An Accuracy–Response Time Capacity Assessment Function That Measures Performance Against Standard Parallel Predictions

James T. Townsend
Indiana University

Nicholas Altieri
University of Oklahoma

Measures of human efficiency under increases in mental workload or attentional limitations are vital in studying human perception, cognition, and action. Assays of efficiency as workload changes have typically been confined to either reaction times (RTs) or accuracy alone. Within the realm of RTs, a nonparametric measure called the workload capacity coefficient has been employed in many studies (Townsend & Nozawa, 1995). However, the contribution of correct versus incorrect responses has been unavailable in that context. A nonparametric statistic that is capable of simultaneously taking into account accuracy as well as RTs would be highly useful. This theoretical study develops such a tool for two important decisional stopping rules. Preliminary data from a simple visual identification study illustrate one potential application.

Keywords: capacity, efficiency, response time, accuracy

Supplemental materials: http://dx.doi.org/10.1037/a0028448.supp

The quest for a yardstick for processing efficiency has long been something of a Holy Grail for many in the psychological sciences in general research (e.g., Hick, 1952; Jastrow & Cairnes, 1891; Neisser, 1967; Pachella, 1974), in specific research domains such as visual attention (e.g., Kahneman, 1967; Schneider & Shiffrin, 1977), and even in studies investigating cognitive impairment or mental pathologies (e.g., Bundesen & Habekost, 2008; Neufeld, Carter, Boksan, Jette, & Vollick, 2002). A number of general reviews and discussions involving varying degrees of breadth, although concerning fundamental principles of capacity and attention, have appeared over the years (e.g., Kahneman, 1973; Navon & Gopher, 1979; Neufeld, Townsend, & Jette, 2007; Norman & Bobrow, 1975; Palmer, Vergheese, & Pavel, 2000; Schweickert & Boggs, 1984; Townsend & Ashby, 1978).

Moving to more precision in our language, we define workload as the number of entities that require processing of some sort. The entities specified by the investigator often refer to objects such as features, words, angles, and so on. We can roughly partition methods and models derived from experimental/cognitive psychology analyzing performance as a function of workload into the following:

1. Descriptive use of statistics, such as the means, variances, and hazard functions. An example would be Group A is faster and more skilled than Group B because the mean reaction time (RT) of Group A is significantly smaller than that of Group B and the probability correct in Group A is larger than that of Group B.

2. Principled models based on specific probability distributions or stochastic processes. The model predictions are functions of parameter sets that may be fit to data or whose properties as functions of the experimental design may be tested.

3. The employment of statistical procedures and formulas connected to canonical properties of major model classes, such as parallel versus serial processing models or limited versus unlimited versus super capacity.

The third approach comprises the major focus in the present study, although we discuss some of the alternative strategies in the Discussion. Our theory-driven methods for dealing with mental efficiency as a heavier burden of work is imposed have been almost entirely devoted to RTs (but see exceptions in Townsend & Ashby, 1978, 1983). The present study offers a significant extension that envelops and synthesizes both accuracy and RT-dependent variables in a unified framework.

A central distinction is that our proposed methodology is based on a battery of measures that supersede any particular parameterized system or model—they are distribution and parameter free. Although we cannot provide a review here, our RT-based workload capacity functions, previously termed C(t), with appropriate specific indices,
have proven useful and robust in measuring workload capacity in many perceptual and cognitive situations (e.g., Richards, Hadwin, Benson, Wenger, & Donnelly, 2011; Townsend & Wenger, 2004). We should observe early on, that although “capacity” tends to connote ability, power, and so on, observable indicants might well be influenced by decisional and motivational variables.

One important aspect of defining capacity in terms of a function that can vary with time, as opposed to a single number (e.g., mean RT), is that, in principle, the latter can reflect real dynamic systems that may vary in their processing efficiency across time. This concept of a within-trial varying capacity appears to be relatively novel and will be entertained further in what follows. Variation in capacity across time could occur due to changes in the input, or actual alterations in resources or between-process interactions, or even external events such as pre- or post-stimulus masks.

RT-based capacity functions have also been productive in joining our other techniques to draw inferences about the nature of the underlying processing channels (e.g., Eidels, Houpt, Altiieri, Pei, & Townsend, 2011). On the other hand, it is ultimately theoretically unsatisfying to possess one theory (or set of theories) and methodologies (or set of methodologies) for RTs and separate sets of theories and methodologies for accuracy. This factor was undoubtedly a major propellant for the detailed process models put forth by investigators over the years.

Our major goal in this study is thus to construct a methodology that combines accuracy and RT in a fashion that continues to employ, as a benchmark or measuring stick, what is commonly known as the class of ordinary parallel processing systems. This extension is rooted in what we refer to as the “standard parallel model,” which includes our former statistics in the limit as accuracy becomes perfect. Perhaps most important, we endeavor to open up a more articulated set of portraits of the interplay of accuracy and RT. Readers who are familiar with our theory of capacity applicable to RT-only data may wish to skip to the subsequent sections.

**Background and Theory of Capacity Applicable Only to Response Times**

Much of our laboratory’s theory and methodology is founded on four major characteristics of human information processing:

1. Efficiency as workload varies: unlimited versus limited capacity.
2. Mental architecture: parallel versus serial processing versus more complex architectures.
3. Decision or stopping rules.
4. Independence versus dependence of channels, items, and/or stages of processing.

All of these are critical, and although logically independent, they interact in sometimes surprising ways when instantiated in various fashions in a functioning system. We placed capacity first here as that will be our main focus.

The term capacity has frequently been employed to capture the idea of efficiency of processing (e.g., Kahneman, 1973; Townsend & Ashby, 1978). Of course, it often is used to represent the more static concept of “capaciousness of memory” rather than to stress efficiency across time, with which we do not deal here. In addition, readers will be aware of its specialized meaning in Shannon’s mathematical theory of information (Shannon & Weaver, 1949). Our usage will continue our tradition of tying it into speed and, now, accuracy as well.

One point has been clear for some time: Descriptions or predictions regarding efficiency or capacity in terms of speed inexorably involve other aspects of processing from the above set of characteristics, such as architecture, stopping rule, and subsystem interactions (e.g., Schweickert & Townsend, 1989; Townsend, 1974; Townsend & Ashby, 1978; Townsend & Thomas, 1994). Thus, to return to our focal theme of workload, we observe that how increases in the processing burden affect capacity will inevitably depend on these other characteristics. In particular, an effective theory should be sensitive to the stopping rule because that alone can affect speed of completion.

Over the past decade or so, we have engaged in the development of a theory and related methodology of capacity for RTs alone that capture all these major aspects of a system (Townsend & Nozawa, 1995, 1997). One way of thinking about this approach is to start from a question of capacity as workload changes. It might well be asked, “relative to what?” The cognitive sciences are relatively bereft of measuring instruments akin to, say, those of physics.

This question led us to found our measure on a specific class of systems. This class is composed of all systems that process items simultaneously, in a stochastically independent way (with zero correlation), and whose channels are impervious to the workload burden. The latter stipulation means that no matter how many channels are working, the speed on any particular channel is unaffected. We call this type of system an “unlimited capacity, independent, parallel system.” Subsequently, we will refer to this kind of model as a “standard parallel” system. In a sense, it is as prototypical as a serial system that is based on the same processing time distributions at each stage, with independent stage times, and whose individual distributions do not change as the workload is altered (e.g., the famous serial model of Sternberg, 1969). We refer to the latter as a “standard serial” system.

We therefore founded our capacity measurement methodology on the standard parallel class of models (Townsend & Nozawa, 1995). The fundamental concept is to run the experiment with a specific “basic” workload, usually $N = 1$, where $N$ is the number of subprocesses involved in a mental task. It is then possible to make predictions for heavier workload $N > 1$, under various assumptions concerning architecture, stopping rules, and so on. These predictions are free of particular probability distributions or their parameter values, a feature that makes the methodology quite powerful and robust. By making our foundational measurement tool standard parallel processing, we were able to define three important subclasses:

1. “Unlimited capacity processing” is processing that is equivalent to that of standard parallel processing. Here, $C(t) = 1$ for all times $t$.
2. “Limited capacity processing” means processing that is slower than that of standard parallel processing. In this case, $C(t) < 1$ indicates limited capacity at any such value of $t$. In these instances, standard parallel processing can be rejected in favor of either a parallel model with negative dependencies or limited resources (e.g., Townsend & Wenger, 2004) or serial processing depending on the task demands and observed capacity values (e.g., Townsend & Nozawa, 1995). For example, if $C(t)$ hovers around
3. “Super capacity processing at time $t^*$ is processing that is faster than standard parallel processing at a given point in time. This seemingly remarkable event occurs at any value of $t$, such that $C(t) > 1$ (of course, taking statistical aspects into account). Super capacity may be produced by coactive mechanisms, which are basically parallel channels that feed into a final common pathway. Another attractive possibility is the class of parallel models that possess facilitatory interactions (both types of systems are discussed further below; also see Townsend & Wenger, 2004). Super capacity is sometimes revealed in experiments where holistic perception is facilitated (e.g., words and faces; Wenger & Townsend, 2006; emotional expressions, Innes-Ker, 2003; perceptual learning of meaningless feature strings, Blaha, 2010; Blaha & Townsend, 2008).

Discussion of these models will resurface in the presentation of the exemplary data. However, a few words may help to explicate matters as follows. First, and as mentioned briefly in the introduction, because the $C(t)$ functions are measuring performance across time, it makes sense to note whether, say, the observed system is qualitatively invariant for the entire observation interval. That is, we can inquire if the system is always limited, always unlimited, or even super capacity for all observed times $t$ (within statistical error of course). But, the characteristics of the system combined with aspects of the input signal may reveal variability in capacity, for instance, early super capacity followed by merely unlimited or even limited capacity. This detailed kind of information may aid in experimental identification of the underlying system and how it responds to different kinds of input strength and information content. Of course, a summary measure such as the average $C(t)$ across observed RTs may be quite useful but naturally obscures the informative real-time trajectory.

Next, because of the involvement of architecture, possible channel or item dependencies, and decisional stopping rules, it is necessary to specify all of these in garnering predictions from various model types. An immediate implication is that even presupposing the baseline parallel architecture and assuming channel independence, the specified $C(t)$ formula must also take into account the decisional stopping rule, that is, when there will be sufficient information for the system to cease and make a correct response. Thus, we next consider the stopping rule explicitly.

Processing all items, features, objects, and so on (i.e., exhaustive = maximum completion time = last-terminating completion time = conjunctive processing) naturally takes longer, everything else being equal, than being able to stop after the first item is completed (i.e., minimum completion time = first-terminating = disjunctive processing). Thus, the primary stopping rules investigated so far are AND (conjunction) and OR (disjunction) designs. In redundant target OR designs, where there can be more than one target in a display or memory, for example, the identification of any of them can lead to a correct response (e.g., Egeth, Folk, & Mullin, 1989; Miller, 1982, 1986; Townsend & Colonius, 1997). An AND design requires completion of all the designated information, several items, features, and so on. A special case of some import is the feature-complete identification design (e.g., Ashby & Townsend, 1986). In these instances, all possible combinations of feature or dimension levels, including featural absence, make up the stimulus set and a unique response (e.g., naming the levels on each dimension) is assigned to each stimulus pattern. A note on terminology: Although targets are “redundant” in the case that correct identification of any single one of them leads to a correct response, we will employ that term along with the equivalent “multiple” for both types of stopping rules.

Any modern treatment of capacity in RT theory or methodology should include the valuable inequalities that have been developed. These and other branches of research sometimes use different terminology from ours, but unless otherwise noted, the concepts map one-on-one into our theory. The first development of inequalities appeared in the OR type of experimental design and includes both upper bounds, indicating superior performance, and lower bounds, indicating subpar performance (according to standards shortly to be made evident).

The superior bound described by Miller (1982), is the upper limit that could be expected on the basis of separate decisions (in each channel) parallel processing. Basically, a major underlying postulate, though unstated at the time (see Ashby & Townsend, 1986; Luce, 1986), was that the marginal processing time distribution on each channel would be invariant whether presented alone or in the context of one or more other signals (e.g., dimensions, features). This stipulation is referred to as “the assumption of marginal RT context invariance” or “context invariance” for short. Let A stand for one target and B for the other. When AB is written in a formula, for example as a subscript, it implies that both A and B were presented together, and when A or B appears alone it means that either was presented by itself. We can express the Miller inequality as

$$P_{AB}(\text{WINNER OF RACE’S TIME} \leq t) = P_{AB}(\text{MIN}(T,T) \leq t)$$

$$= F_{AB}(t) = F_A(t) + F_B(t)$$

$$= P_A (A \text{ finishes by time } t \text{ when working alone}) + P_B (B \text{ finishes by time } t \text{ when working alone}).$$

This inequality has been employed in scores of studies to provide evidence of processing that is somehow “better than normal parallel processing.” Miller called performance that exceeds the race bound “coactive.” The notion of coactive processing came to suggest a pooling of information from separate channels into a subsequent flow upon which a decision was based, and mathematical characterizations of coactivation came into being (e.g., Colonius & Townsend, 1997; Diederich & Colonius, 1991; Miller & Ulrich, 2003; Schwarz, 1989; Townsend & Nozawa, 1995). As Townsend and Nozawa (1995) showed, such models always predict performance superior to ordinary parallel processing, regardless of the particular distributions associated with the individual parallel channels.\(^1\)

Furthermore, specializing our capacity function to include the OR designation, $C_p(t)$, we demonstrated that if $C_p(t) > 1$ for even

\(^1\) Up to quite recently, the RT-based $C(t)$ measures and bounds, as well as the decisional stopping rules, have had to be dealt with somewhat independently, particularly with regard to their mutual relationships. Townsend and Eidekels (2011) proposed a set of transformations and associate spaces for a common portrayal and comparison of these and related RT-based entities. However, it will behoove us here to adhere to the traditional terminology.
a small interval of time when \( t \) is small, the Miller inequality will be violated. And, if the Miller inequality is violated at any par-
ticular value of time \( t \), then \( C_s(t) > 1 \) at that time \( t \). The inequality is thus closely related to our capacity measure as one would hope, but they are not at all equivalent. For instance, Miller’s inequality is satisfied trivially for large \( t \) (thereby giving no information in those larger intervals concerning capacity), whereas \( C_s(t) \) delivers useful information for the entire statistically useful range of RTs. For later processing times, capacity can be quite large without violating the bound. Also, \( C_s(t) \) is a continuous measure offering a measure of capacity at all \( t \) including unlimited or limited capacity.

On the other end of performance in OR designs lies Grice’s inequality (e.g., Grice, Canham, & Gwynne, 1984). Again, assuming that marginal channel distributions are invariant whether one or two channels are operating, the Grice inequality can be written as

\[
P_{AB}(\text{WINNER OF RACE’S TIME } \leq t) = P_{AB}(\text{MIN}(T_A, T_B) \leq t)
= P_{AB}(t) \geq \text{MAX}(F_A(t), F_B(t)).
\]

Interestingly, the Grice bound indicates, in a sense, more limited capacity than Miller’s bound indicates super capacity. That is, assuming roughly equal channel speed, \( C(t) \) will drop to the level of the Grice bound, \( C_s(t) = \[1/2\] \). This is the level of efficiency (or lack thereof) attained by a fixed capacity parallel model (i.e., one that divides up its available capacity among the operating channels). Interestingly, this rather low level of efficiency is also that predicted by standard serial processing when it terminates on the first item finished (e.g., Townsend & Wenger, 2004). Thus, if \( C_s(t) \) is inferior to the Grice bound, very limited capacity is implied but mild to moderate limited capacity can occur without violating that bound.

Another facet of \( C_s(t) \) and the Miller and Grice bounds is intriguing. First, a convention in the literature is to define the redundant signals effect (or redundancy gain) at time \( t \) as \( F_{AB}(t) - \text{MAX}[F_A(t), F_B(t)] \), for \( \text{MAX}[F_A(t), F_B(t)] \leq 1 \). That is, the difference, if any, between the observed RT cumulative frequency function when both targets (i.e., \( A \) as well as \( B \)) are presented, minus the largest of the two single-target cumulative frequency functions, is calculated. But, this is just the advantage or disadvantage of the double-target trials over the best of the single-target trials, the latter being the Grice bound. Hence, that bound is violated if and only if there is no advantage over attending to the better of the two targets. Note again that a system can be quite limited in capacity and yet still produce some redundancy gains if the Grice inequality is not violated. We next take up the case where, for \( N = 2 \), both items must be processed to always produce a correct response, the exhaustive or AND type of task.

The analogue to \( C_s(t) \) in this case of AND designs is called \( C_A(t) \). As briefly anticipated earlier, it would be inappropriate to try to use the same index for capacity both for AND and OR paradigms even though the standard parallel model is used as the measuring instrument, as it were, for both (and any) stopping rules. The reason is that OR processing times can occur faster than AND processing times for purely statistical reasons. The statistical pattern associated with a particular stopping rule must be taken into account. Yet, the logic and structure of \( C_A(t) \) again have as a consequence that it is greater, equal, or less than 1 when capacity is superior, equal to, or less than standard parallel processing respectively (Townsend & Wenger, 2004).

Colonius and Vorberg (1994) derived bounds for AND processing that parallel those for the OR situation. Both bounds assume context invariance just as do the Miller and Grice bounds. The upper level of performance is then \( P_{AB}(T_A \leq t \text{ AND } T_B \leq t) \leq \text{MIN}(F_A(t), F_B(t)) \). The lower bound on performance appears on the left of the following expression and the upper bound on the right-hand side. The middle term is the probability for the combined A and B presentations (along with instructions that both must be processed):

\[
F_A(t) + F_B(t) - 1 \leq P_{AB}(T_A \leq t \text{ AND } T_B \leq t) = P_{AB}(\text{MAX}(T_A, T_B \leq t) \leq \text{MIN}(F_A(t), F_B(t))
\]

Just as with the Miller inequality, as long as the data points are not outliers, then any violation of an upper or lower bound in the AND task may be interpreted as “highly super” and “very limited” capacity respectively, at those time points where \( C_A(t) \) is above or below the bounds. This usage will be continued in our expansion to performance measures, which take both accuracy and RT into account.

**Toward Assessment Measures for Accuracy and Response Times: Combined Accuracy and Response Times Measured by Standard Parallel Predictions**

The introduction of accuracy requires us to concentrate on the presentation of a pair of stimuli, say \( A_1 \) versus \( A_2 \). Because of our application to multiple target experiments below, we immediately designate one of the pair as the “target” and the other as the “nontarget,” although this is certainly not necessary in the general case. It will be assumed in the following that in addition to single- and double-target trials, trials are included in which no target is presented. However, the theoretical developments focus on predictions from the single- to double-target trials, just as in the case of RT-only presented above.

Specific process models make detailed assumptions about how a particular target comes to be processed correctly versus incorrectly. However, in every type of model, it is assumed that evidence or activation is accumulated in the correct channel versus an incorrect channel. Independent race models posit that separate channels accumulate supportive information for the correct versus incorrect response in a statistically independent fashion. Poisson race models are a case in point (Eidels et al., 2011; Smith & Van Zandt, 2000; Townsend & Ashby, 1983). On the other hand, random walk or diffusion processes are equivalent to channels with complete negative dependence such that any increment in one channel occurs if and only if an equal decrement occurs in the other (e.g., Link & Heath, 1975; Ratcliff, 1978). Naturally, intermediate dependencies are possible (e.g., Townsend & Wenger, 2004).

At this point, we begin to construct the mathematical notation required to truly understand the developments from here on, although their direct involvement arrives a bit later. Consider the A process for deciding whether a single A-type of target is present or not, and suppose for illustration that the target A is indeed present. We wish to have in hand a basic expression that captures the idea that the target affirmation process finishes (i.e., “yes, target A is
present”) before a negation process concludes (i.e., “no, target A is absent”).

The expression \( P(T_{AC} \text{ is less than } T_{AI} \text{ and also less than a fixed time value } t) = P(T_{AI} > T_{AC} \leq t) \) specifies the likelihood that the correct decision is made on A sometime before or equal to time = \( t \). This expression includes the necessary stipulation that the incorrect process loses the race with the correct process. Naturally, it is generally computed from the integral \( P(T_{AI} > T_{AC} \leq t) = \int P(T_{AC} = t') < T_{AI} \text{ at } t' \text{ from } 0 \text{ to } t' = t. \) All our constructions will follow this principle. It is desirable to be inclusive of all such models plus models that may lie in the future. Therefore we write the likelihood, say, that a target A is finished correctly and at exactly time \( t \) as \( P_{AI}(T_{AC} = t < T_{AI}) \). Note that this likelihood function could be based on a race between the correct and incorrect decision channels, which is a diffusion process or some other stochastic process.

The reader may observe that, as in the RT-only theory, the double-target trials will be the type on which we concentrate: The statistical functions that play the analogous role to \( C(t) \) will possess a form brought forth from the single-target data that is interpreted according to the parallel predictions and a separate observed part from the double-target trials. In exploring computational models, the latter will be based on various assumptions on the processing of the double-target trials. Because both targets are present, each main A and B process will be processing in a “correct” channel as well as an “incorrect-no” channel. We will simplify the expressions by simply referring to the correct versus incorrect subprocesses.

Now, consider the OR case outlined above on trials assuming that both targets are present, for instance, with visual stimuli such as a dot appearing above (A) and a dot appearing below (B) a central fixation. Suppose that the process terminates when either major target-present decision process, A (e.g., presence of dot above fixation) or B (presence of dot below fixation), reaches threshold. Incorporating incorrect responses and accuracy into the capacity equation requires extending the concept of the “race” (that exists between two channels) to the correct versus incorrect subprocesses within a channel. Of course, as indicated previously, the competition between correct and incorrect within a process, A for example, may or may not be a race in the usual sense. It will be immaterial in our developments what the architecture at this level is. It could, for instance, be serial, a diffusion process, a random walk, or a true race.

For illustration, we assume momentarily that a true race between correct (present; subscript 2) versus incorrect (absent; subscript 1) exists in A and B processes. Figure 1 provides a pictorial depiction of such a race in domains A and B for a hypothetical visual detection experiment.

Conversely, in an AND task observers are instructed to make a “yes” response if and only if, say, both dots are present in the display. In the nontarget as well as the single-target trials, they are instructed to make a “no” response. A “yes” response in this framework is supposed to occur when the correct process finishes before the incorrect process completes in Channel A and the correct process in Channel B finishes before the analogous incorrect process. Hence, a correct response is made when correct processing in Channel A terminates and incorrect processing in Channel A has not terminated by time \( t \), and when the same holds true in Channel B.

Before we introduce the quantitative measures, it is important to emphasize again that one aspect of our approach concerns the nonstatic nature of these measures, even within a single observer. That is, our use of the term and our corresponding measures are not intended to suggest that capacity denotes a unitary fixed construct of an observer. Consider the following as an example: Suppose an assembly line worker is charged with a specific task. Suppose that whenever this worker assembles parts quickly, he or she tends to make a larger than acceptable amount of errors. In such instances, capacity may be low and otherwise be indicative of “inefficiency.” However, when this worker slows down, he or she makes extremely few errors and works “efficiently.” Even within this one individual observer then, we can have evidence for high capacity for some points in time and low capacity for others. In the context of upper and lower bounds on efficiency, it is therefore possible for an observer to violate the lower bound for some time points and the upper bound for others. Although capacity has often been conceptualized as a unitary, fixed property of memory or system-level performance, the dynamic approach recasts it as an adaptive property. As cognitive resources, input strength, or interactions between processes change over time, changes may therefore be observed in the system’s capacity.

As soon as we broach the question of capacity in the context of both accuracy and speed, we are confronted by issues of connotations from the term “capacity.” High capacity is “good” and low capacity is “bad.” Probably most investigators would agree that speed and correctness together are “good” or somehow indicative of increased efficiency, just as \( d' \) is accorded that distinction in adaptations of signal detection theory to speed-accuracy issues (e.g., McElree & Dosher, 1989). Incorrectness, even with speed, is usually considered less attractive, as is slowness even with considerable accuracy. However, is it better to be fast if one is inaccurate and thereby “get it over with” or to be slow and thus not output errors at a high rate?
Although fast and incorrect responses should be considered inefficient in some sense, this question cannot be answered in a context-free manner.

Returning to the example of the assembly line worker, if we were to analyze incorrect responses, which are fast and plentiful for early response times, the system will exhibit evidence for high capacity. In this instance, high capacity for incorrect responses (i.e., more and faster incorrect responses than predicted) may be construed as a form of inefficiency. Conversely, capacity calculated on the correct response times may be low due to the damage to accuracy. Thus, although the different measures of capacity may appear to contradict each other in some instances (high capacity in one quadrant vs. low capacity in another), they are in actuality providing mutual information regarding the state of the system. The accuracy-adjusted capacity coefficient thus aims to provide a generalized taxonomy that can assess efficiency across both correct and incorrect trials and across faster versus slower response times.

Therefore, our theoretically neutral strategy is to simply define the statistical functions in the various cases. In fact, our statistics will be written so as to indicate the extent to which logical combinations such as “correct and slow” are more probable in the data than predicted by the standard parallel model. In certain instances, this neutral strategy will be stated in the form of the reciprocal of the accuracy values. These instances will be pointed out. These statistical measures can then be employed by the investigator to describe speed and RT under varying conditions, across individuals and so on, or to test all the specific predictions of contending process (e.g., computational) models.

First, we shall concentrate on being (a) incorrect and fast, (b) correct and fast, (c) incorrect and slow, and (d) correct and slow. When it makes sense, we will also discuss in qualitative terms what large versus small assessment coefficient values mean in terms of value or benefit for each of these quadrants. Because both incorrect and correct responses will be assessed, the system can essentially be in one of the four states delineated above. In that latter case we can assess (a) the probability that an incorrect response is made at or before time t, (b) the probability that a correct response is made at or before time t, (c) the probability that an incorrect response will be made but has not yet completed by time t, and (d) the probability that an correct response will be made but has not yet completed by time t.

Obviously, these cases are not all statistically independent. For example, the sum of a and b is the probability that some response, whether correct or incorrect, is made at or before time t. The sum of a and c yields the probability that an incorrect response is made, and so on. However, for purposes of completeness and symmetry and reader comprehension, we will offer the primary statistics for all four possibilities. On the basis of our fundamental strategy of constructing the capacity measuring tool on predictions from the standard parallel class of models, we find immediately that each component of the equation denotes a “defective” (i.e., does not integrate to 1) cumulative frequency function (technically, the cumulative distribution function) modulated by the accuracy level in that channel.

The next step will be to take the −log of both the observed and predicted likelihoods and then form a ratio analogous to $C_{\text{OR}}$ or $C_{\text{AND}}(t)$. The likelihoods are analogues of the $H(t)$ and $K(t)$ functions introduced by Townsend and Nozawa (1995) and Townsend and Wenger (2004), respectively. These components of the $C_{\text{OR}}(t)$ ($H(t)$) and $C_{\text{AND}}(t)$ ($K(t)$) functions consist of logarithmic transformed survivor and cumulative distribution functions (of RTs) and essentially denote the amount of “work completed” up until time t. In the case of approximately perfect accuracy, the log function turns the multiplication on the right side of the $C_{\text{AND}}(t)$ (and the correct and fast case for the AND stopping rule) into addition. This portion of the equation corresponds to the predictions of a parallel independent model without capacity limitations or facilitations.

As intimated earlier, there is an unavoidable degree of subjective judgment involved in deciding what goes in the numerator and what goes in the denominator in some cases. We will employ formulas when appropriate, which reduce to the original capacity C(t) functions as accuracy becomes very high. However, in certain cases, such as when we view the likelihood that a trial is inaccurate but fast, the defined ratio will be simply a convention and it is naturally up to the investigator to interpret the accompanying statistic.

Due to the potential model-specific or subjective interpretation of each ratio, we adapt the value-free notation of $A_{ij}(t)$ ($A$ for “assessment”), with i, j, being suitable subscripts to denote the specific comparison implied in the ratio. Thus, $A_{\text{CF}}(t)$ will signify the comparison of the (log of) observed joint “correct and fast” to the standard parallel prediction and similarly for the other combinations of accuracy and speed. A will be referred to verbally as the “assessment” statistic, function, ratio, or coefficient, equivalently.

There are two targets on each of the trials subject to the present analyses. We partition the response types into “correct” and “incorrect.” This is trivial when there are but two stimuli and responses for each target possibility. Another postulate is the fairly natural one that, as the instructions are to “respond if and only if either of the targets is present,” both channels must deliver an incorrect (i.e., nontarget) decision if the observer is to respond incorrectly in the OR case.

In order to keep the magnitude of the developments under control, we will focus on double-target trials and the comparisons with the standard parallel predictions forwarded from the single-target conditions where only one target is presented. This tactic allows us to employ a more elementary notation because we can simply specify correct versus incorrect channels, with the fixed interpretation that correct channels are attached to the target-present decisions and vice versa for the incorrect channels. This focus is in accord with virtually all studies of redundant target perception, but obviously, if the investigator’s purview is expanded to include, say, specific targets and nontargets, performance on no-targets-present trials, and so on, an even richer landscape for model exploration and testing will emerge. Experimental designs may even be augmented to include more than two targets in the display, although these cases may require the researcher to consider the effects that multiple targets have on working memory, as processing resources may change under greater memory load.

In the following section, we introduce the logic and provide derivations for the formulas for the correct and fast case for the OR and AND stopping rules. Derivations and more in-depth discussion for the incorrect and fast, incorrect and slow, and incorrect and slow cases appear in the supplemental materials. The vital capacity
formulas are shown in Table 1. Although we deemed it optimal overall to juxtapose the theoretical sections on OR and AND designs, some readers may wish to read, say, the theoretical OR section concomitantly with the exemplary OR and to do similarly in regard to the AND sections.

**OR Stopping Rule**

**OR-Correct and Fast**

The probability that the correct process in Channel A OR Channel B terminates at or before time \( t \), while the incorrect process is still going on, can be constructed by observing that this event can occur through the following sum of likelihoods: A being correct and fast while B is incorrect, plus B being correct and fast while A is incorrect, plus A being correct and fast while B is correct and slow, plus B being correct and fast while A is correct and slow, plus both being correct and fast. The general equation for standard parallel processing is given in Equation 1 at the bottom of the page. Equation 1 holds true under parallel independent model predictions.

Next, \( A^{OR}_{CF}(t) \) can be obtained by taking the \(-\log\), which provides a measure of work completed up until time \( t \). The logarithm does not neatly simplify the equation by splitting up multiplication into addition as it does in the case of the canonical capacity coefficient (see Townsend & Nozawa, 1995). The function when one is correct in Channel A OR Channel B is shown in Equation 2 at the bottom of the page. In Equation 2, \( A^{OR}_{CF}(t) \) increases as accuracy and/or speed increases relative to the standard parallel model. It is one case where the investigator may feel confident in associating the result \( A^{OR}_{CF}(t) > 1 \) with super capacity in the sense of qualitatively better than standard parallel model performance.

**AND Stopping Rule**

Recall that the AND stopping rule requires observers to respond “yes” if and only if the target is present in both channels. We shall once again introduce the case in which observers are correct yet fast.

**AND-Correct and Fast**

A correct response is achieved in an AND task if and only if one correctly recognizes the target in Channel A and the target in Channel B. The equation below states that the probability of making a correct response in the redundant target condition (while being fast) by time \( t \) equals the probability that the target was correctly identified by time \( t \) in Channel A (and the incorrect response has not reached threshold) and that the target was correctly identified by time \( t \) in Channel B. The probability of being correct in Channel A AND Channel B while the incorrect processes are still accruing information is given by Equation 3:

\[
\int_0^t P_{AB}(T_{ABC} = t' < T_{AB})dt' = \int_0^t P_A(T_{AC} = t' < T_A)dt' \cdot \int_0^t P_B(T_{BC} = t' < T_B)dt' \tag{3}
\]

The assessment coefficient \( A^{AND}_{CF}(t) \), for the AND stopping rule when the observer is both correct and fast, is provided below:

\[
A^{AND}_{CF}(t) = \frac{\log \left[ \int_0^t P_A(T_{AC} = t' < T_A)dt' \right] + \log \left[ \int_0^t P_B(T_{BC} = t' < T_B)dt' \right]}{\log \left[ \int_0^t P_{AB}(T_{ABC} = t' < T_{AB})dt' \right]}
\]

In this case, as accuracy becomes very high, \( A^{AND}_{CF}(t) \) approaches \( C_{AND}(t) \) in the RT-only theory (e.g., Townsend & Wenger, 2004) and thus seems legitimately associated with the label super capacity or superior redundant target performance.

**Performance Bounds for Workload Assessment Functions**

As discussed previously, the traditional upper and lower bounds essentially place limits on the performance of certain classes of parallel models. The typical setup is to allow positive or negative channel dependence as long as the marginal distributions are invariant. That is, the marginal processing time distribution when
### Table 1
Assessment Coefficient Functions for Quadrants I Through IV for the OR Experimental Design and the AND Experimental Design

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>OR Experimental Design</th>
<th>AND Experimental Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>[\mathcal{A}<em>1(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
<td>[\mathcal{A}<em>1(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
</tr>
<tr>
<td>II</td>
<td>[\mathcal{A}<em>2(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
<td>[\mathcal{A}<em>2(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
</tr>
<tr>
<td>III</td>
<td>[\mathcal{A}<em>3(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
<td>[\mathcal{A}<em>3(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
</tr>
<tr>
<td>IV</td>
<td>[\mathcal{A}<em>4(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
<td>[\mathcal{A}<em>4(\tau) = \frac{\log \left[ \int_0^\tau T</em>{A_1} = \tau &lt; T_{A_2} \right] + \log \left[ \int_0^\tau T_{B_1} = \tau &lt; T_{B_2} \right]}{\log \left[ \int_0^\tau T_{A_2} = \tau &lt; T_{B_1} \right]}]</td>
</tr>
</tbody>
</table>

**Note.** This table shows each of the assessment coefficient functions for Quadrants I through IV for (A), the OR experimental design, and (B), the AND experimental design. The interpretation of the assessment value changes depending on the quadrant the researcher wishes to analyze. For instance, an assessment value > 1 in the correct and fast condition indicates efficient processing in the redundant target condition, whereas a value of > 1 in the incorrect and fast condition indicates inefficient processing due to the presence of more/faster errors.
running along with others is unchanged from when it operates alone. The reader will notice that this amounts to—in fact is equivalent to—an assumption of unlimited capacity (Colonius & Townsend, 1997; Colonius & Vorberg, 1994; Miller, 1978; Townsend & Nozawa, 1995).

Violations of upper or lower bounds suggest one of the following (and this list is not intended to be exhaustive):

1. The assumption of cross-channel independence has been violated in such a way that the marginal probabilities were affected (e.g., Eidels et al., 2011; Townsend & Wenger, 2004).

2. Architecture other than parallel is in action.

3. The system’s reservoir or supply of capacity is depleted or overfilled when the number of acting channels changes.

4. The system allocates its resources in such a way as to produce disparities in the data from standard parallel processing.

It appears that the situation may be more complex than those when only RT is considered, and even there the bounds can sometimes be made more precise (see, e.g., Colonius & Vorberg, 1994). Therefore, we can make no guarantees that the bounds constructed here will necessarily be optimal for all purposes, but they will offer some indication of what may be deemed extreme deviations from ordinary parallel processing.

The bounds for the exemplary correct and fast cases (for the OR and AND rules) will now be provided. The other bounds can be readily derived using similar logic. Derivations can be found in the supplemental materials.

OR: Correct and Fast

The equation for the upper bound when the observer is correct and fast in an OR design is

$$A_{CF}^{OR}(t) \leq \log \left[ \int_0^t P_A(T_{AC} = t' < T_A)dt' \right] + \log \left[ \int_0^t P_B(T_{BC} = t' < T_B)dt' \right]$$

$$= \log \left[ \int_0^t P_A(T_{AC} = t' < T_A)dt' \cdot P_B(T_{BC} = t' < T_B) \right]$$

Next, the lower bound can be written as

$$A_{CF}^{OR}(t) \geq \log \left[ \int_0^t P_A(T_{AC} = t' < T_A)dt' \right] + \log \left[ \int_0^t P_B(T_{BC} = t' < T_B)dt' \right]$$

The upper and lower bounds can likewise be obtained for the corresponding AND cases by using logic similar to the derivations shown above. The upper and lower bounds for the correct and fast quadrant will now be shown for the AND design.

AND: Correct and Fast

For the cases where the participant is both correct and fast in an AND experiment, the upper and lower bounds for a parallel independent model of unlimited capacity are given by the two equations at the bottom of this page. Just as in the case of RT-alone data, the bounds can aid in evaluations of how limited or how super performance is in comparison to standard parallel processing.

Decomposing Accuracy and Response Time

Assessment Functions

The assessment functions capture the overall performance, combining accuracy and RT, as a function of workload, relative to standard parallel predictions. These assessments play a vital role in testing predictions of various types of processing models. In addition, further information can be garnered by decomposing the separate accuracy plus conditional RT contributions to the joint accuracy–RT statistical functions. Considering as a prime example from the AND decision statistics, $P(\text{Correct and } T \leq t) = \int_0^t P_A(T_{AC} = t' < T_A)dt'$, we can rewrite this expression as

Upper bound:

$$\log \left[ \int_0^t P_A(T_{AC} = t' < T_A)dt' \right] + \log \left[ \int_0^t P_B(T_{BC} = t' < T_B)dt' \right]$$

$$= \log \left[ \int_0^t P_A(T_{AC} = t' < T_A)dt' \cdot P_B(T_{BC} = t' < T_B) \right]$$

Lower bound:

$$\log \left[ \min \left\{ \int_0^t P_A(T_A = t)dt' \cdot P_B(I) \right\} \right] + \log \left[ \int_0^t P_B(T_B = t')dt' \cdot P_A(I) \right]$$

$$= A_{CF}^{AND}(t)$$
\[ \int_{0}^{1} P_{ab}(T_{ABC} = t' < T_{ab}) dt' = P(\text{Correct}) \cdot P(T = t|\text{Correct}). \]

This simple transformation allows our original assessment function to be reexpressed in the following way,

\[
\log[\frac{\int_{0}^{1} P_{a}(T \leq t|\text{Cor.}) \cdot P_{a}(\text{Cor.}) dt + \int_{0}^{1} P_{a}(T = t|\text{Cor.}) \cdot P_{a}(\text{Cor.}) dt}{\int_{0}^{1} P_{a}(T \leq t|\text{Cor.}) dt}],
\]

where the distinct roles played by accuracy and conditioned response time statistic are revealed. This thinking permits a decomposition into analogous assessment ratios based only on accuracy or conditioned response time statistics, respectively. Then, for instance, the observed \( P(\text{Correct}) \) can be compared with the standard parallel prediction:

\[
[\log \int_{0}^{1} P_{a}(T_{AC} = t' < T_{ab}) dt'] + \log \log \int_{0}^{1} P_{a}(T_{BC} = t' < T_{ab}) dt'.
\]

Alternatively, a ratio-assessment function analogous to our original one but based only on the conditional statistics can be written as follows:

\[
[\log \int_{0}^{1} P_{a}(T_{AC} = t' < T_{ab}) dt'] - \log \log \int_{0}^{1} P_{a}(T_{BC} = t' < T_{ab}) dt'.
\]

In fact, it can be seen that this statistic is identical to the original C(t) function for the AND stopping rule when accuracy approaches 100% (Townsend & Wenger, 2004)! The subscripts A and B of course denote processing rate/accuracy in that particular channel.

There are many purposes that can be fulfilled in such decompositions that go beyond our scope. However, these statistics could be very useful in analyses of speed-accuracy trade-offs. Consider again the AND case and assume that processing is governed by independent counters; one for the correct decision and one for the incorrect decision in each channel, as in Townsend and Ashby (1983, Chapter 9). Next, assume that when the double-target condition occurs, the processing speeds are (or would be) unaffected in the two channels. Still, probability correct would itself suffer because it is the product of the two single-target probabilities correct. Hence, suppose that the observer adjusts her or his decisional criteria so that accuracy remains constant at approximately the average of the two single-target accuracies at the expense of faster RTs (the classical speed-accuracy trade-off). This outcome can be immediately probed by the present decomposition where the new probability correct will not be equal to the standard parallel prediction but rather to the average of the two single-target accuracies.

Similarly, suppose one analyzes the contribution of RTs separately from accuracy. This strategy calls for conditioning on correct responses and assessing the speed of response times in the double-target condition relative to parallel race model predictions. For the correct and fast AND case, this essentially involves utilizing the AND capacity equation below originally proposed by Townsend and Wenger (2004). Because we are assuming that the observer adjusted the decisional criteria in such a way to maintain accuracy levels at the expense of faster RTs, we should expect the RT-only formula to yield the following,

\[
\log \frac{\int_{0}^{1} P_{a}(T \leq t|\text{Cor.}) dt + \int_{0}^{1} P_{a}(T = t|\text{Cor.}) dt}{\int_{0}^{1} P_{a}(T \leq t|\text{Cor.}) dt} < 1,
\]

which indicates slower responses.

The other cases (e.g., incorrect and slow) can be similarly checked for their qualitative and quantitative predictions. Before moving on, we simply note that the above instance of decomposition-inference is only one of many that can be contemplated. For instance, it is possible that the flow of resources to the separate channels from central reservoirs could be changed in going from \( N = 1 \) to \( N = 2 \), due to divided attention. This by itself would cause alterations in the A(t) functions.

Finally, Townsend and Wenger (2004) argued that RT-only capacity measures are robust up to an error rate of around 30% (and maybe larger). The conditional RT-only measure may be calculated using the RTs from correct (or incorrect responses) regardless of the number of responses available (in certain cases where there are few responses, the measure may not be entirely robust). Thus, the conditional RT-only measures tend to be robust in the sense that they appear to be relatively unaffected by modest differences in error rate; consequently, they can be calculated in cases when error rate is very low and also when it is high. These issues will come to light in the section where we discuss data obtained from a visual detection study employing both OR and AND stopping rules.

We now move to illustrate our new assessment tools with an illustrative experiment in visual detection. We will find that the bounds and decomposition transformation can aid in deciphering experimental results.

### An Illustrative Study

The A(t) assessors can provide a rich quilt of analyses. These can be used for a number of interwoven purposes. Here we focus on relatively broad and illustrative questions that might be of interest in theoretical or applied settings. However, we envision future implementations to more specific computational and analytic models. This outlook is discussed further in the General Discussion.

### Suggested Major Applications

1. Provide a global overview of how a change in workload affects speed and accuracy relative to ordinary parallel processing or other types of architecture, stopping rules, or channel or stage interactions.

2. Falsify or provide affirmation to broad hypotheses and predictions from large classes of models.

3. Test specific, parameterized models against the A(t) data patterns.
4. Employ not only the double-target data and the $A(t)$ functions but also the single-target and distractor (or blank) data explicitly rather than only through their utility via the $A(t)$ statistics.

Because this section is intended for illustration and not for tests of specific hypotheses, models or theories, only a limited exploration is set out. We will assay the following issues in our exemplary experiment. These would primarily fit under applications (1) and to some extent (2) just above.

**Initial and Generic Questions**

1’. Does performance overall meet the level of efficiency required by standard parallel processing with an appropriate decisional stopping rule?

2’. Following up on (1’), do we see accuracy and/or RT diverge with regard to the parallel standard at each time point? That is, if we observe deviations from race model predictions at one or more points, were they primarily due to changes in accuracy, RTs only, or a combination of RT and accuracy?

3’. Are there relatively obvious signs of speed–accuracy trade-offs (Link & Heath, 1975; Pachella, 1974; Thurstone, 1927; Vickers, 1979)? For example, when workload is incremented, does accuracy meet or exceed standard parallel expectations but with a slowing down incommensurate with the latter? Such behavior could be caused by changes in decisional criteria.

4’. Are there interesting differences between the joint $A(t)$ functions and the RT-only $A_c(t)$ functions conditioned on correct (or incorrect) response? Do the latter differ for correct versus error data?

Admittedly, these latter aims might be only the initial stages of more powerful investigations that are geared toward the broader scope and great depth of fundamental applications listed above. With regard to 3’, “Are there relatively obvious signs of speed-accuracy trade-offs?” an ancillary but intriguing question arises. Is it possible to witness potential speed–accuracy trade-offs even when accuracy, in the double-target condition, is very high? Many investigators have called inference constructed only on RT, without consideration of accuracy, into doubt.

We will see that the tentative answer is “yes, it is possible at least in some circumstances, to diagnose speed–accuracy trade-off in high-accuracy data.” Now, higher statistical power may obviously be called for, especially when investigators wish to test hypotheses at the individual level. Still, our results are provocative.

The elucidatory experimental data were obtained from a participant in a visual detection task involving the detection of one to two dots on a computer screen. Data were gathered in both an OR and an AND condition. The experiment reported below was designed to be analogous to the basic visual detection design reported by Townsend and Nozawa (1995) and more recently by Townsend and Eidel (2011). We refer the interested reader to these papers for an in-depth description of the experimental methodology, although a summary is provided below.

**Method**

**Design and stimuli.** This visual detection experiment involved the presentation of a dimly lit dot (subtending to 1° of a visual angle) above, below, or both above and below a central fixation point on the blank, black screen on a computer monitor. One fourth of trials consisted of (a) one dot presented above (top-only), (b) one dot presented below (bottom-only), (c) one dot presented above and below the fixation (redundant target), and (d) target absent trials. Hence, all trial types occurred with equal probability (see Mordkoff and Yantis, 1991, for a discussion of contingencies). In the redundant target trials, two dots were displayed on a vertical line and were spaced equally above and below a central fixation point at an elevation of ±1°. The stimuli were mixed within blocks. The entire visual target detection task (both OR and AND conditions) required four 1-hr experimental sessions to complete, over 4 consecutive days. The task was administered in a dark room after a period of 10 min of dark adaptation.

At the onset of each trial, a fixation point (subtending to .05° of visual angle/viewing distance of 50 cm; luminance of .067 cd/m²) was presented on the center of the screen for 500 ms, followed by a blank, black screen for 500 ms before the stimulus appeared. The stimulus appeared on the screen for 100 ms or until a response was given and was subsequently replaced by a blank screen. The participant was instructed to respond as quickly and as accurately as possible. Response sampling was initiated at the onset of the stimulus display and continued for 4,000 ms. The intertrial interval was set to 1,000 ms. In the OR portion of the task, participants were required to make a “yes” response with their dominant hand on a mouse pad if a dot appeared above, below, or both above and below a central fixation point and a “no” response otherwise. In the AND portion of the task, participants were required to make a “yes” response if and only if a dot appeared above and below the central fixation point and a “no” response otherwise.

Each experimental session began with a practice block consisting of 100 trials, followed by five experimental blocks of 160 trials each, with a 2-min break between blocks. The order of trials was randomized within a block. The total number of experimental trials, including target absent, was 3,200 (1,600 OR and 1,600 AND trials).

**Example Results and Discussion**

**OR design.** The proportion correct responses for this participant in the OR portion of the task were .97 (redundant target), .69 (top-only), and .92 (bottom-only). The predicted redundant target error rate, given the accuracy level in the single-target conditions, equals .69 + .92 − .69 × .92 = .975, which happens to be very close to the observed accuracy rate of .97. The false alarm rate in the target absent condition was slightly below 3%.

The assessment results for the OR task are shown in Figure 2. RTs were binned, and empirical cumulative distribution and survivor functions were computed across both correct and incorrect responses (see the supplemental materials). Results for the accuracy-adjusted assessment coefficient are shown for each of the four quadrants. Additionally, the RT model predictions obtained from conditioning on correct (and incorrect) responses are shown across quadrants. The conditional $A_c(t)$ functions are similar to the decomposition functions (see Decomposing Accuracy and Response Time Assessment Functions),

1°.
except they contain only the conditional RT data and predictions (as in the RT-only capacity coefficient).

The standard parallel predictions (in this case, the prototypical race type of model) for accuracy are shown separately in the lower right quadrant of the figure. The separate RT-only and accuracy data allow one to “decompose” speed and accuracy and observe their respective contribution to the data. Decomposing RTs and accuracy in this manner thus provides the reader with immediate information regarding whether any observed deviations from standard parallel race model predictions result from changes in speed, accuracy, or both.

The upper and lower bounds for the OR condition (and AND condition in the following figure) are displayed in each quadrant for comparison purposes. Violations of these bounds would suggest that the upper or lower limit of parallel independent race model predictions has been exceeded, indicating either parallel interactive or coactive (e.g., Miller, 1982) processing.

**Correct.** First observe that probability correct was virtually equal to the standard parallel prediction. Nonetheless, the A(t) results for the correct and fast responses in Figure 2 indicate that the combination of speed and accuracy fell substantially below what is predicted by that yardstick model, as A(t) < 1. Still, the lower bound was not violated for any point in time.

Importantly, we can observe through A_c(t) and the fact that the observed and predicted accuracies were almost identical that the correct times were quite sluggish compared with parallel predictions. The correct and slow analyses in the bottom right panel are consonant with those for correct and fast. The slowness suggested in the correct and fast case is even heightened in the correct and slow case through A_c(t) as well as A(t), where both are seen to violate the upper bound, meaning that slowness exceeds that expected on the basis of the measuring instrument.

**Incorrect.** Quadrants containing responses that are incorrect due to misses can offer additional insights into the processing dynamics of the system. Although the absolute number of error trials was small, it turned out that t tests confirmed a significant difference (at the .05 level) between the standard parallel predictions and the data. Hence, though we should take the following probes of the error data with a fairly massive grain of salt, they may bear consideration.

When the observer was responding “incorrectly and fast,” A(t) was mildly greater than 1 but lay far below the upper bound for all time points. Similarly, the incorrect and slow magnitude was severely less than 1. Together, these two quadrants suggest that misses were made in a hurried fashion, particularly in comparison to the hit RTs.

Amalgamating the accuracy data in the OR design and neglecting RTs could provide a misleading picture because accuracy (alone) is pegged at the level predicted by a standard parallel system, that is, a set of parallel channels operating independently and with unlimited capacity. Such a conclusion might be made, falsely, in the absence of the integrated RT tests. The above results are intriguing. We can provide a tentative qualitative “model” that appears to capture the major characteristics of the OR data as follows:

The fact that the overall accuracy was equal to the standard parallel predictions in conjunction with the slower correct RTs and faster misses is provocative. The slower correct responses could be caused by a raised target-decision criterion. However, this could only happen in conjunction with the observed hit accuracy if sensory signal–noise ratio is actually better than standard parallel processing (i.e., somewhat super capacity). The other mechanism that could assist in maintaining a strong hit profile is if the nontarget criterion were also stiffened. However, in contrast to that possibility, misses were relatively fast.

In fact, qualitatively the combination of improved signal–noise ratio along with the slower responses would presumably leave the miss times about the same, but they too were more speedy than expected. Hence, it appears likely that the criterion for nontargets (i.e., blank stimuli) was moderately lowered. Along with the modestly speeded hits, this simply again indicates that sensory integrity was likely better than standard parallel.

Overall, then, these preliminary results suggest super capacity processing that is reachable through coactivation (e.g., Colonius & Townsend, 1997; Schwarz, 1994) or nondirective, facilitatory parallel channels (Eidels et al., 2011). In addition, there appears to have been a difference in the criteria settings in the double- versus single-target trials. If confirmed, such a finding suggests that an observer might be able to control decisional criteria based on early sensory information within a trial. Evidence for such facility has surfaced before (e.g., Townsend, Hu, & Kadlec, 1988). It is important to note that the overall picture here could not have been acquired through accuracy or response times alone.

**AND design.** A dramatic divergence from the OR results is seen in workload-based performance alterations in the AND task. Whereas accuracy taken in the context of RTs in the OR double-target condition was improved (over single-target performance) to an extent better than that predicted by standard parallel processing, in the AND task, even the proportion of correct responses was harshly degraded: .97 (top-only), .98 (bottom-only), .58 (double target). Thus, the obtained double-target accuracy, .58, was strikingly reduced from that expected from the standard parallel model, which was .97 × .98 = .95.

The decomposed accuracy and RT results along with the global assessment coefficient for the AND task are displayed in Figure 3. Once again, the decomposed conditional RT (i.e., A_c(t)) and accuracy results assist one in determining whether deviations from standard parallel predictions in the A(t) formula result from changes in conditional RT, accuracy, or both.

![Figure 2](image_url). Figure showing the assessment statistics for the OR data. For comparison purposes, the conditional capacity measure in which reaction times (RTs) are conditioned on accuracy (or inaccuracy), labeled A_c(t), is also shown. L. = lower; U. = upper; Pred. = predicted; Obs. = observed.
sensory capacity was also present. Due to the magnitude of the decrement in hits, that degraded expected by the standard parallel model. However, we suspect that, partly responsible for the deterioration in accuracy from that speeded-up performance. It appears that the nontarget criteria were when compared with the single-target trials resulting in decisional criteria were relaxed both for targets and for nontargets when the participant was incorrect, A(t) was consistently greater than 1 whether fast or slow. Espying the incorrect and fast versus incorrect and slow expressed in terms of the conditional A_c(t) data, we see that the "miss" redundant target RTs were substantially faster than standard parallel model predictions. In fact, rather ubiquitous upper bound violations were observed in Figure 3 in the case of the fast "miss" redundant target RTs were substantially faster than standard parallel model predictions. In this case, the A(t) function makes a contribution by helping resolve this apparent speed–accuracy result because it makes use of combined accuracy and RT data. The qualitative bird's-eye view of the AND data suggests that decisional criteria were relaxed both for targets and for nontargets when compared with the single-target trials resulting in speeded-up performance. It appears that the nontarget criteria were lowered more than the target criteria. The latter finding could be partly responsible for the deterioration in accuracy from that expected by the standard parallel model. However, we suspect that, due to the magnitude of the decrement in hits, that degraded sensory capacity was also present. Intriguingly and qualitatively, the A(t) analyses again suggest that observers may be capable of dynamically altering their decisional criteria depending on whether a single- or double-target stimulus is presented. Moreover, it certainly appears that the target rate may have suffered relative to the nontarget rate (a form of degradation akin to that of signal-to-noise ratio) when two targets, rather than a single target, were presented. This result dramatically contrasts with the OR analyses.

Illustrative Answers to the Initial and Generic Questions

1’. Does performance overall fall below, meet, or exceed the level of efficiency required by standard parallel processing with an appropriate decisional stopping rule at each time point? Provisional answer: No, in neither the OR nor the AND design does this individual meet standard parallel expectations. In both paradigms, the RTs, the accuracy, or both are at odds with the parallel predictions.

2’. Digging deeper into Question 1’, do we see accuracy and/or RT diverge with regard to the parallel standard? That is, if we observe deviations from race model predictions at a certain point, were they primarily due to changes in accuracy, RTs only, or a combination of RT and accuracy? Provisional answer: The patterns in each of OR and AND, separately, are consistent within each paradigm, yet they are non-trivial and differ rather dramatically across these two response rules. In particular, accuracy per se was equal to standard parallel predictions in the OR design but was much degraded in the AND design.

3’. Are there relatively obvious signs of speed–accuracy trade-offs? For example, when the workload is incremented, does accuracy meet or exceed standard parallel expectations but with a slowing down incommensurate with the latter? Such behavior could be caused by changes in decisional criteria. One must tread carefully here. Within a race class of models, raising both criteria will be expected to increase accuracy. However, simply raising the criterion for targets and leaving alone that for nontargets, or even lowering it, will tend to lower hits and increase misses.

Provisional answer: In the OR design there was no evidence of speed–accuracy trade-off of the traditional variety for the reasons just mentioned. In the AND design, it appears that both target and nontarget criteria may have been relaxed, which could have somewhat damaged accuracy—this could be ordinary speed–accuracy trade-off but in the wrong direction. Nonetheless, accuracy fell so far that limitations in processing capacity (e.g., perceptual signal–noise ratio type of effects) are suspect.

4’. Are there interesting differences between the joint A(t) functions versus the RT-only A_c(t) functions conditioned on correct (or incorrect) response? Do the latter differ for correct versus error data?

From one point of view, it would be more theoretically satisfactory if target, as well as nontarget, criteria did not vary depending on the stimuli presented and this assumption is taken as a postulate in many model tests and comparisons. However, there is evidence, albeit from accuracy alone, that decisional criteria can change depending on the complexity of the particular stimulus presented, even within the same task and block of trials (Townsend, Hu, & Evans, 1984; Townsend, Hu, & Kadlec, 1988).
Provisional answer: Absolutely; in all cases, contrasting and informative divergences were discovered. The $A(t)$ functions offered a quick read of overall performance relative to parallel processing, but the decomposition via $A_\alpha(t)$ and probability correct yielded illuminating diagnoses of accuracy versus RTs.

As a final point to these illustrative analyses, we remark on the rather vast difference between the OR versus AND results. The decision rule clearly invoked a dramatic effect on the efficiency of processing that appears to involve both decisional and sensory components. We can summarize the findings as follows:

1. In the OR design, accuracy was maintained at the rather high level expected from the standard parallel model, but the slow correct responses and fast incorrect responses suggest that the target criterion was raised and the nontarget criterion was lowered. These results suggest that the underlying sensory observations were even stronger than standard parallel predictions and that accuracy was brought down a bit due to the criteria alterations in moving from the single-target to the double-target condition.

2. In the AND design, accuracy fell precipitously. However, when RTs were conditioned on either correct or incorrect and fast, they were more frequent than standard parallel predictions whereas slow RTs were infrequent.

We frankly did not expect the severe degradation in accuracy found in the AND data relative to the OR data. Indeed, with more complex stimuli such as letter string versus words and scrambled features versus coherent faces, we have often discovered evidence for configurally caused super capacity in the AND as compared with OR designs (e.g., Wenger & Townsend, 2006). It should be highly interesting to learn if an individual’s system characteristics commonly differ so greatly simply with a change of required decision rule. Of course, the present and further conjectures concerning the information processing by our observer will likely be substantially aided by inclusion of the target-absent data, which had to be ignored in these initial explorations.

Finally, we cannot declare with any certainty presently whether any extant models will be able to accommodate the above data, either qualitatively or quantitatively. Standard parallel processing was certainly falsified in both experimental conditions. Sensory processing appears to have been superior in the OR design and quite poor in the AND condition. With regard to the exploratory analyses so far implemented, it appears that race-type or diffusion-type models should be able to handle the qualitative aspects at least, but that remains to be demonstrated.

General Discussion

A Brief Consideration of Other Approaches to Capacity Assessment

The notion of human performance efficiency under varying workload is ubiquitous in the cognitive sciences, and as one would expect, diverse proposals have emerged over the years to quantify this important concept. The present study cannot offer a general review of the large literature pertaining to performance and efficiency, especially since it spans many quite distinct disciplines and topic areas. However, within realms close to the present investigators’ domain, it is possible to locate models and their analyses within which workload variations are a central part of experimental design and where the design and tests have been specifically constructed to test what we classify as unlimited capacity parallel models. We discuss a few of the most germane of these below.

Of course, in contrast, there exist many models in which workload variations are present, empirically or theoretically (i.e., within a discussion of model predictions in general), but the purposes diverge primarily from that just mentioned. A few examples arbitrarily selected from a substantial literature are as follows.

Bundesen and colleagues (e.g., Bundesen & Habekost, 2008) have successfully employed their mathematical theory of visual attention (TVA) with a broad spectrum of behavioral, neuroscientific, and pathology data. The basics of the model are parallelism, independence of channels, and, in the majority of workload manipulations, limited capacity. In addition, the model is able to represent interstimulus resemblances through structures drawn from Luce’s similarity choice model (Luce, 1963; Townsend & Landon, 1982).

Ratcliff (1978) provided general tests of his limited-capacity parallel Wiener diffusion model in a short-term memory study (see also Smith & Van Zandt, 2000). Treisman and colleagues (e.g., Treisman & Gormican, 1988) and Wolfe and colleagues (e.g., Wolfe, 1994, 2007) have developed and tested an extensive set of models, some of which possess unlimited capacity parallel properties, in addition to their sophisticated serial models. Palmer, Horowitz, Torralba, and Wolfe (2011) assessed the ability of a distinct set of parameterized distributions to accommodate search data as a function of set size.

Our present approach, focusing on tests and potential classification of performance relative to the class of unlimited capacity models, is somewhat closer to several other lines of attack. In fact, Sternberg’s 1966 study basically assessed all standard serial models against the class of standard parallel models as we have defined them here.3 Thornton and Gilden (2007) employed simulations involving diffusion processes of what appeared to be either standard serial or parallel models and applied them to a variety of visual search tasks. A number of visual stimuli allowed parallel processing, but some required serial allocations of resources.

Wenger and Townsend (2006) provided simulations of general classes of parallel versus serial models in an extensive study of words, faces, and random arrangements of letters or facial features.4 Parallel models always won handily. However, capacity was limited in some circumstances but was found to be super in others. For instance, stimulus aggregates of features or letters that had to be processed exhaustively (i.e., an AND requirement) and were made up of a coherent gestalt tended to produce super capacity parallel processing.

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3 The standard serial class of models is defined by the assumptions that (a) processing proceeds sequentially, with each item or subprocess completing before the next begins; (b) the duration of each subprocess is defined by the same probability distribution on time; (c) the duration random variables on each subprocess are stochastically independent of one another (see Townsend & Ashby, 1983, for further details).

4 The models were all based on stochastic dynamic linear systems, and the only assumption concerning serial processing was that it was sequential without overlapping durations. Parallel processing assumed only simultaneous processing, but channel interactions were sometimes allowed to occur. Wiener diffusion models form a very special case of this class of models.
McElree and Dosher (1989) combined accuracy and RT in the context of speed-accuracy trade-off functions plus an ordinary search design. A parallel model made up of Ratcliff (1978) variety diffusion processes provided good fits and explanations. Busey and Palmer (2008) estimated accuracy thresholds in a study of the independence of identification versus localization information. They concluded that there were unlimited capacity and independence of the two types of perception and certainly evidence against a dominance of one type over the other.

A hefty literature that has only rarely intersected with the memory or visual search literatures per se follows the line opened by Miller (1978, 1982) and Grice and colleagues (e.g., Grice, Canham, & Boroughs, 1984). There the emphasis has always been on testing the various RT inequalities, which we view as the boundaries of limited or super capacity (e.g., Townsend & Eide., 2011). We have repeatedly referred to the main concepts and certain offshoots from that literature in the earlier text, so we refrain from dwelling on those concepts here. However, the key theme of the present theory stems from the fundamental idea of implementing very general stochastic assessments in order to test simple predictions that are made, or not, by entire classes of models. The upshot is the production of statistical measurements that test, for example, the standard parallel model class, but that also provide a detailed pattern of data to assess any candidate model. We have so far not located any parameter and distribution-free methodologies with which the present approach would directly compete.

Summary and Conclusion

Until fairly recently, when the concept of processing capacity was developed into a systematic framework and applied to factorial experiments, measures and tests that were simultaneously nonparametric and dynamic have been scarce (but see Townsend & Ashby, 1978). The RT-based workload capacity metric that we began developing in the late 1980s exploited the design principles of generalized parallel independent processing as a benchmark for redundancy gain (Neufeld et al., 2007; Townsend & Nozawa, 1988, 1995, 1997; Townsend & Wenger, 2004; Wenger & Townsend, 2000, 2001).

Our goal in the present study was to take the core of the RT-capacity approach and extend it in such a way as to merge accuracy with RT for a more complete system of measurement of performance across changes in workload. This was accomplished in a parameter-free manner. The accuracy-adjusted measure is intended to provide a significant step toward closing the gap between the domains of response times and accuracy. Researchers can now assess the relative contribution of redundant target information when the observer is incorrect and fast, correct and fast, incorrect and slow, and correct and slow. Our results apply to any situation where the workload changes and is not, of course, restricted to loads of one versus two items.

In fact, any logical stopping rule (e.g., searching for a single target in a display of $n - 1$ distractors or an exclusive OR paradigm) can be subjected to our analyses. For instance, visual search paradigms in which RT alone has been the focus (e.g., Atkinson, Holmgren, & Juola, 1969; Townsend & Roos, 1973; Treisman & Gormican, 1988; Wolfe, 2007) or, less frequently, accuracy (e.g., Dosher, Han, & Lu, 2004; Palmer et al., 2000) can now be invested with conjoined RT and accuracy assessments and concurrent model tests.

Of course, experimental results that are at odds with the parallel predictions might, in general, be traceable to a number of potential sources, including alterations in resources or allocations, various types of failures of stochastic independence, nonparallel architectures such as serial, or even the use by a participant of inappropriate stopping rules. Some of these might be disconfirmed or tentatively affirmed by patterns in the A(t) data. We again take note of the fact that we have had to neglect the “no-target-at-all” trials in both the OR and the AND paradigms. It is evident that consideration of statistics from the no-target-at-all trials, not to mention more detailed characteristics from single-target trials, is expected to deepen the penetration of our qualitative battery of analyses.

A major feature of A(t) analogous to that of C(t) is that of being dynamic. That is, they depict workload efficiency as a function of time. Although alterations of capacity across conditions are reasonably common, this type of varying observable resource dynamic is not. For instance, even though the Miller (1978) and Grice et al. (Grice, Canham, & Boroughs, 1984; Grice, Canham, & Gwyne, 1984) bounds are founded on stochastic RT distributions, the interpretations are traditionally expressed as categorical. That is, the data obey the bounds, suggesting “ordinary” parallel processing or not, in which case coactivation is typically inferred.

We propose that the temporal details of the A(t) functions, as well as their constituents (e.g., probability of being correct and faster than some time, t), may demonstrate their value in uncovering underlying cognitive machinery. As one of several possibilities, observe that contemporary parameterized stochastic models often assume that one or more of their parameters (e.g., rate of processing) are randomly selected on a trial-to-trial basis. It may be that the dynamic portrait found in the A(t) indices could be affected, and possibly predicted, by that “mixing” operation. And, it also could be that studying time-series statistics across trials in an experiment might shed further light on such mechanisms. An important limitation of our nonparametric A(t) functions is that the individual quantities (e.g., the correct and fast A(t)) cannot immediately discriminate effects of changes due to decision-criterial influences versus those due to true energy or signal-to-noisylke influences. Here is where specific parameterized models can contribute.

For example, Eidels, Donkin, Brown, and Heathcote (2010) extended the linear ballistic accumulator (LBA) model to the double factorial, redundant-targets paradigm (Townsend & Nozawa, 1995). They then provided an in-depth comparison of the C(t) workload capacity functions with the LBA parameters. A key finding was that the LBA rate of processing, but not the decision criteria, was highly correlated with C(t). In addition, like other parameterized models, the LBA takes account of the base time, which is presently unreachable by the A(t) indices. We do envision that, as in our informal attempts at deciphering the A(t) data from the illustrative experiment, the expressed pattern of all the A(t) indices may prove quite distinct with regard to criteria versus energylke effects.

Finally, it should be noted that both single-target and double-target data and the A(t) statistics can in principle be analyzed with regard to questions of optimality, questions that we have had to bypass in this initial investigation. It is hoped that the A(t) and
AC(t) analyses, along with their constituent stochastic elements, can serve as a first-order methodological sieve for candidate models in both a qualitative and a quantitative manner.

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Received September 16, 2011
Revision received February 6, 2012
Accepted February 6, 2012 ■