Set-up for the next two problems: You are in charge of loading a cargo plane with aid packages for a UN relief effort. The plane is to be loaded with two types of package, described as follows:

A) The A package has a volume of 15 cubic feet, weighs 180 lbs, and contains (among other things) 110 lbs. of grain and 20 lbs. of powdered milk.

B) The B package has a volume of 18 cubic feet, weighs 280 lbs, and contains (among other things) 170 lbs. of grain and 25 lbs. of powdered milk.

The plane can hold a maximum of 39,000 lbs of cargo, and has a maximum capacity of 3000 cubic feet. You want to ship at least 2000 lbs. of powdered milk. How many A-type and how many B-type packages should be loaded onto the plane in order to maximize the amount (in lbs.) of grain shipped, but stay within the weight and size limits and meet the requirement for powdered milk? (Don’t worry that the actual answer may involve fractional loads, and don’t worry about actually getting the load to fit geometrically - the problem is solved without regard to these considerations in order to get an estimate about loading the plane).

In setting up this problem, let (fill in whatever lines you will need):

\[ x = \# \text{ A packages loaded onto the plane} \]
\[ y = \# \text{ B packages loaded onto the plane} \]
\[ z = \# \text{ } \]
\[ w = \# \text{ } \]

1) (5 pts.) For the linear programming problem corresponding to this set-up, what is the objective function (in terms of the variables you defined above)?

Answer: 110x + 170y

2) (10 pts.) For the linear programming problem corresponding to this set-up, list the appropriate constraint equations below. (List only those that are formally called for by the text above).

\[ x \geq 0 \quad y \geq 0 \quad 180x + 280y \leq 39,000 \quad 15x + 18y \leq 3,000 \quad 20x + 25y \geq 2,000 \]
3) (10 pts.) A hat holds four $1 dollar bills and two $5 dollar bills. Two of the bills are drawn out at random, one after the other, without replacement. What is the probability of drawing a $1 dollar bill, and then a $5 dollar bill? You may want to draw a tree to help with the next problem as well.

4) (10 pts.) A hat holds four $1 dollar bills and two $5 dollar bills. Two of the bills are drawn out at random, one after the other, without replacement. Find the expected amount of money drawn out. (Let X = $ value of the two bills drawn. Find E(X).)

5) (10 pts.) A hat holds four $1 dollar bills and two $5 dollar bills. A bill is selected at random and then replaced. This is repeated two more times for a total of 3 draws with replacement. What is the probability on getting at least one $5 dollar bill?

This is a Bernoulli process where the probability of drawing a $5 bill on each draw is 1/3 (i.e. 2 out of 6). The complement of the event of getting at least one $5 bill is getting no $5 dollar bills.

\[
1 - (2/3)^3 = 19/27
\]

probability of getting no $5 bills in 3 draws.
6) (10 pts.) A weather balloon is launched. At precisely noon tomorrow, there is a probability of .6 that the balloon will be over the Nile Valley and a probability of .4 that it will be over the Sinai Peninsula. The probability of rain in the Nile Valley at noon tomorrow is .8. The probability of rain in the Sinai Peninsula at noon tomorrow is .3. Tomorrow at noon, telemetry from the balloon shows that it is raining at its current location. What is the probability that the balloon is over the Nile Valley?

\[
\begin{array}{c|cc|c}
\text{Event} & \text{N} & \text{R} & \text{P}\text{.} \\
\hline
\text{Nile Valley} & \cdot .8 & = .48 & \text{N} \\
\text{Sinai Peninsula} & \cdot .2 & = .12 & \text{R} \\
\hline
\text{Total} & .6 & .4 & 1
\end{array}
\]

\[
\text{Pr}[\text{N}|\text{R}] = \frac{\text{Pr}[\text{N} \cap \text{R}]}{\text{Pr}[\text{R}]} = \frac{.48}{.48 + .12} = \frac{4}{5}
\]

7) (10 pts.) A club has 7 people, exactly two of which are female. If two people are selected at random from the club, what is the probability that they are both female?

There are \( C(7,2) \) ways to select two of the people. ONE of these ways is to select the two females.

\[
C(7,2) = \frac{7 \cdot 6}{2 \cdot 1} = 21
\]

Answer: \( \frac{1}{21} \)