \rightarrow \gamma q + X)} = \sum_f x_1 q(x_1, \mu^2) x_g G^{(2)}(x_g, q_\perp) H_{qg \rightarrow \gamma q}.
\]

Remarks:
- Independent CGC calculation gives the identical result in the correlation limit.
- Direct measurement of the Color Dipole gluon distribution.
DY correlations in $pA$ collisions

[Stasto, BX, Zaslavsky, 12]

$M = 0.5, 4\text{GeV}, Y = 2.5$ at RHIC $dAu$. $M = 4, 8\text{GeV}, Y = 4$ at LHC $pPb$.

- Partonic cross section vanishes at $\pi \Rightarrow \text{Dip at } \pi$.
- Prompt photon calculation [J. Jalilian-Marian, A. Rezaeian, 12]
There is no sign of suppression in the $p + p$ and $d + Au$ peripheral data.

The suppression and broadening of the away side jet in $d + Au$ central collisions is due to the multiple interactions between partons and dense nuclear matter (CGC).

Probably the best evidence for saturation.
Dijet processes in the large $N_c$ limit

The Fierz identity:

$$= \frac{1}{2} \quad \text{and} \quad = \frac{1}{2}$$

Graphical representation of dijet processes

\[ g \rightarrow q\bar{q}: \]
$$= \frac{1}{2} \quad \text{and} \quad = \frac{1}{2} N_c$$

\[ q \rightarrow qg \]

\[ 2 \quad = \quad = \frac{1}{2} \quad = \quad = \frac{1}{2} N_c \]

\[ g \rightarrow gg \]

The Octupole and the Sextupole are suppressed.
Gluon+quark jets correlation

Including all the $qg \rightarrow qg$, $gg \rightarrow gg$ and $gg \rightarrow q\bar{q}$ channels, a lengthy calculation gives

$$\frac{d\sigma(pA\rightarrow\text{Dijet}+X)}{d\mathcal{P}\cdot S.} = \sum_q x_1 q(x_1, \mu^2) \frac{\alpha_s^2}{s^2} \left[ \mathcal{F}^{(1)}_{qg} H^{(1)}_{qg} + \mathcal{F}^{(2)}_{qg} H^{(2)}_{qg} \right]$$

$$+ x_1 g(x_1, \mu^2) \frac{\alpha_s^2}{s^2} \left[ \mathcal{F}^{(1)}_{gg} \left( H^{(1)}_{gg\rightarrow q\bar{q}} + \frac{1}{2} H^{(1)}_{gg\rightarrow gg} \right) \right.$$ 

$$+ \mathcal{F}^{(2)}_{gg} \left( H^{(2)}_{gg\rightarrow q\bar{q}} + \frac{1}{2} H^{(2)}_{gg\rightarrow gg} \right) + \mathcal{F}^{(3)}_{gg} \frac{1}{2} H^{(3)}_{gg\rightarrow gg} \right],$$

with the various gluon distributions defined as

$$\mathcal{F}^{(1)}_{qg} = xG^{(2)}(x, q_{\perp}), \quad \mathcal{F}^{(2)}_{qg} = \int xG^{(1)} \otimes F,$$

$$\mathcal{F}^{(1)}_{gg} = \int xG^{(2)} \otimes F, \quad \mathcal{F}^{(2)}_{gg} = - \int \frac{q_1 \cdot q_2}{q_{1\perp}^2} xG^{(2)} \otimes F,$$

$$\mathcal{F}^{(3)}_{gg} = \int xG^{(1)}(q_1) \otimes F \otimes F,$$

where $F = \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-i q_{\perp} \cdot r_{\perp}} \frac{1}{N_c} \langle \text{Tr} U(r_{\perp}) U^\dagger(0) \rangle x_g$.

Remarks:

- All the above gluon distributions can be written as combinations and convolutions of two fundamental gluon distributions.
- This describes the dihadron correlation data measured at RHIC in forward $dAu$ collisions.
Comparing to STAR and PHENIX data

Physics predicted by C. Marquet. Further calculated in [A. Stasto, BX, F. Yuan, 11]

For away side peak in both **peripheral** and **central** \( dAu \) collisions

\[
C(\Delta \phi) = \frac{\int |p_1 \perp|, |p_2 \perp| \frac{d\sigma^{pA \rightarrow h_1 h_2}}{d\sigma^{pA \rightarrow h_1} dy_1 d^2 p_1 \perp d^2 p_2 \perp}}{\int |p_1 \perp| \frac{d\sigma^{pA \rightarrow h_1}}{d\sigma^{pA \rightarrow h_1} dy_1 d^2 p_1 \perp}}
\]

\[
J_{dA} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{\sigma_{dA}^{\text{pair}}}{\sigma_{dA}^{\text{pair}}} \frac{\sigma_{pp}^{\text{pair}}}{\sigma_{pp}^{\text{pair}}}
\]

- Using: \( Q_{sA}^2 = c(b)A^{1/3} Q_s^2(x) \).
- **Physical picture**: Dense gluonic matter suppresses the away side peak.
Conclusion:

- The establishment of an effective factorization for the collisions between a dilute projectile and a dense target.
- DIS dijet provides direct information of the WW gluon distributions. Perfect for testing saturation physics calculation, and ideal measurement for EIC and LHeC.
- Modified Universality for Gluon Distributions:

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× ⇒ Do Not Appear. √ ⇒ Appear.

- Two fundamental gluon distributions. Other gluon distributions are just different combinations and convolutions of these two.
- The small-x evolution of the WW gluon distribution, a different equation from Balitsky-Kovchegov equation; [Dominguez, Mueller, Munier, Xiao, 11]
- Dihadron correlation calculation agrees with the RHIC STAR and PHENIX data.