

BIOLOGY 1230: BIOLOGY I LABORATORY

FALL SEMESTER 2000

**COUGAR COOLING**

September 18, 2000

**Instructor Version**

# OSMOSIS AND COOLING: THE SAME PROCESS?

## Introduction: Some Simple Questions

Why are there no fish in the Great Salt Lake? Will global warming prevent some animals and plants from living in areas where they now are found? Why do we get thirsty on a hot day? Why is extreme thirst a common symptom in diabetics with severe insulin deficiency?

And most importantly: Does a single physiological explanation provide full or partial answers to these questions? The answer is **yes!** *OSMOSIS* is involved in all of these questions.

Can we go even further and use a quantitative explanation of osmosis to **predict** how much water the diabetic must drink to survive?, or how much water must be added to the Great Salt Lake before fish can survive?, or where a certain kind of animal will live if environmental temperatures rise? Answering these questions is the purpose of this series of lab exercises.

## Ionic Regulation

Osmosis is the process by which water moves from high water concentration to low water concentration across a semi-permeable membrane. We wish to understand the quantitative relationships between water concentration and rates of water flow into (or out of) cells. This will help us understand the special adaptations of organisms that live in extreme environments, such as the Great Salt Lake. The Great Salt Lake is interesting because it has no outflow of water except through evaporation. As a result, it is nearly 7 times as salty as the ocean! The water content of typical cells is much less salty than this.

### Quick Quiz

1. Will water tend to leave or enter normal cells, if they are placed in Great Salt Lake water?
2. With respect to such a cell, is the GSL hypotonic or hypertonic? (Your textbook defines these terms.)

## Modeling Water Movement

To predict phenomena like diabetes and the effects of global warming, we must understand the forces that unbalance the cell's internal environment. Prediction requires rules by which we can extrapolate from the known to the unknown. Mathematical equations are one set of rules that have proved useful for this purpose in almost all sciences, biology included.

We will ease into this activity by starting with another biological process that may appear to be completely different. This is the process of cooling. We start here, because everyone has experienced how a cold drink warms up on a hot day, or how we get cold on winter days when we haven't worn warm clothes. After studying one process (cooling), we will apply what we learn to another process (osmosis).

To begin this project, go to the next page for Section: *When Did the Cougar Die?*



## When Did the Cougar Die?

Many states in the western U.S. regard cougars (also known as mountain lion, puma, panther) as game animals with a designated hunting season. Typically, hunting seasons occur in the fall and winter with hunting allowed only during daylight hours. Wildlife Conservation Officers (COs) have the task of verifying that hunters kill cougars only during the specified times. Typically, this is accomplished by requiring successful hunters to visit a check station where legal cougars are tagged and illegal cougars are confiscated with dire consequences for the hapless hunter.



Go to the next Section to help the Conservation Officer answer the question.

## Step 1: Building a Conceptual Model

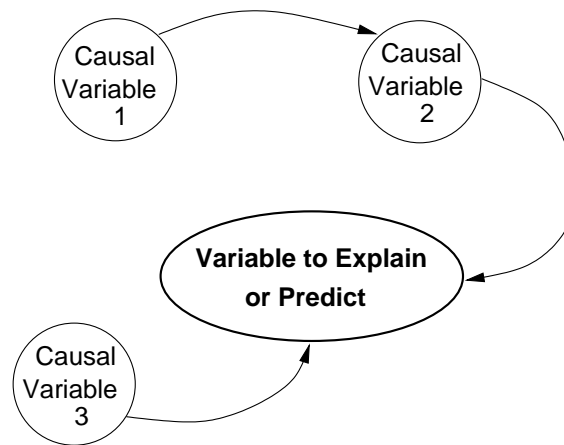
A hunter arrives with a cougar at a check station one hour after the legal time for hunting. The hunter claims that the cougar was taken legally and the late arrival is due to the time it took to load the animal into the truck and drive to the check station. Is there a way that the wildlife officer can verify the claim of the hunter?

### Your Job

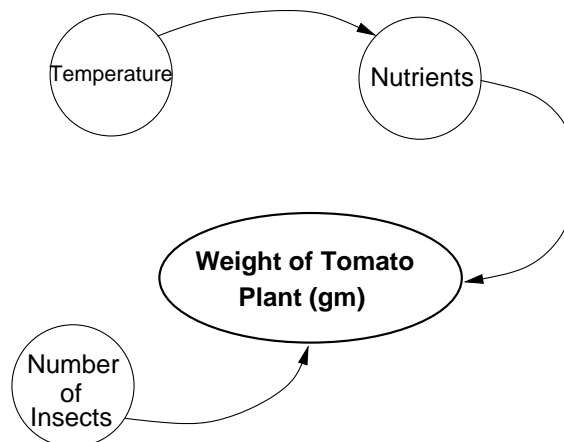
To help the officer answer this question, your task is to create a *mathematical model* which will predict the time of death.

To start, with your lab bench group draw a **Concept Map** of this problem.

A concept map is a diagram that illustrates the possible major factors that you believe influence a variable of interest. An ellipse at the center of the map represents this variable and the causal chain of influences are circles and arrows.



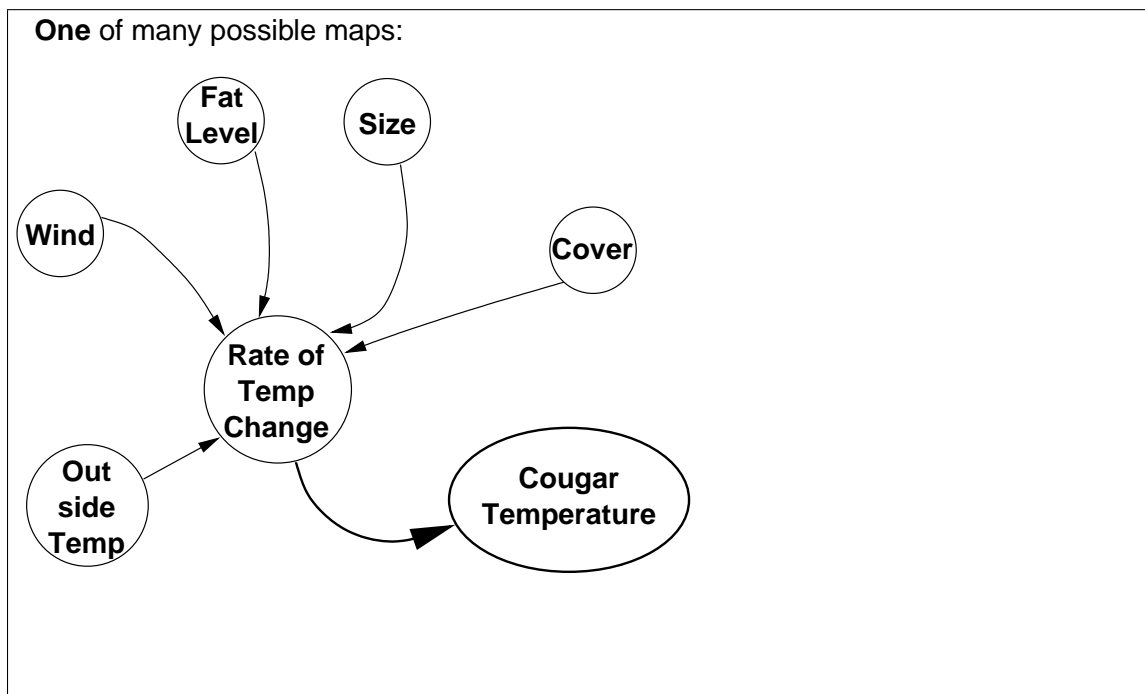
For example, suppose you were interested in the factors that influence the size of tomatoes. You might draw a concept map like the following.



Answer the following questions as you do your drawing of the concept map of a cooling cougar.

1. What are the most important factors that we should use in trying to predict the time of death?
2. What quantitative information (i.e., data) can the CO collect at the check station that will help her decide if the hunter should be cited?
3. What qualitative relationships can be identified among factors? For example, is there positive or negative **feedback** present? How is feedback acting in the conceptual model you have created? Can you draw a picture or graph of this?

Box 1. Draw a concept map below.



When you have completed the map, show it to your instructor for suggestions (and feedback!).

Wait for the instructor to re-convene the class for a discussion of your ideas and those of other groups.

## Step 2: Guessing Relations Among Variables

### Temperature Functions

Now that we agree on a conceptual model of the problem, we're ready to identify some relationships among the variables before writing an equation.

We want to quantify the relationship between a body's temperature and time. Mathematically, this means we want to describe a *functional* relationship between time and temperature. The cougar's temperature will be the dependent variable. The independent variable might not be time explicitly, because as time passes during cooling many other variables might also be changing. These other variables may have more explanatory power than "time", or may be the physical **cause** of the temperature change.

### What We Know

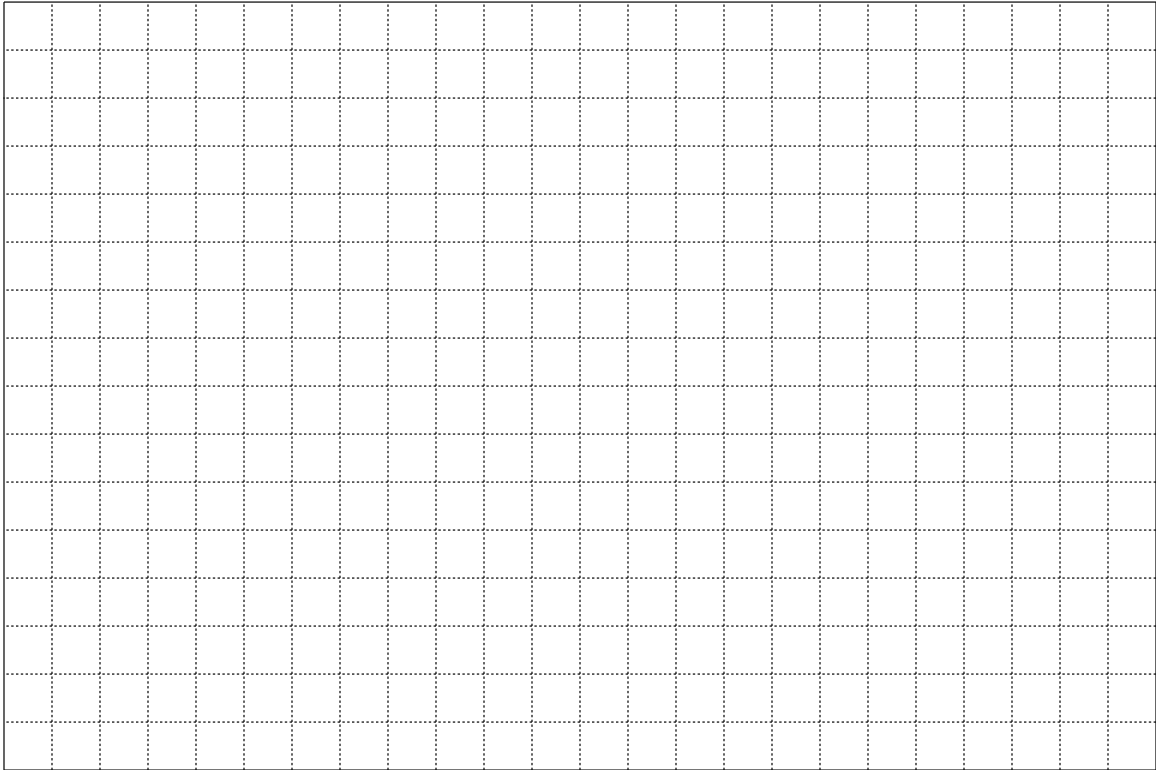
- As soon as the cougar is killed, the body temperature will change in the direction of the ambient air temperature surrounding the cougar.
- If the air temperature is colder than the body temperature of the cougar, the body temperature will decrease with time. If the air temperature is higher, the body temperature will increase with time.
- The air temperature will be cooler than the body temperature of a live cougar, because most cougar hunting is done in the fall.

### Guess a Curve!

Use the graph paper supplied to guess some patterns for the cooling over time. You should try several different curves to show the decreasing temperature.

Before asking your instructor for help, plot your own curves based on the following questions.

1. What are the axes labels? What are the x axis and the y axis? Which is the dependent variable and which is the independent variable?
2. What are reasonable ranges for the 2 axes? What is the body temperature of a living cougar? Is it greater than or less than a human's temperature?
3. What are some possible curves for the two variables? Will the y-axis values go up, then down? Always down? How will the **slope** of the temperature curve change with time? Will the **slope** be constant, increasing, decreasing?)
4. Will the temperature reach a minimum? What will the change over time be very near the minimum?



Now that you have some possibilities; guess which of these best illustrates the qualitative behavior we should expect to see in the cooling body. Write down which one you think is most likely and why on a separate piece of paper. Keep this guess in mind as you proceed through this laboratory exercise. Later we will test your intuition. If you are proven wrong, you will have to think a bit more about the process.

When the instructor re-convenes the class, present your ideas.

### Step 3: Performing the Experiment

The largest percentage of the body weight of the cougar is water. As a first approximation of the cougar, we will study how a volume of water cools as a function of time. Our first observations will measure the temperature of a beaker of water as it cools.

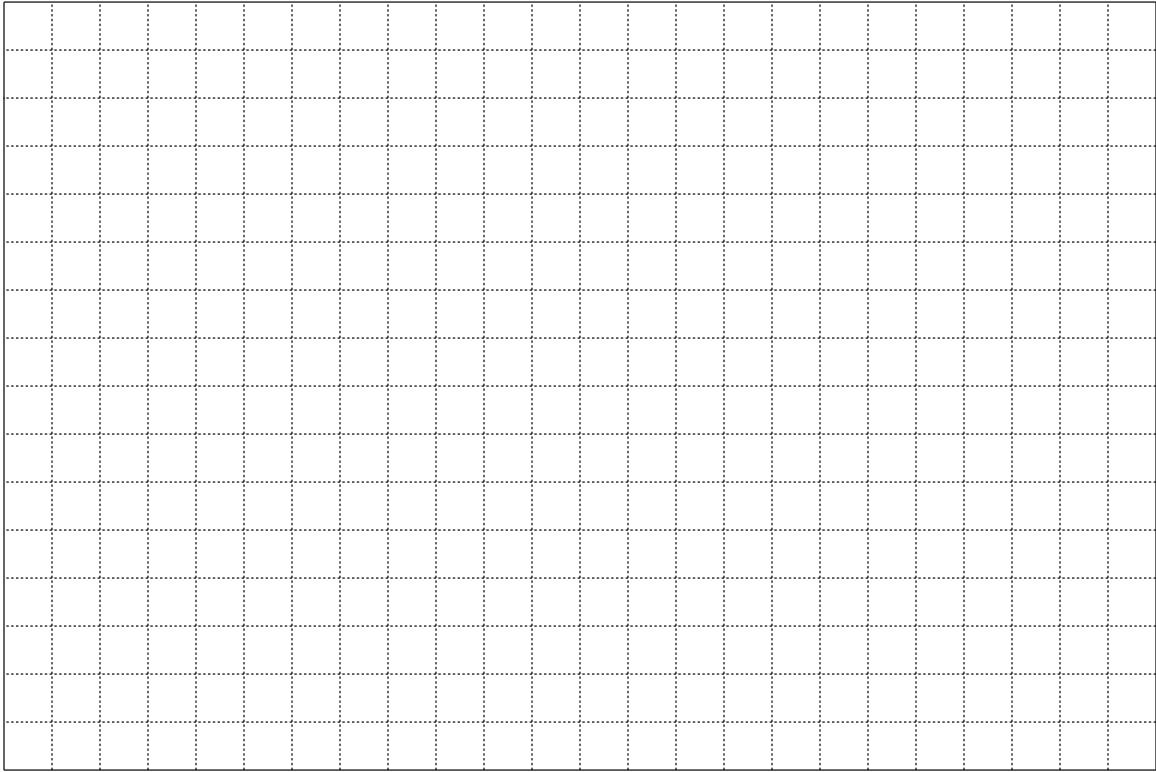
The materials you will need are:

1. a glass beaker that can hold more than 400ml of water
2. a tub or tray to hold ice water
3. a thermometer showing a centigrade scale
4. a stopwatch
5. a sheet of paper for recording measurements (attached)
6. a stirring rod

The steps in the experiment are:

1. Equilibrate the thermometer by filling the glass beaker with warm water and inserting the thermometer. Set this aside for about 4 minutes.
2. While the thermometer is equilibrating, from the front or side of the lab, fill a shallow, plastic container with a slurry of crushed ice and cold water.
3. Pour the slurry into your plastic tub or tray.
4. After the thermometer has equilibrated (4 minutes), empty the beaker and quickly perform the next step.
5. Pour 300ml of warm water from the constant temperature bath into the glass beaker. Insert the warm beaker into the ice slurry. The ice slurry should be equal to the height of the warm sample water.
6. Immediately after inserting the beaker in the ice slurry record the beaker temperature. On your data sheet (attached), record the time, the initial temperatures of the slurry and the sample; in a fourth column record the difference in the two temperatures.
  - (a) Make sure that you suspend the thermometer in the water so that the thermometer does not contact the sides of the beaker.
  - (b) Gently stir both the slurry and the sample water by making four slow passes through the two media with the thermometer. (Why?)
7. Measure the temperature every 60 seconds for 10-15 minutes and record the temperature and the difference between the temperature of the water and the ice. Your instructor will tell you when you have enough data.
8. Stir both the slurry and the sample after each observation as described above.





Wait for the instructor to re-convene the class to discuss how to analyze the data.

## Step 4: Analyzing the Data

At this point, we have a graphical model of how temperature will change with time. We now have to test our hypothesis with experimental data. Our hypothesis is only *qualitative*: a graph of how temperature decreases with time. To really help the conservation officer, we must also produce a *quantitative* hypothesis that predicts exactly how cold the cougar will be after the period of time since the hunter killed the animal. To produce a quantitative prediction we need an equation.

To begin, let's let  $T$  be temperature and  $t$  be time. We'll need some more variables later, but this will do for now.

So, what should the equation look like?

Well, you might guess that it should look like:

$$T = f(t)$$

### Quick Quiz

State in words what the above equation says in symbols.

This is not a bad idea, but what is wrong with it? Think hard about this and write a good answer on a piece of paper.

Okay, so you know that we can't just write down an equation off the top of our head (even if it might be right in the end). In science, we try to get the right answer for the right reason. And that means we have to think about why the curve has the shape it does.

### Your Job

Write answers to these questions:

1. For the data you just collected and plotted, what happens to the change in temperature over time? Describe what you see in simple words.
2. Why does the temperature change behave this way? Is it just because the water beaker is getting old and cold as time passes? Or, is the temperature change related to the current temperature?
3. State in words a hypothesis that relates temperature change with temperature. Here is one possibility, but it's wrong and you can do better. *Temperature change decreases in proportion to the current temperature.*
4. List the variables you might need. Give them mathematical symbols like  $T$  and  $t$  above.

After you think you have a good possibility, wait for the other groups to finish and then present your idea to the class.

## Step 5: Computing the Rate of Change

The data records the behavior of the temperature of the cooling material over a period of time. (See Figure 1 below.) Your data should indicate that the **rate of cooling** is

Box 2: Rate of cooling in words

“The **rate of cooling** is changing with the **difference** between the temperature of the sample beaker and the surrounding ice.”

This pattern is the key to creating a mathematical equation for temperature. In order to write an equation, we will need the following definitions.

Fig 1 reminds us of what the data look like. The x-axis is time and the y-axis is temperature.

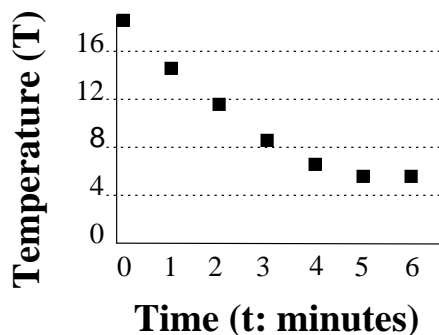


Figure 1: Temperature over time.

### Definitions:

1. Since the process is time dependent, we will use the variable  $t$  to keep track of the period of time on which the process occurs (e.g. 20 minutes for the experiment).
2. The variable  $T$  will be the temperature of the sample water during the cooling process. This means that  $T$  is dependent on the variable  $t$  and we denote this dependence,  $T = T(t)$ .
3. From the class discussion, the rate of cooling seems to be very important. We will keep track of this quantity with the variable,  $r$ . We have observed that the rate is dependent on temperature. We can denote this dependence as  $r = r(T)$ , or in words: "the rate of cooling is a function of temperature".

- We also need some constants to keep track of the environmental conditions (in our case ice water). We will let  $T_a$  represent the temperature of the ice bath (or, ambient air if this were a real cougar). The initial temperature of the body is the temperature at time 0 and will be denoted by  $T_0$ . Note that  $T_0 = T(0)$  by the definition above.
- For the data points, we will use  $t_i$  to denote the time of the  $i$ -th sample. For example, in Fig. 1  $t_3$  is 3 and  $T_3 = 9$  (approximately). We can translate this as  $T(t_i) = T_i$  to be consistent with the definitions above.

**Quick Quiz**

From Fig. 1, what is  $t_6$ ? What is  $T_6$ ? (Write down your answer so that the instructor can check it.)

Using these useful definitions, we can answer the question: How does the temperature rate of change depend on temperature?

First, we need to know how to define the rate of change. Actually, this is tricky, but we can compute an approximate value for the rate of change by computing the slope of the line joining two successive data points. See Figure 2 below.

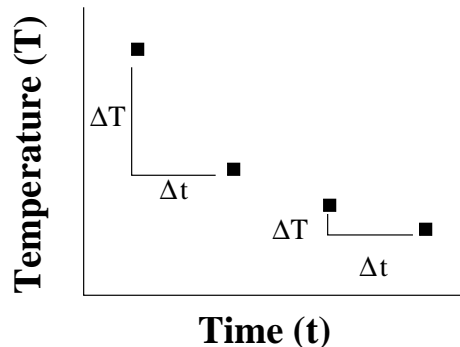


Figure 2. Temperature over time showing the temperature rate of change:  $\Delta T/\Delta t$ .

The slope of each of these lines gives an approximation of the rate of change of temperature with respect to the independent variable,  $t$ .

For example, suppose we had data different from Fig. 1 or 2 in which the first two data points in the sample were (Time, Temperature): (0.03,21.26) and (0.45,13.41), we can compute an approximate rate of change between the points. The value is

$$\text{slope on } [t_0, t_1] = \text{rate of change} \approx \frac{T_1 - T_0}{t_1 - t_0} \approx \frac{13.41 - 21.26}{0.45 - 0.03} \approx -18.69$$

We can do this calculation at any point along the x-axis. Suppose at the other end of the data we had:

$$\text{slope on } [t_0, t_1] = \text{rate of change} \approx \frac{T_{11} - T_{10}}{t_{11} - t_{10}} \approx \frac{2.08 - 2.13}{4.45 - 4.04} \approx -0.12$$

As predicted, the rate of heat loss at the end is much smaller than the rate of heat loss at the beginning.

**Quick Quiz**

Using approximate values from Fig. 1, what is the rate of change of temperature between time 2 and 3?

Now go to the next page and do these calculations on your own data.

## Step 6: Tabulating Your Data

At this point, we want to make a complete tabulation of our data so that we can see the patterns and relationships that the data reveal. These patterns will help us write the final equation we need to solve the conservation officer's problem.

You have a data sheet with 4 columns: (1) time, (2) temperature of ice water, (3) temperature of the water sample, and (4) the difference between the two temperatures. Add a fifth column for the rate of change of temperature. Do the following work on the data set from the experiment that you conducted.

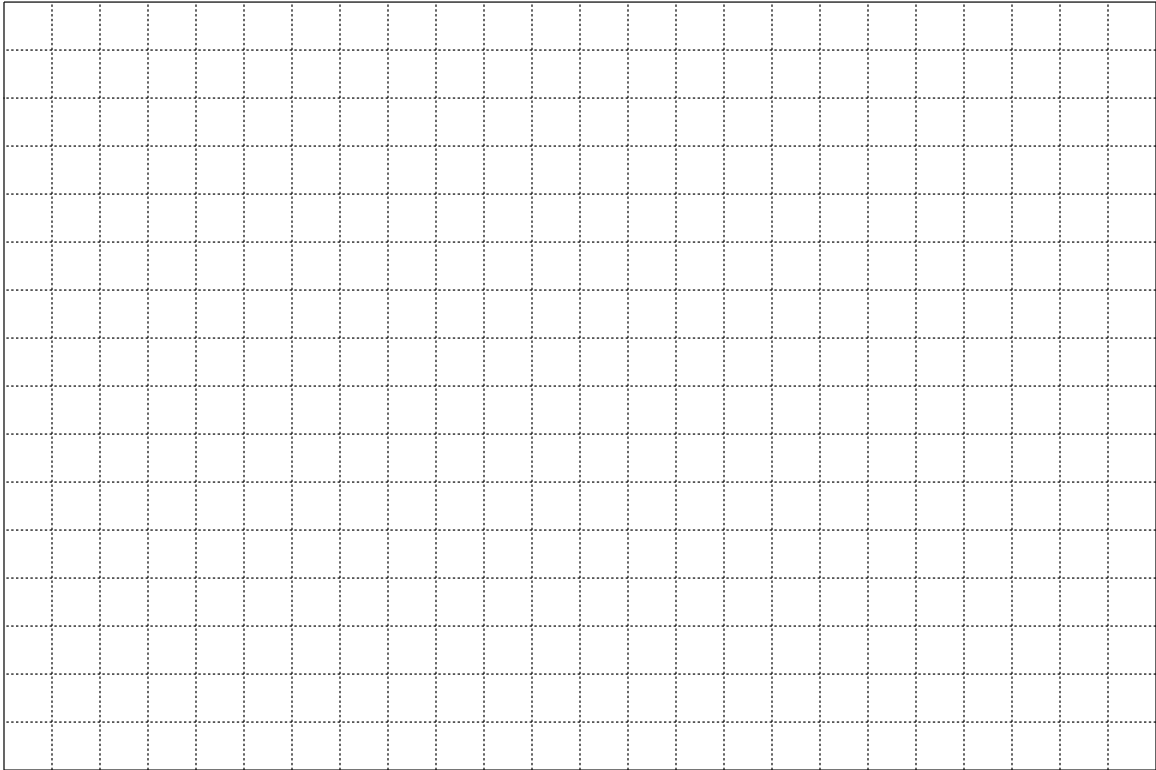
### Your Job

Tabulate the rates of change:

1. Compute the approximate rate of change of the temperature on each time interval and record this in a fifth column of your original data sheet.
2. Record the values on the same row as the starting temperature of the interval.

These 5 columns of numbers are everything we know about cooling water. Are there any interesting patterns?

Use the graph paper on the next page to try out some combinations. **Hint: Look in Box 2 to remind yourselves of our basic hypothesis."**



When the instructor re-convenes the class, present your results.

## Step 7: Identifying Quantitative Parameters

### Coefficient of Thermal Conductivity

In Step 5, we guessed that the rate of temperature change would be a function of temperature and we said:  $r = r(T)$ , where  $r$  is the amount of temperature change. Following this last class discussion, we now have an exact equation for  $r(T)$ . This equation is a straight line with negative slope and an intercept of 0. We called the slope  $-D$ . You should write the equation in Box 3 below.

Box 3: Model of temperature rate of change.

$$r(T) = -D \cdot (T - T_a)$$

This equation was revealed in the class discussion just completed, where we called  $D$  the ...

Box 4:  $D$  in words

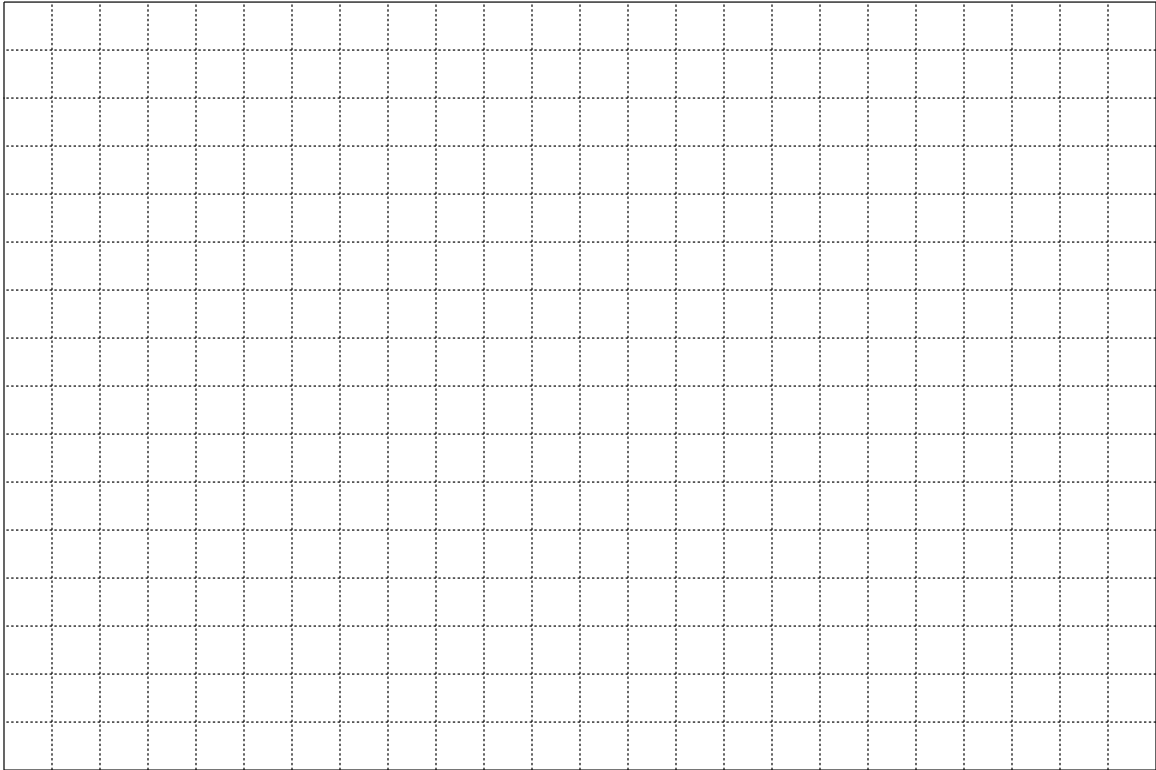
thermal conductivity

### The Value of $D$

This is good, but we have a problem: **What is the value of  $D$ ?** In other words,  $D$  is an unknown **parameter**, and we must **estimate** its value using our data.

### Your Job

1. Design a plan to estimate  $D$ . (**Hint: Look at Box 3.  $D$  is unknown, but do we know  $r(T)$  and  $T - T_a$ ? Can you plot pairs of  $r(T)$  and  $T - T_a$  to get a simple graph?**)
2. Once you see how to estimate  $D$ , calculate  $D$  using your data. Check with your lab instructor that you have the right method.



### Quick Quiz

1. Is the model a good description of these data? Why or why not? Write down the value of  $D$ .
2. What is the physical meaning of  $D$ ? What are the units of  $D$ ?
3. Would  $D$  have a different value if we wrapped the sample beaker in an insulator like foam? If yes, would  $D$  be larger or smaller? What if the beaker was made of glass? What if the sample container was not a beaker but a long skinny cylinder immersed in the ice water?

When the instructor re-convenes the class, discuss how you estimated  $D$  and your answers to the above questions.

## Step 8: Computing Temperature from the Model

Your instructor will lead the class through this material.

So far, we've learned that the rate of cooling depends on the current temperature according to the following equation:

**Box 5: Model of rate of temperature change.**

$$r(T) = -D \cdot (T - T_a)$$

The physical law that this embodies is known as

**Box 6: Name of model of temperature change.**

Newton's Cooling Law

We've also learned how to estimate D from real data.

This is great, but it's **not enough!** It doesn't solve the wildlife officer's problem! For that we also want to be able to predict the temperature of a particular dead cougar.

Here is how we do that.

1. The rate equation tells us how much temperature will be lost in one unit of time (1 minute). If we know the temperature at time  $t = 0$  minute, then subtracting the temperature lost in 1 minute will give me the temperature at time  $t = 1$  minute. Like this:

Box 7: Temperature at next time in words

$$\text{Temp}_{\text{minute } 1} = \text{Temp}_{\text{minute } 0} - D \times (\text{Temp}_{\text{minute } 0} - \text{Temp}_{\text{ambient}})$$

or

next minute = previous minute - temperature lost in one minute

or to use our symbols:

Box 8: Equation for temperature at time 1

$$T_1 = T_0 - D(T_0 - T_a)$$

This can be repeated (or **iterated**) to get  $T_2$ :

Box 9: Equation for temperature at time 2

$$T_2 = T_1 - D(T_1 - T_a)$$

2. Unclear? Here's another way of looking at it.

*If you have 10 coins in a box at time  $t=0$  and you remove coins at the rate of 2 coins per minute, how many coins will you have in the box after 1 minute? [10-2=8, of course]. Continuing: if you have 8 coins at time  $t=1$ , how many will you have at  $t=2$ ?*

3. Examine the equations in Boxes 8 and 9. Observe how the value on the left side of Box 8 is inserted in the right side of Box 9. This is a general pattern and here is the general formula for temperature at any time  $t$ .

Box 10: General equation for temperature at next time

$$T_{t+1} = T_t - D(T_t - T_a)$$

Now we're ready to use this equation to solve a **mystery** beaker. Wait for the instructor to give you the assignment.

## Step 9: The Mystery Beaker

So far, we have developed the rate equation and have made estimates of  $D$  from measurements on one beaker. We also derived an equation that predicts future temperatures. We want to use the equation to make predictions on a new beaker that has not been measured before. This is the real problem that faces the conservation officer. She can not use the temperature curve for the old, stuffed cougar that they dragged out for her Fisheries and Wildlife course on "Cougar Cooling". To get an estimate of time of death for this new cougar, she has to apply the model to a new situation.

To give you practice in also doing this, we're going apply the cooling equation to a **mystery** beaker.

Here is your problem:

"When was the beaker of warm water placed in ice water?"

### A Pseudo-Cougar Kill

Putting a warm beaker in ice is analogous to killing the cougar. Determining how long ago that occurred is analogous to the problem facing the conservation officer.

We did this in the following way. Unbeknownst to you, at some time in the past, we secretly heated a beaker identical to yours to a high initial temperature and allowed it to cool in an ice bath identical to yours for some period of time. At the end of that time, we measured the temperature of the beaker.

### Your Job

The instructor will tell you what the starting temperature was. Your assignment is to use the cooling model to determine the "time of death" of the mystery beaker. I.e., how long was the beaker in the ice?

Use your calculator and (optionally) graph paper. Save your answer to compare with the answers from the other groups.

When the instructor re-convenes class, compare your answer with those of the other groups.
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## Step 10: In-Class Exercises

These solved problems are similar to the ones you will be assigned as homework by your instructor. If you have time, you may work on them in class or do them on your own time.

- Our basic model works for any cooling body (biological or otherwise) including deer as well as cougars. Below are real cooling data for a field-dressed deer weighing 25-38 kilograms in air with temperatures in the range -12 to -4 C. Hours are hours after death (note the unequal time intervals). Temperatures were taken from the head of the deer. What is  $D$  for this deer?

Answer: Calculate and plot the rates versus temperature. Estimate the slope of the line visually to get  $D$ . Using just two points on the curve:  $T=38$  and  $T=7$  with rates 6 and 1 respectively, then  $D = (6-1)/(38-7) = 0.16$ . Students should plot the data, visually estimate the line, and use that to get the slope.

Cooling Curve	
Hours	Temperature
0	38
1	32
2	27
3	23
4	20
5	17
6	14
7	11
8	9
9	7
12	> 5
13	> 4
18	> 2
22	> 1
30	> -1

Rate vs Temperature	
Temp	Rate
38	6.0°/h
32	5.0°/h
27	4.2°/h
23	3.5°/h
20	3.1°/h
17	2.7°/h
14	2.4°/h
11	1.8°/h
9	1.2°/h
7	0.67°/h
5	0.55°/h
4	0.40°/h
2	0.25°/h
1	0.20°/h

- (a) In the above data, what are the units of  $D$ ?

Answer:  $\Delta\text{RATE}/\Delta\text{TEMP} = \text{°C/h} / \text{°C} = 1/\text{hour}$ .

(b) If the increments in time had been minutes, what would be the units of  $D$ ? Answer: 1/minute

- If the starting temperature of a body is 38°C, ambient temperature is 10 °C, and  $D = 0.2/\text{h}$ , what is the temperature after 6 hours?

Answer. Iterate

$$T_{i+1} = T_i - D(T_i - T_a)$$

six times to get:

Hours	Temperature
0	38.0
1	32.2
2	27.8
3	24.2
4	21.4
5	19.1
6	17.3

4. In the above problem, if the observed temperature had been  $15.5^{\circ}\text{C}$ , how much time elapsed since death?

Answer: Iterate the equation until temperature is less than  $15.5^{\circ}\text{C}$ . At  $t = 7$ , temperature =  $15.8$  and at  $t = 8$ , temperature =  $14.7$ . So, 8 hours had elapsed.

5. Assume that the ambient temperature is  $4.4^{\circ}\text{C}$ , sunset occurred 3 hours previously, and a deer just brought to the check point has  $D = 0.16$  with a living body temperature of  $37.8^{\circ}\text{C}$ . What is the critical temperature for this deer to be legally killed? In other words, above what temperature will there be evidence for an illegal kill?

Answer: Iterate the recursion equation 3 times (3 hours). The resulting temperature is the critical value. Here are the temperatures:  $37.8$ ,  $32.4$ ,  $27.9$ ,  $24.2$ . Any carcass above the last temperature was illegal.

Congratulations! You're done! Your instructor will tell you where to find some real homework problems for you to do on **your own**.