Gas Pressure

Gas molecules inside a volume (e.g. a balloon) are constantly moving around freely. During this molecular motion they frequently collide with each other and with the surface of any enclosure there may be (in a small balloon that would be many thousands of billions of collisions each second).

Figure 1: The internal gas pressure in a balloon, $P_B$, is given by the impacts of moving gas molecules, as they collide with the skin of the balloon from the inside.

The force of impact of a single one such collision is too small to be sensed. However, taken all together, this large number of impacts of gas molecules exerts a considerable force onto the surface of the enclosure: the gas pressure (see Figure 1).

The larger the number of collisions per area of enclosure, the larger the pressure:

$$P = \frac{F}{A} \left[ \frac{N}{m^2} = Pa \right]_{SI}$$

The SI-unit of pressure is Pascal [Pa], but in Meteorology it is accepted to use millibars [mb], where $100 \text{kPa} = 1000 \text{mb}$. The direction of this gas pressure force is always perpendicular to the surface of the enclosure at every point.
Atmospheric Pressure – Pressure Profile

In the example of the balloon (above), there is not only gas inside the balloon (exerting pressure from the inside), but there is also gas (air) on the outside, exerting pressure onto the outside surface of the balloon (Figure 2).

Figure 2: The atmospheric pressure outside a balloon, $P_A$, is given by the impacts of moving gas molecules, as they collide with the skin of the balloon from the outside. Because collisions occur on all sides of the balloon, the atmosphere exerts a pressure from all sides.

The rate, at which the skin of the balloon is bombarded by air molecules, is dependent on how tightly the gas molecules are packed, or on the gas density:

$$\text{(Gas) Density} = \frac{\text{Mass of gas}}{\text{Volume}} \quad \text{or} \quad \rho = \frac{m}{V} \quad \left[ \frac{\text{kg}}{m^3} \right]_{\text{SI}}$$

Since gas is compressible, its density depends on the force that is used to compress it (see Figure 6-2 in Lutgens and Tarbuck, 7th Ed., 1998). In the atmosphere, the force that compresses the air at the surface is just the weight of all the air in the atmospheric column above it (Figure 3).

$$\text{Atmospheric Pressure} = \frac{\text{Weight of Overlying Air}}{\text{Unit Area}}$$
At the surface the atmospheric pressure is on average $P_0 = 101.3$ kPa = 1013 mb

Figure 3: Surface pressure and density in an air column

The higher we go in the atmosphere, the less air remains in the column above us. Thus, the atmospheric pressure always decreases with height. Similarly, air density decreases with height, because the overload to compress the air gets less and less, as we go higher.

Clearly, at the surface, the air is densest. Thus, as we rise from the surface through the first kilometer of the atmosphere, we leave a lot of dense air below us: the overload (and thus the density and pressure) decreases quickly. As a result, the air in the next km is already much less dense than below, and the overload (density and pressure) decrease more slowly. This ever slower decrease of the overload, of density and pressure, is continued for every consecutive vertical stretch of atmosphere.

In fact, at 6 km height, we already have approximately half of the mass of air below us. Thus, the atmospheric pressure at 6 km height is only half that at the surface (i.e., $P_{6\text{km}} = 500$ mb). Between 6 and 12 km the overload (and the pressure) is halved again: only ¼ of the surface pressure is left. This consecutive halving gives rise to the following (approximate) pressure profile, starting at a surface pressure of $P_0 = 1000$ mb:

<table>
<thead>
<tr>
<th>height (km)</th>
<th>Pressure (mb)</th>
<th>Fraction of $P_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000 (= $P_0$)</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>500</td>
<td>1/2</td>
</tr>
</tbody>
</table>
The shape of this profile-curve is called a **negative exponential decrease** (compare Figures 1.12, Lutgens and Tarbuck, 7th Ed., 1998). The density profile behaves in a similar way.

\[
\frac{P(z)}{P_0} = \exp\left\{ -\alpha(z - z_0) \right\}
\]

where \(\alpha\) is a temperature dependent parameter.

**Hydrostatic Pressure – Mercury Barometer**

The principle that the pressure at a given level is equivalent to the weight of the overlying column is not only true for air, but for *fluids* (gases and liquids) in general. The pressure generated by an overlying column of fluid is thus termed the **hydrostatic pressure**.

The upper boundary of the air column that gives rise to atmospheric pressure is the vacuum of space. Being rather light, the mass of a column of air with a 1 cm\(^2\) cross section is almost exactly 1 kg. If a much heavier liquid substance is used to balance this air column, only a relatively small length would be needed. In addition, because the density of liquids does not change with height (most liquids are incompressible), such an equivalent liquid column has a well defined upper boundary (below a vacuum).

One of the heaviest liquids at room temperature is *mercury* (Hg) and the height of the Hg-column that is equivalent to normal pressure (1013 mb) is only 760 mm long (29.92 “). For this reason, columns of mercury, “hanging” in an inverted vacuum tube, can be used as practical instruments to measure atmospheric pressure (see Figure 6-4, Lutgens and Tarbuck, 1998).

If water were used instead of mercury, the height of the column equivalent to normal pressure would be 10.33 m - not a very practical length of tube to work with.

**The Gas Laws**

The example of the gas-filled balloon can also be used to explore the basic gas laws (see also Appendix D, p. 414, Lutgens and Tarbuck, 1998).

In the following, lets assume that the balloon is tight, so that the amount or mass of air in it stays the same: \(m = \text{const}\). With density being the ration of mass per volume, the gas density of the balloon thus varies only with its volume (when mass is held constant).

If we squeeze the balloon, we compress the air and two things will happen:

- the air pressure in the balloon will increase.
• the density of the air in the balloon will increase.

Since density is mass over volume, and the mass stays constant, the rise in density means that the volume of the balloon decreases: pressure goes up; volume goes down. This finding is expressed more precisely by

**Boyle’s Law** (after Robert Boyle, Irish scientist, around 1600)

Boyle’s law states that, *at a constant temperature, the volume of a given mass of gas varies inversely with pressure*. For two states of pressure ($P_1$, $P_2$) and two corresponding volumes ($V_1$, $V_2$), this is stated mathematically:

$$P_1 \cdot V_1 = P_2 \cdot V_2$$

If $P_2 = 2 \cdot P_1$, it follows that $V_2 = 1/2 \cdot V_1$.

In Lab 3 (see Lab Manual, p 26) we are performing another experiment with a balloon that illustrates the principle expressed in

**Charles’s Law** (after Jacques Charles, French scientist, around 1790)

By warming the balloon up, we increase the speed of the moving gas molecules inside it. This in turn increases the rate at which the gas molecules bombard the skin of the balloon. Because the balloon’s skin is elastic, it expands upon this increased pushing from inside, and the volume taken up by the same mass of gas increases with temperature. In consequence, the density [=mass/volume] decreases with rising temperature. Cooling the balloon down again will make the balloon shrink.

Thus Charles’s law states that *at a constant pressure, the volume of a given mass of gas is directly proportional to its (absolute) temperature*. For two states with temperatures ($T_1$, $T_2$) and two corresponding volumes ($V_1$, $V_2$):

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

It must be noted that in this case (and whenever temperature appears in a multiplication or a division) the **absolute or Kelvin scale must be used** for temperature.

**The Ideal Gas Law or Equation of State**

The example used to illustrate Charles’s law probably does not follow Charles’s law exactly. It is very likely that, during the heating process, when the rate of collisions by the gas molecules increased, the pressure increased as well as the volume. Thus, in practical situations all three variables involved in Boyle’s and Charles’s law are linked and both principles are in action at the same time:

- Pressure: $P$
- Temperature: $T$
Density: $\rho = \frac{m}{V}$

These variables describe the state of the gas at any one time and are combined in the single relationship known as the ideal gas law or the equation of state:

$$P \cdot V = m \cdot R \cdot T \quad \text{or} \quad P = \rho \cdot R \cdot T$$

where R, the constant of proportionality, is known as the universal gas constant ($= 287 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$)

When mass and pressure are held constant, the gas law collapses to Charles’s law. When mass and temperature are held constant, it is equivalent to Boyle’s law. Thus, the gas law combines the two laws.

The equation of state or ideal gas law is one of the most fundamental relationships linking the three variables temperature, pressure and density that describe the thermodynamic state of the atmosphere. Some application of the gas law is involved in most practical problems in meteorology.