EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting 4 aces?

QUESTION: How many 5 card hands have 4 aces (i.e., what is X)?

\[
Pr[4 \text{ aces}] = \frac{X}{C(52,5)}
\]

To see this, lay the deck out on a table, and start making up all the 5 card hands with 4 aces. First, you will want to select the 4 aces.

\[
X = 48
\]

\[
Pr[4 \text{ aces}] = \frac{X}{C(52,5)} = \frac{48}{C(52,5)}
\]

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a full house? [Full house: 3 cards of one value and 2 cards of another.]

\[
Pr[\text{Full house}] = \frac{X}{C(52,5)}
\]

\[
= \frac{48}{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48} = \frac{48}{2,598,960} = \frac{1}{54,145}
\]
EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a full house?  
[Full house: 3 cards of one value and 2 cards of another value.]

QUESTION: How many 5 card hands have 3 cards of one value and 2 of another (i.e. what is \( X \))?

\[
\begin{align*}
\text{Pr[Full house]} &= \frac{X}{C(52,5)} \\
X &= C(13,2) \times C(4,3) \times C(4,2) \\
&= 3,744 \\
\end{align*}
\]
**EXAMPLE:** You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a full house?
[Full house: 3 cards of one value and 2 cards of another value.]

\[
\text{Pr[Full house]} = \frac{X}{C(52,5)} = \frac{3,744}{2,598,960}
\]

\[
X = \binom{13}{2} \times \binom{4}{3} \times \binom{4}{2} = \frac{6}{4165}
\]

**EXAMPLE:** You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a flush?
[Flush: All 5 cards the same suit.]

**SOLUTION:**

\[
\frac{X}{C(52,5)}
\]

To find \(X\), try making up all flush hands:

1) Choose suit: 4 ways to do this
2) Choose 5 cards of that suit: \(C(13,5)\) ways to do this
3) There are \(X = 4 \times C(13,5)\) possible hands that are flushes.
EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a flush?
[Flush: All 5 cards the same suit.]

SOLUTION:
\[
\frac{4 \cdot C(13,5)}{C(52,5)}
\]

EXAMPLE: You are dealt a 5 card poker hand from a 52 card deck. What is the probability of getting a pair?
[Pair: Exactly 2 cards match in value.]

SOLUTION:
\[
\frac{X}{C(52,5)} = \frac{\text{# of 5 card hands with 2 cards same value}}{C(52,5)}
\]

To find X, try making up all “pair” hands:
1) Choose 4 values: \(C(13,4)\) ways.
2) Choose one of the 4 values for a pair: \(C(4,1)\) ways.
3) Choose two cards of “pair” value: \(C(4,2)\) ways.
4) Choose one card of “singleton” value: \(C(4,1)\) ways.
5) Choose one card of “singleton” value: \(C(4,1)\) ways.
6) Choose one card of “singleton” value: \(C(4,1)\) ways.

\[
X = C(13,4) \cdot C(4,1) \cdot C(4,2) \cdot C(4,1) \cdot C(4,1) \cdot C(4,1)
\]

\[
= \frac{X}{C(52,5)} = 715 \cdot 4 \cdot 6 \cdot 4 \cdot 4 \cdot 4
\]