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Chapter 40

New Theoretical Results on Testing Self-Terminating vs. Exhaustive Processing in Rapid Search Experiments

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Two important questions in cognitive psychology concern how a subject “searches” through arrays of items to find a specific, predetermined item. One question involves how comparisons between a single (often visually) presented “probe” or “target” and a list stored in memory take place, a memory search (e.g., Sternberg, 1966). The second question deals with visual search, which involves the comparison processes between a display of several items and a single “target” item stored in memory (e.g., Atkinson, Holmgren, & Juola, 1969; Townsend & Roos, 1973). The difference between visual and memory search is illustrated in Figure 1. Mean reaction times to determine that a target is present (on “positive” trials) or absent (on “negative” trials) are used to try and determine the underlying processes involved in the search.

It is very difficult, however, to determine how the comparison process takes place between the target item and the items in the search set, especially

when using mean reaction time data as the sole basis for determination. For instance, the complications arising from the tests of serial (one at a time) and parallel (simultaneous) processing are well known (e.g., Anderson, 1976; Townsend, 1972; Townsend, 1974; Townsend, 1976; Vorberg, 1977). Several procedures have been developed in recent years that are capable of diagnosing among a large class of serial vs. parallel models (Snodgrass & Townsend, 1980; Townsend, 1984; Townsend & Ashby, 1983). On the other hand, less is known concerning "self-terminating" vs. "exhaustive" search, although Ashby (1976), Townsend and Ashby (1983) and Townsend and Roos (1973) have made some progress on the problem. We say that a search is exhaustive if the subject must search through all items in the search set before making a positive or negative response, while self-terminating search takes place if the subject can discontinue the search after the detection of the target.

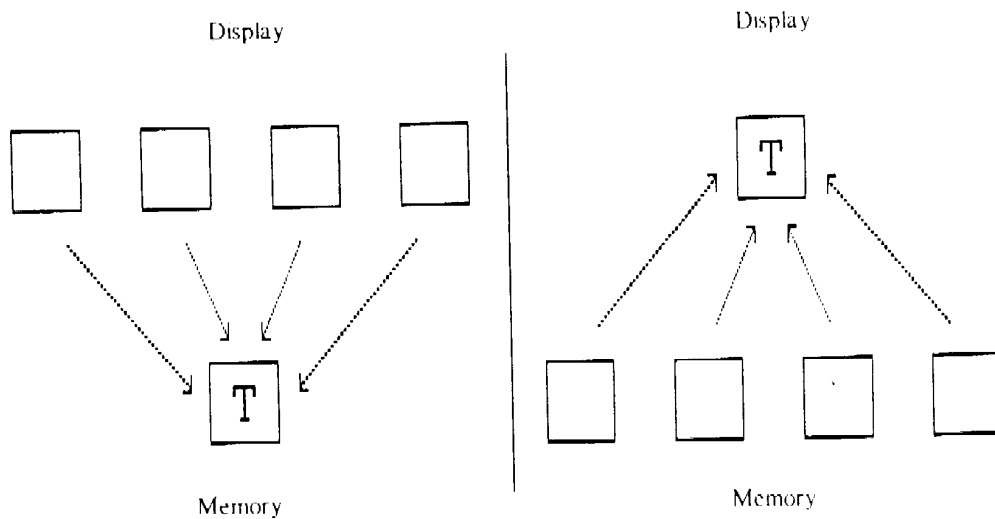


Figure 1. Visual (left) and memory search (right). Memory search takes place between several items stored in memory and single target presented visually, while visual search takes place between a single target held in memory and several items presented between a single target held in memory and several items presented visually.

Observation of mean reaction time with increased load has been the most popular method to test exhaustive and self-terminating processing, probably because of its ready intuition and the close tie between increasing mean reaction time and efforts to test serial and parallel processing (Sternberg, 1966). But, there are other techniques which can be used to make the exhaustive vs. self-terminating distinction. These include the use of redundant targets (e.g. Baddeley & Ecob, 1973; Bjork & Estes, 1971; Egeth, Folk & Mullin, 1988), reaction time variances (Schneider & Shiffrin, 1977), or keeping the expected number of items processed constant (Ashby, 1976; Townsend & Ashby, 1983). The emphasis in this paper is on the observation of slope differences between reaction time functions for target present (positive) and target absent (negative) trials, that is, parallelity vs. nonparallelity of positive and negative trial functions, to test for self-terminating or exhaustive processing. We will also

consider the implications of exhaustive processing on position effects: the effects of target placement on mean reaction time (see below).

Consider the standard serial model as an example. The standard serial model postulates that the comparison process between the target and the items in the search set takes place serially, where mean processing times (comparison times) are equal for all target and nontarget items (Sternberg, 1966). This model predicts linear, increasing mean reaction time functions, since increasing the load by the addition of a single nontarget item to the search set produces a constant amount of processing time added to the overall mean reaction time.

The standard serial self-terminating model predicts that the positive mean reaction time function increases only half as fast as the negative process function; on the average the target will be found half-way through the comparison process and processing will then stop. This prediction depends, of course, on either random placement of the target among nontarget items by the experimenter, or a random pattern of search on the part of the searcher. Alternatively, the standard serial exhaustive model predicts equal-sloped mean reaction time functions, because the comparison process always continues through the entire search set, even after discovery of the target. These results depend entirely on the exact specifications of one's model, however. Many self-terminating models may not predict the two-to-one slope (rate of increase) ratio between positive and negative mean reaction time functions (Townsend & Roos, 1973), just as some exhaustive models may not predict equal slopes.

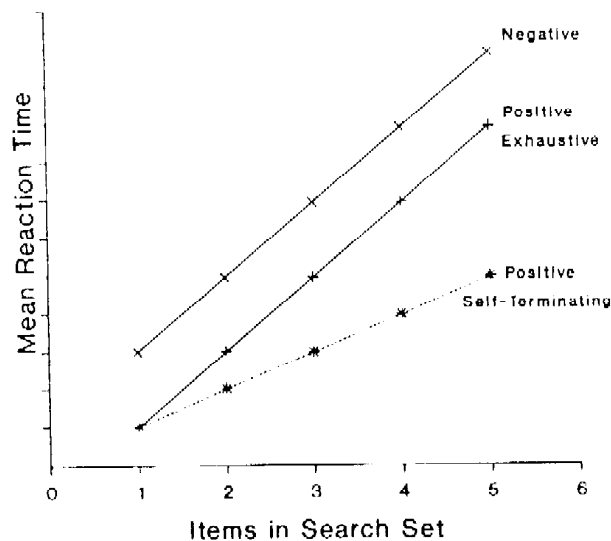


Figure 2. The standard serial model. The self-terminating search model predicts a positive mean reaction time function that has half the slope of the negative mean reaction time function.

Assuming that errors are not playing a major role in the experiment then the following postulate seems reasonable for the standard serial model. When mean reaction time is plotted separately for positive trials and for negative trials, the mean reaction time function for negative trials should be indicative of

an exhaustive search. The subject must make an exhaustive search of all the items in the negative search set to be able to respond correctly that the target item is not present. If the slope (rate of increase) of the positive function is half that of the negative function, we might wish to take that as evidence for self-terminating search as suggested above. If, on the other hand, the two functions are equal-sloped, with equal rates of increase, an exhaustive search model might be favored (see Figure 2). (For more background on the self-terminating vs. exhaustive processing question, see Ashby, 1976; Townsend, 1974, or Townsend & Ashby, 1983).

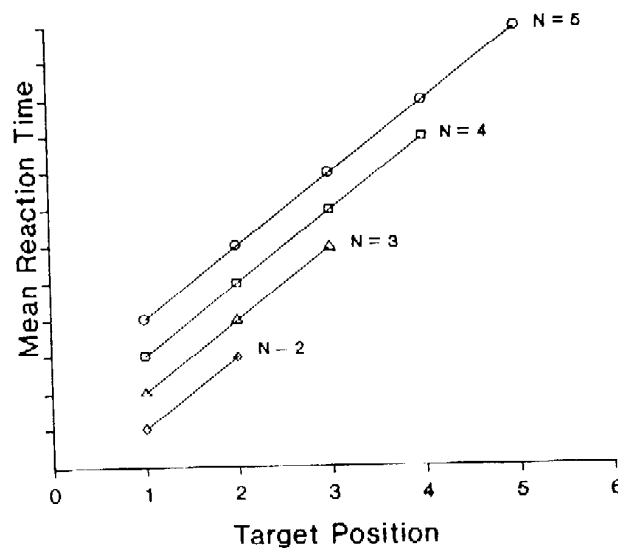


Figure 3. Position effects in the standard self-terminating model with a fixed order of processing (position 1 to position 5). For clarity, the position curves for different values of N have been separated, rather than plotted as a single line.

This sort of test is very easy to apply. However, overall mean reaction time data can mask other important aspects of the data which may be helpful in distinguishing between different processing models. For instance, the standard serial model above assumes that the individual mean processing times are equal for each of the items and positions in the search set. This is a strong and questionable assumption. Equal average processing times across all items implies that there can be no effect of target placement on mean reaction time unless search is self-terminating. We call the influence of target placement a "position effect¹"; so, for the standard serial self-terminating model, mean reaction times will be faster when the target is processed very early in the

¹ Our "position effect" has often been referred to as a "serial position effect." In an attempt to forestall any confusion between "serial" processing and "serial" position, and to generalize the concept of "position" to nonlinear display presentations, we adopt the shorter term "position effect" in this paper.

search, and slower when the target is processed near the end of the search. We can consider target placement in two ways: the target placement in the search set (either the set of items held in memory or the set of items presented visually), and the effects of target placement on the processing path, that is, the actual order in which items in the search set are processed, or processing location. If the serial processor were to always proceed from search set position one to position two, and on to position n , mean reaction time will increase when plotted against target placement in the search set (Figure 3). Position effects are manifested by an interaction of target search set location with processing path in self-terminating serial processing. In parallel systems, the speed at which an individual search set position is processed determines the position effects.

In this paper, we will present several new results concerning position effects and the form of the mean reaction time functions. However, let us first outline the present status of the use of mean reaction time functions and position effects to discriminate between self-terminating and exhaustive search strategies. Ashby (1976) and Townsend and Roos (1973) showed that reasonable serial and parallel self-terminating models can predict equal-sloped mean reaction time functions (itself a phenomenon previously attributed only to exhaustive serial process models, e.g., Sternberg, 1966, 1975) yet still permit position effects across target and nontarget items (which cannot be predicted by standard exhaustive serial models). It was demonstrated at that time that plausible process models (for example, models based on reasonable strategies, limited capacity and differential allocation of processing attention) of either the self-terminating or exhaustive variety could accommodate the often reported equal slopes of the positive and negative reaction time functions. These same models could handle the occasionally cited differential position effects which were earlier thought to imply self-termination, a result presumably at odds with the finding of equal slopes.

The upshot was (and is) that equal slopes are not very diagnostic with regard to exhaustive or self-terminating processing. However we know much less about the diagnosticity of unequal slopes (i.e., a slope ratio greater than one). Also, position effects, while certainly compatible with self-terminating processing, can be predicted to some extent by exhaustive models. To what extent exhaustive models can predict strong position effects (as compared to the predictions of self-terminating models) has never been determined. This paper will be devoted to the issue of the relative slopes of the negative and positive functions and to the prediction of such position effects.

The overall plan will be to consider serial exhaustive models and within that class, to address first the slope ratio question and second the question of ultimate position effects. Then we will consider parallel exhaustive models, presenting our somewhat more limited results with regard to these two questions.

Serial Exhaustive Models

Our strategy will be to work from more restricted cases to the most general. Hopefully, this strategy will whet the reader's intuition for the underlying dynamics of the process.

The Slope Ratio Issue

Let us then first formalize the standard serial model with exhaustive processing. To calculate the mean serial reaction times for a negative or positive trial, we need only add up the mean processing time for all of the items in the search set. Notice that "processing times" refer to the time taken for an individual comparison between target and search set, whereas "reaction times" refer to total time taken by the subject to make a response.² Call the mean reaction time for a negative trial with n items $E_n[RT^-]$, and for a positive trial with n items $E_n[RT^+]$. If, as above, we assume that all items, target and nontarget, are processed with the same individual mean processing time, then mean total reaction time for n nontarget items ($E_n[RT^-]$) is just $nE[T]$, where T is the random processing time for an individual nontarget item. Likewise, the mean reaction time for $n-1$ nontarget items and one target ($E_n[RT^+]$) is $nE[T]$, since targets and nontargets are processed identically, and all nontargets will also be processed in this exhaustive search. The rate of increase of these two functions is the same, predicting equal-sloped mean reaction time functions. This model can never predict any slope ratio other than one-to-one between the positive and negative mean reaction time functions.

The Before vs. After Model

Now allow the individual mean processing times to vary a bit. Assume that positive and negative processes take place in exactly the same way until the discovery of a target in the positive process. After the target is processed, all nontarget items can be processed with a different duration (not necessarily longer or shorter, just different). By making the distinction between "before" and "after" target processing, this model can predict a very limited range of position effects, but also fails to predict a slope ratio different from one-to-one.

² We refer to the total mean comparison or processing time for all items as mean reaction time. For the present, we assume that the average "residual" processing time, such as the time required for stimulus encoding, motor responses, etc., is invariant (or of negligible variance) across different processing loads. The addition of this constant term to the total mean processing time does not affect our derivations or conclusions.

Before a target is processed, let the random processing time for a nontarget be $T(B)$. Let the random target processing time be $T(t)$, and let random processing time for nontargets processed after the target be $T(A)$. Therefore, processing times vary with the order in which items in the search set are processed; in this way, we predict position effects in various types of displays (linear, circular, etc.) depending on the processing path taken through the search set. For a search set consisting of n items, there are $n!$ possible processing paths that could be taken. We will let the probability of a single processing path j ($j = 1, 2, \dots, n!$) be represented by P_j . We assume for this model that all processing paths are equiprobable, that is, that the probability of any processing path is $1/n!$. The target item then appears in one of the n different processing locations along a path with probability $1/n$.

For negative trials with n items, mean reaction time is just $nE[T(B)]$, since the processor is looking for a target up until the last nontarget item. Mean reaction time for a positive trial can now depend on when or where the target is found:

$$\begin{aligned} E_n[RT^+] &= 1/n \sum_{i=1}^n E[RT^+ | \text{target is in the } i^{\text{th}} \text{ search set position}] \\ &= 1/n \sum_{i=1}^n [(i-1)E[T(B)] + E[T(t)] + (n-i)E[T(A)]] \\ &= (n/2)(E[T(B)] + E[T(A)]) + E[T(t)] - (1/2)(E[T(B)] + E[T(A)]) . \end{aligned}$$

Notice that the slope of the positive function is half that of the negative function when and only when $E[T(A)]$ is zero. If $E[T(B)] = E[T(A)]$ the equal slope result is produced. $E[T(A)]$ is zero when all nontargets after the target are left unprocessed, that is, a self-terminating search, or, unreasonably, when nontargets found after the target are processed with infinite speed.

Equal Average Increments Model

More generally, we may allow for position effects across targets and nontargets, across search set position and processing location. This allows a tremendous amount of freedom in the exhaustive model. Let $E[T_{ij}^-]$ be the mean processing time for a nontarget in search set position i on processing path j , and P_j represent the probability of processing path j as defined above. Likewise, let $E[T_{ij}^+]$ represent the mean processing time for a target item in the same serial position i , and on the same processing path j . Processing time for an individual item is now allowed to vary with search set position (i.e., where in the search set it is located), processing location (i.e., where on the processing path it is located), and its identity as a target or a nontarget.

At this point, we make the assumption of equal average increments (Townsend & Ashby, 1983; Townsend & Roos, 1973). The equal average

increments assumption states that the average target or nontarget processing times across search set position and processing order is a constant with respect to n . That this average value might change with n could suggest that somehow the processor could know ahead of time how many items there are to be processed, and where a target is going to be found in any given display. Alternatively, it is not difficult to conceive of a serial (or parallel) model where individual processing times increase as load increases, perhaps due to a fixed amount of capacity spread across all items to be processed, or that an earlier subprocess could deliver more items for comparison but in a more degraded form. The assumption of equal average increments rules out such possibilities. Models which satisfy the constraint of equal average increments, however, are still very general. Reaction time is allowed to vary with both search set position and processing location, allowing for all conceivable position effects, without making restrictive assumptions about system capacity.

The assumption of equal average increments is defined by the equation

$$E[T] = \sum_{j=1}^{n!} P_j (1/n \sum_{i=1}^n E[T_{ij}]) .$$

Mean processing time $E[T]$ for an individual target or nontarget item is independent of overall load. Notice that this definition refers only to the processing times across individual target or nontarget items, and not to all the items intermixed within a search set. The processing times can vary with search set position, processing location, or both, but do not depend on search set size. Thus, in a limited capacity system, reaction time could increase with n across trials, but only because there are more items to be processed. The item processing times will be unaffected by the load change on the average. This prevents the processor from knowing how many items are in the display before processing all of them; for example, a system would not be able to change the processing rate for an item within a trial because the number of items to be processed had changed from the previous trial.

Using the assumption of equal average increments, we can now examine a general serial exhaustive model, where mean negative reaction time for n nontarget items is expressed as

$$\begin{aligned} E_n[RT^-] &= \sum_{j=1}^{n!} P_j E[RT^- | \text{processing order } j] \\ &= \sum_{j=1}^{n!} P_j \sum_{k=1}^n E[T_{kj}^-] \\ &= nE[T^-] . \end{aligned}$$

$E[T^-]$ is the weighted average of the individual nontarget processing times, assumed constant with respect to n .

For positive trials, we must examine the mean reaction time for any trial conditioned on where the target is found in the search set and where the target is placed in the search set. Let Q_j^* represent the probability (determined by the experimenter) that the target is placed in search set position j^* , where j^* varies from one to n . The overall mean reaction time is expressed as

$$E_n[RT^+] = \sum_{j=1}^{n!} Q_j^* P_j E[RT^+ | \text{processing order } j, \text{ target placement } j^*]$$

$$= \sum_{j=1}^{n!} P_j [Q_{j^*} \sum_{j^*=1}^n [\sum_{i \neq j^*}^n E[T_{ij}^-] + E[T_{j^*j^*}^+]]]$$

We need to examine the change between $n=1$ and $n=2$ to show that a two-to-one slope ratio is impossible, even under these general conditions.

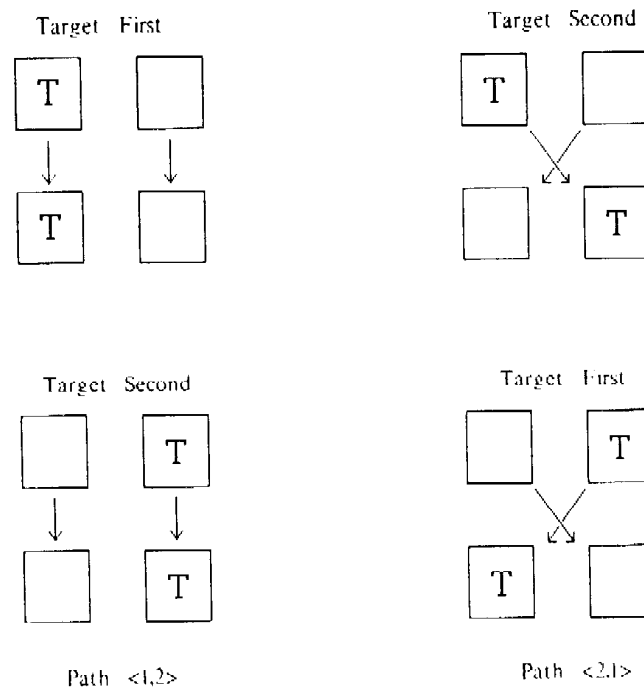


Figure 4. The two possible processing paths and different target placements in the search set. For either processing path $\langle 1,2 \rangle$ or $\langle 2,1 \rangle$, the target can be processed first or second in the search depending on its original position in the search set. For example, the target is processed first on path $\langle 1,2 \rangle$ if it is first in the search set. On path $\langle 2,1 \rangle$, the target is processed first if it is second in the search set.

The reasoning for larger n is completely similar. In the case where $n=1$, $E_1[RT^+]$ is just $E[T^+]$, the average processing time for a target item. When $n=2$, we must consider two possible target placements and two possible pro-

cessing paths. These considerations can be seen in Figure 4. For processing path $\langle 1,2 \rangle$, that is, search set position 1 processed first and search set position 2 processed second, the target can be placed in either the first or second position in the search set. Assume this is done by the investigator with probability 1/2. So, it can be processed first or second on this processing path depending upon where it is placed. Likewise, for processing path $\langle 2,1 \rangle$, the target can be placed in either the first or second position. Considering all possibilities we write

$$\begin{aligned} E_n[RT^+] &= P_1\{(1/2)(E[T_{11}^+] + E[T_{21}^-]) + (1/2)(E[T_{11}^-] + E[T_{21}^+])\} \\ &\quad + P_2\{(1/2)(E[T_{12}^+] + E[T_{22}^-]) + (1/2)(E[T_{12}^-] + E[T_{22}^+])\} \\ &= E[T^+] + E[T^-] . \end{aligned}$$

The change in reaction time from $n=1$ to $n=2$ for the positive case can then be written as

$$\Delta E[RT^+] = E[T^+] + E[T^-] - E[T^+] = E[T^-] .$$

The negative case is simpler, since we know that $E_n[RT^-] = nE[T^-]$. We can write the change in reaction time from $n=1$ to $n=2$ as

$$\Delta E[RT^-] = 2E[T^-] - E[T^-] = E[T^-] .$$

Because $\Delta E[RT^-] = \Delta E[RT^+]$, a two-to-one slope is impossible.

As noted above, this result extends to the case where there are n items in the display, since $E_n[RT^+] = (n-1)E[RT^-] + E[T^+]$. Since the rate of change between both positive and negative trials is $E[T^-]$ for all n , mean reaction time curves cannot be anything but equal-sloped.

Unrestricted Exhaustive Serial Models

We now wish to examine the case where individual processing times are allowed to vary with no restrictions. In the present demonstration, we employ the reasonable postulate that, on the average, processing times do not become faster as the processing load increases. This outcome would imply a supercapacity process which is rarely found in the human information processing literature.

To begin, let us look at the change in the reaction time from $n-1$ to n items by averaging over search set position and processing location. In the positive case,

$$\Delta E[RT^+] = [(n-1)E[T^-,n] + E[T^+,n]] - [(n-2)E[T^-,n-1] + E[T^+,n-1]]$$

and in the negative case

$$\Delta E[RT^-] = nE[T^-,n] - (n-1)E[T^-,n-1] .$$

Notice that the average individual processing time $E[T,n]$ is now a function of the number of items n in the search set, whereas in the case of equal

average increments, we assumed that it was independent of search set size.

For a given n , relate the change in positive reaction time $\Delta E[RT^+]$ to the change in negative reaction time $\Delta E[RT^-]$ by the ratio $n/n-1$, that is,

$$[n/n-1]\Delta E[RT^+] = \Delta E[RT^-] .$$

In this way, we specify that the change in the positive function is less than the change in the negative function by a ratio that is less than or equal to two. If we can show that this ratio is impossible, it will follow immediately that a two-to-one ratio (or for that matter, any ratio greater than or equal to $n/(n-1)$) is impossible.

By substituting the values for $\Delta E[RT^+]$ and $\Delta E[RT^-]$ into the above expression, we see that

$$\begin{aligned} E[T^+, n] - E[T^+, n-1] &= [(n-2) - \{(n-1)^2/n\}]E[T^-, n-1] \\ &= \{-1/n\}E[T^-, n-1] , \end{aligned}$$

or that the positive process must be of supercapacity, since the change in the average target processing time from $n-1$ to n must be negative. Processing time for target items must therefore be getting faster with increasing load to produce a negative reaction time function which increases more rapidly than the positive reaction time function, as given by the assumption

$$[n/n-1]\Delta E[RT^+] = \Delta E[RT^-] .$$

Ultimate Search Set Position Effects

To investigate further capacity restrictions within the serial exhaustive model, we now allow for the strongest reasonable position effects on positive trials. With "strong search set position effects," we allow the average conditional reaction time for a positive trial (conditioned on the placement of the target item within the search set) to equal the reaction time for a negative trial such that, if the target is placed in the j^{th} position in the search set, then the mean conditional reaction time for that trial will equal the mean reaction time for a negative trial with j items; $E_n[RT^+ | \text{target is } j^{\text{th}}] = E_j[RT^-]$. This is not an unreasonable assumption, since these are exactly the same position effects that would be predicted by the standard serial self-terminating model with a fixed search path starting at position one and proceeding through to position n (see Figure 3). While the specific calculations that we present here concern increasing position effects, that is, primacy effects, the results are identical for strong recency effects, assuming that the search takes place in the opposite direction through the search set.

Assume that we have already averaged over processing location, so that T_j denotes the processing time for an item in the j^{th} position in the search set. The model on which the present results are based permits average processing times to vary with search set position or position on the processing path. However, as the processing load is increased by adding new items to the search set, the

times for the previous search set positions are unchanged. Thus, for all search set sizes n greater than or equal to j , T_j^- will be the average negative processing time for search set position j . Then we can express the concept of strong search set position effects by the equation

$$E_n[RT^+ | \text{target } j^{\text{th}}] = \sum_{i \neq j}^n E[T_i^-] + E[T_j^+] = E_j[RT^-] = \sum_{i=1}^j E[T_i^-].$$

Solving for $E[T_j^+]$ we find that

$$E[T_j^+] = E[T_j^-] - \sum_{i=j+1}^n E[T_i^-].$$

So that $E[T_j^+]$ will always be positive, it must be the case that

$$E[T_j^-] > \sum_{i=j+1}^n E[T_i^-],$$

for all j . So, for example, $E[T_1^-]$ must be greater than $E[T_2^-] + \dots + E[T_n^-]$ to produce the position effect. Likewise, $E[T_2^-]$ must be greater than $E[T_3^-] + \dots + E[T_n^-]$. Therefore, $E[T_i^-]$ is always considerably greater than $E[T_{i+1}^-]$, and $E[T_i^+]$ is very much less than $E[T_i^-]$. Notice also that $E[T_n^-]$, the change in negative reaction time from $n-1$ to n , goes to zero as n gets large. This means that this model predicts a negative reaction time function which is nonlinear, negatively accelerating toward an asymptote with large n . These effects are systematically extreme to the point of being absurd.

Defining the capacity of a system as the behavior of the mean individual processing time under increasing load, then these results also imply that the nontarget process is of supercapacity. The value for the mean individual nontarget processing time $E[T^-]$ as defined by

$$E[T^-] = (1/n) \sum_{j=1}^n E[T_j^-]$$

is strongly decreasing with n , that is, average processing time is speeding up under increased load.

We have examined a rather general serial exhaustive model which predicts strong position effects. We made no assumptions on processing capacity or on slope ratio, yet we showed that the exhaustive properties of this model make the prediction of strong position effects together with linearity of the mean reaction time functions and limited capacity processing impossible. When we allowed for position effects and examined conditions under which the positive

function increases more slowly than the negative function, we found that the target processes must be of supercapacity.

We have shown that even in very general cases it is unreasonable to expect a serial exhaustive process to produce significant slope differences, or even strong position effects. We will now investigate the parallel exhaustive case.

Parallel Independent Process Models

Serial models were for many years the most preferred explanation of human information processing. The major reason was probably the close relation of serial processing to the ubiquitous digital computer and the inevitable computer metaphors (see Roediger, 1980, for an excellent discussion of the use and misuse of metaphors in psychology). Another ancillary reason may be that for most purposes, mathematical representations of serial processes are also more tractable (for certain exceptions, see Vorberg & Ulrich, 1987). This is certainly the case here. Our results for parallel systems are more limited in scope than those for the serial systems. Yet, we believe they are reasonably convincing and bear the same message: slope ratios (negative to positive) of greater than one are unnatural, if not absurd, for most exhaustive systems. Another conclusion is that strong search set position effects lead to awkward or impossible consequences in exhaustive models.

While discussing serial models, mean individual processing times were used. More stochastic "power" is required for our theorem-proving machinery in the parallel situation (e.g., Townsend & Ashby, 1978; Townsend & Ashby, 1983, Ch.8). In the independent parallel case, we will use the individual processing time distribution functions. We will use the notation $G^+(t,n)$ and $G^-(t,n)$ to refer to the processing time distributions of targets and nontargets under load n respectively. Observe that within-trial temporal effects such as "before" vs. "after" are absent because of the assumption of independence on separate items.

The Slope Ratio Issue

As in the serial model case, we will look at the behavior of the distribution functions as n ranges from 1 to 2, varying the capacity requirements on the target or nontarget processes and varying the initial ($n=1$) conditions of the process. We will see that the two-to-one slope ratio which we are trying to create will interact with the capacity of the independent process and the nature of the distribution functions. As in most of the serial demonstrations, any slope ratio greater than one will usually lead to an ill outcome. The two-to-one ratio is especially of note, however, because of its association with the self-terminating version of the standard serial model. Its use in some of our proofs will also simplify the notation, lessening the number of symbols to remember.

Notice first that mean reaction time for a single nontarget item is

$$E_1[RT^-] = \int_0^{\infty} (1 - G^-(t,1)) dt ,$$

(see e.g., Townsend & Ashby 1983, pp.92-93, p.170) and mean reaction time for two nontarget items is

$$E_2[RT^-] = \int_0^{\infty} (1 - [G^-(t,1)]^2) dt .$$

Similarly, mean reaction time for a single target item is

$$E_1[RT^+] = \int_0^{\infty} (1 - G^+(t,2)G^+(t,1)) dt ,$$

and mean reaction time for one target and one nontarget is

$$E_2[RT^+] = \int_0^{\infty} (1 - G^-(t,2)G^+(t,2)) dt ,$$

when search is exhaustive. The change from $n=1$ to $n=2$ in the negative process can then be expressed as

$$\Delta E[RT^-] = \int_0^{\infty} (G^-(t,1) - [G^-(t,2)]^2) dt \quad (1)$$

and in the positive process as

$$\Delta E[RT^+] = \int_0^{\infty} (G^+(t,1) - G^-(t,2)G^+(t,2)) dt . \quad (2)$$

Both Processes Are Unlimited in Capacity

Let both target and nontarget processes be of unlimited capacity, that is, $G^-(t,1) = G^-(t,2) = G^-(t)$ and $G^+(t,1) = G^+(t,2) = G^+(t)$ (see, e.g., Townsend & Ashby, 1983, Ch.8). This means that the individual processing time for a target or nontarget is independent of overall load, the same restriction that we examined in the serial case. We will show that, under these conditions, positive decisions must be slower than negative decisions. This result is rarely seen in typical memory and visual search tasks (e.g., Atkinson, et al., 1969; Briggs & Blaha, 1969; Clifton & Birenbaum, 1970; Townsend & Roos, 1973; Schneider & Shiffrin, 1977). Because our model has no mechanism by which to predict such a result, for example, no "rechecking" procedure as suggested in visual "same" vs. "different" judgments (e.g., Bamber, 1969; Krueger, 1978), we will conclude that the two-to-one slope ratio is impossible in this case.

The change from $n=1$ to $n=2$ for negative trials is

$$\Delta E[RT^-] = \int_0^{\infty} G^-(t)(1 - G^-(t)) dt .$$

In the same way, the change in the positive process from $n=1$ to $n=2$ is

$$\Delta E[RT^+] = \int_0^{\infty} G^+(t)(1 - G^-(t)) dt .$$

A two-to-one slope ratio (but any ratio greater than one will produce the same result, as noted) requires that $\Delta E[RT^-] = 2\Delta E[RT^+]$, or that

$$\int_0^{\infty} G^-(t)(1 - G^-(t)) dt = 2 \int_0^{\infty} G^+(t)(1 - G^-(t)) dt .$$

Therefore, $G^-(t)$ is greater than $G^+(t)$ for all t greater than zero. This suggests that negative processes are actually faster than positive processes in

this case, since this restriction implies that $E[T^-]$ is less than $E[T^+]$. We have already discussed this situation as being very unlikely in visual or memory search. It appears, then, that this unlimited capacity independent parallel exhaustive search model cannot predict a two-to-one slope ratio.

To illustrate this, consider the case where individual processing times are distributed exponentially, i.e., $G(t) = 1 - e^{-v(n)t}$. Denote the processing rate for a target under load n as $v^+(n)$ and the processing rate for a nontarget under load n as $v^-(n)$. The change in the negative function from $n=1$ to $n=2$ is

$$\Delta E[RT^-] = [1/(v^-(2) + v^-(2)) + 1/v^-(2)] - 1/v^-(1) \quad (3)$$

and for the positive (but still exhaustive) function,

$$\Delta E[RT^+] = [1/v^-(2) + 1/v^+(2) - 1/(v^-(2) + v^+(2))] - 1/v^+(1) \quad (4)$$

Since both processes are of unlimited capacity in this case, i.e., independent of load n , processing rates simplify to v^+ and v^- . The change in the negative function $\Delta E[RT^-]$ becomes $1/2v^-$, while the change in the positive function is

$$\Delta E[RT^+] = v^+ / ((v^- + v^+)v^-).$$

Setting $\Delta E[RT^-] = 2\Delta E[RT^+]$ and solving for v^+ gives $v^+ = v^-/3$. Therefore, v^+ is always less than v^- , or $E[T^+]$ is always three times greater than $E[T^-]$.

Only Negative Processes Are of Unlimited Capacity

Let's examine another case where only the nontarget process is of unlimited capacity, but where processing times are equal for targets and nontargets when $n=1$. This is not a crazy assumption, as it is widely thought that the intercept difference between the positive and negative mean reaction time functions is due to subprocesses lying outside the search procedure (e.g., Sternberg, 1975). In any event, the target processing time is now free to vary in any way which might produce a two-to-one slope, as long as the processing time for a single target is equal to that for a single nontarget. We will show that for this case, the target process must be supercapacity, that is, must be faster with increased load, and that this model therefore cannot predict a two-to-one slope ratio.

Let $G^+(t,1) = G^-(t,1) = G(t)$, and $G^-(t,1) = G^-(t,2)$; $G^+(t,2)$ is free to vary in any way to produce a two-to-one slope ratio. Substituting into Equations (1) and (2), a two-to-one ratio is produced if

$$\begin{aligned} \int_0^\infty (G(t) - [G(t)]^2) dt &= 2 \int_0^\infty (G^+(t,1) - G^+(t,2)G^-(t,2)) dt \\ &= 2 \int_0^\infty (G(t) - G(t)G^+(t,2)) dt. \end{aligned}$$

This holds only if $G(t) - G^+(t,1) < G^+(t,2)$, or that the positive process is of

supercapacity. Any model which implies supercapacity must be deemed implausible.

Again looking at an exponential example of this condition, let $v^+(1) = v^-(1) = v^-(2) = v^- = v$, but $v^+(2) \neq v$. Then $\Delta E[RT^-] = 1/2 v^- = 1/2 v$ as before, and substituting into Equation (4) above,

$$\Delta E[RT^+] = v/(v^+(2)(v + v^+(2))) .$$

Setting $\Delta E[RT^-] = 2\Delta E[RT^+]$ and solving for $v^+(2)$ gives

$$v^+(2) = (v/2)(\sqrt{17}-1)$$

so $v^+(2) > v = v^+(1)$, and the target process is of supercapacity.

Limited Capacity Processes

Finally, we will examine the case where the nontarget process is of limited capacity, again setting the processing time distributions for single targets to be equal to single nontargets when $n=1$. In this way, the target process may not need to speed up as much to produce a two-to-one slope ratio as it would if the nontarget process was of unlimited capacity. Intuitively, since the nontarget process is slowing down, perhaps the target process will not need to "work as hard" to produce the two-to-one slope ratio. We will show that under these conditions, the processing time distributions of this model must be very carefully ordered to produce the desired slope ratio.

These conditions stated mathematically are $G^-(t,1) = G^+(t,1) = G(t)$, and $G(t) > G^-(t,2)$. The negative processing time distributions are ordered in this way to produce limited capacity processing for nontarget items, that is, mean processing time which increase as the processing load increases (see Townsend & Ashby, 1983, Ch.8). Substituting into Equations (1) and (2), we observe a two-to-one slope ratio if

$$0 \int_0^\infty (G(t) - [G^-(t,2)]^2) dt = 2 \int_0^\infty (G(t) - G^-(t,2)G^+(t,2)) dt .$$

Therefore, $G^+(t,2) > G^-(t,2)$, i.e., the positive processing time distribution must be greater than the nontarget processing time distribution when $n=2$. This result predicts that mean target processing times will be less than mean nontarget processing times with a load of two. When we look at the implications for larger values of n , we see that $G^+(t,n) > G^-(t,n)$ for the condition that $\Delta E[RT^-] = 2\Delta E[RT^+]$. If we assume that the positive process is also of limited capacity, we can show that the positive and negative processing time distributions are ordered such that $G^+(t,1) = G^-(t,1) > G^+(t,2) > G^-(t,2) > \dots > G^+(t,n) > G^-(t,n)$. Let the positive process become "maximally limited," that is, as limited in capacity as the negative process by letting $G^+(t,n) = G^-(t,n)$. Then, by substitution into the above integral, we can show that $G^-(t,1)$ must equal $G^-(t,2)^2$, or that $G^-(t,1)$ must be less than $G^-(t,2)$. This is a violation of

the original assumption that the negative process be of limited capacity, and hence the positive process cannot be "as limited" as the negative process.

This is not as strong a result as those we have presented in the previous two models, yet it is very difficult to determine the relationships between these functions under such minimal restrictions as distribution ordering. We present below, however, an example of an exponential model which, under these restrictions, must predict supercapacity of the positive process.

We will only consider the case where the negative function is linear in n (Townsend & Ashby, 1983, p.89), e.g., the rates must then satisfy the equation

$$v^-(n) = 1/n \sum_{i=1}^n 1/i .$$

In this case, $v^-(1)=1$ and $v^-(2)=3/4$, and therefore $\Delta E[RT^-]=1$.

We express $v^+(2)$ as $av^+(1)$, so that we can consider the processor to be of limited capacity if $a < 1$, of unlimited capacity if $a = 1$, and of supercapacity if $a > 1$. Substituting into Equation (4), the change in the positive function becomes

$$\Delta E[RT^+] = [\{1/av^+(1)\} + \{4/3\} - \{1/(3/4 + av^+(1))\}] - 1/v^+(1) .$$

Setting $\Delta E[RT^-]=2\Delta E[RT^+]$ and solving for $v^+(1)$ gives

$$v^+(1) = (3/40a)[(8a-5) + \sqrt{64a^2 + 80a - 135}] .$$

To avoid an imaginary $v^+(1)$, a must be greater than .956. If $a < 1$, however, $v^+(1)$ is less than 1. This means that the intercept of the positive function is greater than that of the negative function, implying both slow positive responses and a crossover of the two reaction time functions. Restricting $v^+(1)$ to be greater than or equal to $v^-(1)$ so that positive responses are faster than negative responses, no value of $a \leq 1$ will work. Again, the positive function must be of supercapacity to produce the two-to-one slope ratio.

Ultimate Search Set Position Effects

As in the serial case, we now want to determine what kinds of position effects we could reasonably expect from an independent, exhaustive parallel model. Because of the intractabilities which arise from a "distribution-free" approach to this question, we are forced to consider only the exponential case as a suggestive example.

Recall from the serial case that the strongest position effects could be represented allowing the conditional positive reaction time (conditioned on target placement j within the search set) to be equal to the reaction time for a negative trial with j items. In the parallel case we will make the identical argument, examining the target processing rates for different target placements in terms of plausibility.

We will restrict the negative reaction time function to be linear by the same definition of processing rates as in the case of limited capacity processes above. We will also assume that position has no effect on negative processing rates. This yields $v^-(1)=1$ and $v^-(2)=3/4$ as observed before.

For positive processing rates, let $v_i^+(2)$ denote the processing rate for a target item in display position i . The conditional reaction times for two items can then be expressed as

$$E_x[RT^+ | \text{target is } i^{\text{th}}] = [\{1/v^-(2)\} + \{1/v_i^+(2)\} - \{1/(v^-(2) + v_i^+(2))\}]$$

where $i=1,2$. By replacing the value $3/4$ for $v^-(2)$ in this equation, and setting the conditional positive reaction time equal to the negative reaction time for i items, we can solve for $v_i^+(2)$. When the target is first ($i=1$), $E_1[RT^-]=1$. Solving for $v_1^+(2)$ yields the quadratic equation

$$(1/3)(v_1^+(2))^2 + (1/4)(v_1^+(2)) + (3/4) = 0,$$

which has no real solutions. In other words, no speed of processing given by $v_1^+(2)$ can satisfy the condition. The same procedure for $v_2^+(2)$ gives one positive solution, that $v_2^+(2)$ must equal $3/4$. We see here the parallel analogue to the previous serial model, in that we may observe linearity of the negative function or strong position effects but not both.

Exhaustive parallel models with independent processing times apparently have great difficulty predicting significant slope differences between positive and negative reaction time functions. Similarly, our example suggests that exhaustive parallel models are incompatible with strong position effects and linearity, but more general results on this issue would be desirable.

Summary and Conclusion

We began with serial models, showing that when the search is exhaustive, sizeable slope differences between negative and positive mean reaction time functions are very difficult and usually impossible to predict. Even when allowed a wide range of position effects, where individual processing rates are independent of processing load or varying with processing load, exhaustive search models tend to predict parallel, or at least not steadily diverging, mean reaction time functions. The finding of equal-sloped mean reaction time functions is by no means uncommon (e.g., Atkinson, et al., 1969; Burrows & Okada, 1971; Sternberg, 1966). Townsend and Roos (1973) showed, however, how self-terminating models could predict such a result; these studies, therefore, provide little support for an exhaustive processing hypothesis. Other studies reporting significant slope differences between positive and negative mean reaction time functions (e.g., Briggs & Blaha, 1969; Briggs & Johnsen, 1973; Clifton & Birenbaum, 1970; Schneider & Shiffrin, 1977) are thus very strong evidence against exhaustive processing.

It is further difficult for a serial exhaustive model to predict strong search set position effects especially while preserving linearity of the negative reaction

time function. A serial exhaustive process model predicting strong position effects also predicts that the negative function is very negatively accelerated, or concave, and consequently that the nontarget process is of supercapacity. Some concavity of positive and/or negative mean reaction time functions is not uncommon (e.g., Kristofferson, 1972; Townsend & Roos, 1973), but the above exhaustive model predicts that asymptotically the mean reaction time function levels out to a constant, an unlikely prediction indeed.

Parallel models appear to have similar limitations to serial models. Parallel exhaustive search models that predict slope differences between positive and negative functions imply supercapacity of target processing in many cases. Strong position effects are impossible to predict if the negative mean reaction time function is linear. Although an unlimited capacity, independent exhaustive parallel model could predict some degree of concavity, it could not generally predict the untoward result of a flat asymptote mentioned in the preceding paragraph (c.f., Townsend & Ashby, 1983, Proposition 4.9, p.92). Furthermore, it also could not predict significant differences between positive and negative functions, nor could it explain the conjunction of linearity and strong effects noted in several of Townsend and Roos' subjects.

A sizeable number of memory and visual search studies finding strong position effects has been reported. This causes us to be skeptical of exhaustive processing explanations in rapid memory and visual display search experiments. For instance, Burrows and Okada (1971) found quite dramatic primacy and recency effects in their memory search study. Also, Forrin and Cunningham (1973) found strong recency effects which decayed as the time target (probe delay) increased. Harris, Shaw and Altom (1985) found very strong recency effects in visual search of a large ($n=10$) display.

Perhaps most convincing are the results of Clifton and Birenbaum (1970), who found a clear dichotomy of slope ratios between their subjects, nine of which had equal-sloped mean reaction time functions and the remaining three had two-to-one slope ratios, the negative function increasing twice as the positive function. They found a significant recency effect of target position when probe delay was short, averaging across subject data. These two findings of slope differences and position effects are impossible for any of the exhaustive models we have discussed to predict.

This brief discussion is not meant to serve as an exhaustive review of the vast visual and memory search literature. It is, in fact (and unfortunately), very difficult to condense and compare these works because of the differences in experimental conditions (varied vs. consistent mapping of stimulus materials, amount of practice, sequential or simultaneous presentation of the memory set in memory search, etc.) and the data which were not published in each case. For instance it was rare to find studies that reported both positive and negative mean reaction time functions as well as position effects (much less ancillary statistics such as error rates, reaction time variances, etc.). On the whole, however, the experimental literature suggests that exhaustive processing strategies are implausible in most memory and visual search paradigms. It should be recalled in this context that self-terminating models can accommodate equal or

nonequal slopes. Further, the absence of position effects in a particular study may be due simply to equal probabilities of different search paths in serial processing or equal processing times on different positions in parallel processing, even though processing is self-terminating.

These facts, together with the damage done to the exhaustive hypothesis by the empirical occurrence of significant slope differences and position effects, provide strong evidence in favor of self-terminating and against exhaustive processing.

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