P breaking effects in a quark (nuclear) medium with axial charge

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c. **Parity breaking (PB) in heavy ion collisions**: fireballs with a non-zero topological charge (and/or neutral pion condensate, next talk by Domenec Espriu); polarization splitting for photons and vector mesons in Chern-Simons constant background (axial chemical potential); A.A., D.Espriu, V.A.Andrianov,X.Planells
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Finite volume effects: passing through and reflecting from a boundary A.A., S.S.Kolevatov, R.Soldati

Quantization: Bogoliubov transformation from vacuum to parity breaking medium and back.
Massive MCS electrodynamics (CFJ model)

\[ \mathcal{L}_{MCS} = -\frac{1}{4} F^{\alpha\beta}(x) F_{\alpha\beta}(x) + \frac{1}{2} m^2 A_\nu(x) A^\nu(x) + \frac{1}{2} \zeta_\mu A_\nu(x) \tilde{F}^{\mu\nu}(x) + g.f. \]

In momentum space wave Eqs.

\[
\begin{align*}
\left\{ \begin{array}{l}
\left[ g^{\lambda\nu} (k^2 - m^2) - k^\lambda k^\nu + i \varepsilon^{\lambda\nu\alpha\beta} \zeta_\alpha k_\beta \right] a_\lambda(k) = 0 \\
k^\lambda a_\lambda(k) = 0
\end{array} \right.
\end{align*}
\]

Projection onto different polarizations with the help of

\[
S^\nu_\lambda = \delta^\nu_\lambda D + k^\nu k_\lambda \zeta^2 + \zeta^\nu \zeta_\lambda k^2 - \zeta \cdot k (\zeta_\lambda k^\nu + \zeta^\nu k_\lambda); \quad D \equiv (\zeta \cdot k)^2 - \zeta^2 k^2
\]

Transversal polarizations,

\[
\pi_{\pm}^{\mu\nu} \equiv \frac{S^{\mu\nu}}{2D} \pm i 2 \varepsilon^{\mu\nu\alpha\beta} \zeta_\alpha k_\beta D^{-\frac{1}{2}}; \quad \varepsilon_{\pm}^{\mu}(k) = \pi_{\pm}^{\mu\lambda} \epsilon^{(0)}_\lambda
\]

Scalar and longitudinal polarizations,

\[
\varepsilon^\mu_S(k) \equiv \frac{k^\mu}{\sqrt{k^2}}, \quad \varepsilon^\mu_L(k) \equiv \left( D k^2 \right)^{-\frac{1}{2}} \left( k^2 \zeta^\mu - k^\mu \zeta \cdot k \right)
\]
Energy spectrum and birefringence

Transversal polarizations,

\[ K_{\nu}^{\mu} \varepsilon_{\pm}^{\nu}(k) = \left( k^2 - m^2 \pm \sqrt{D} \right) \varepsilon_{\pm}^{\mu}(k); \]

\[ \omega_{k, \pm} = \left\{ \begin{array}{c}
\frac{\sqrt{k^2 + m^2 \pm \zeta_0 |k|}}{\sqrt{k^2 + m^2 + \frac{1}{2} \zeta_x^2 \pm \zeta_x \sqrt{k_x^2 + m^2 + \frac{1}{4} \zeta_x^2}}}
\end{array} \right\} \zeta_{\mu} = (\zeta_0, 0, 0, 0) \]

\[ \zeta_{\mu} = (0, -\zeta_x, 0, 0) \]

Polarizations of linearly polarized radio waves could be rotated with distance \( L \)!

\[ \zeta_0 \ll |k| \quad k_{\pm} \simeq \omega_k \mp \frac{1}{2} \zeta_0; \quad \Delta \phi_{rotation} = \frac{1}{2}(\phi_L - \phi_R) = \frac{1}{2} \zeta L. \]

Distances are of order the Hubble scale \( \sim 10^{10} \) l.y. and from the analysis of 160 galaxies with linearly polarized radio waves \( \Rightarrow |\zeta| < 10^{-33} \) eV \( \sim 1/R_{Universe} \)

**Large-scale Universe is not birefringent! (at low energies)**
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- **Our special interest:**
  PB background inside a hot dense nuclear fireball in HIC !?
Topological charge

\[ T_5(t) = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \text{Tr} \left( G^j \partial^k G^l - i \frac{2}{3} G^j G^k G^l \right) \]

in a finite volume it may arise from quantum fluctuations in hot QCD medium (due to sphaleron transitions!? [Manton, Klinkhamer, Rubakov, Shaposhnikov, McLerran]) (due to long-range P-odd correlations!? [Zhitnitsky's talk on this workshop]) and survive for a sizeable lifetime in a heavy-ion fireball (i.e. only a little dissipation due to gluon flux (jets))

\[ \langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}} \simeq 5 - 10 \text{ fm}, \]

For this period one can control the value of \( \langle \Delta T_5 \rangle \) introducing into the QCD Lagrangian a topological chemical potential

\[ \Delta L = \mu_0 \Delta T_5, \quad \Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2} \int_0^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr} \left( G^{\mu\nu} \tilde{G}_{\mu\nu} \right) \]

in a gauge invariant way.
Axial baryon charge

Partial conservation of isosinglet axial current broken by gluon anomaly (consider the light quarks only),

$$\partial_\mu J_5^\mu - 2 i m_q J_5 = \frac{N_f}{8\pi^2} \text{Tr} \left( G^{\mu\nu} \tilde{G}_{\mu\nu} \right)$$

predicts the induced chiral (axial) charge

$$\frac{d}{dt} (Q_5^q - 2N_f T_5) \simeq 0, \quad m_q \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q} \gamma_0 \gamma_5 q = \langle N_L - N_R \rangle$$

to be conserved $\dot{Q}_5^q \simeq 0$ (in the chiral limit $m_q \simeq 0$) during $\tau_{\text{fireball}}$. 
Axial chemical potential can be associated with approximately conserved $Q_5^q$ (for $u, d$ quarks!)

$$\Delta L_q = \mu_5^q Q_5^q,$$

to reproduce a corresponding

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle, \iff \mu_5^q \simeq \frac{1}{2N_f} \mu_\theta$$

LPB to be investigated in e.m. interactions of leptons and photons with hot/dense nuclear matter via heavy ion collisions.

- e.m. interaction implies

$$Q_5^q \to \tilde{Q}_5 = Q_5^q - T_5^{em}, \quad T_5^{em} = \frac{1}{16\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} A^j \partial^k A^l$$

- $\mu_5$ is conjugated to (nearly) conserved $\tilde{Q}_5$
Bosonization of $Q_5^q$ following VMD prescription

$$\mathcal{L}_{\text{int}} = \bar{q} \gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2} g_\omega \omega_\mu \Pi + \frac{1}{2} g_\rho \rho_\mu^0 \lambda_3 + \frac{g_\phi}{\sqrt{2}} \phi_\mu \Pi_s,$$

$$(V_{\mu,a}) \equiv (A_\mu, \omega_\mu, \rho_\mu^0, \phi_\mu), \quad g_\omega \approx g_\rho \equiv g \approx 6; \quad g_\phi \sim 8$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} (F_{\mu\nu} F^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} + \rho_{\mu\nu} \rho^{\mu\nu} + \phi_{\mu\nu} \phi^{\mu\nu}) + \frac{1}{2} V_{\mu,a}(\hat{m}^2)_{a,b} V^\mu_b$$

$P$-odd interaction

$$\mathcal{L}_{\text{mix}} \propto -\frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} \text{Tr} \left[ \hat{\zeta}_\mu \, V_\nu \, V_\rho \right] = \frac{1}{2} \text{Tr} \left( \hat{\zeta} \varepsilon_{jkl} \hat{V}_j \partial_k \hat{V}_l \right) = \frac{1}{2} \varepsilon_{jkl} V_{j,a} N_{ab} \partial_k V_{l,b}$$

with $\hat{\zeta}_\mu = \hat{\zeta} \delta_\mu^0$, spatially homogeneous and isotropic background.
Resonance splitting in polarizations

$\zeta = 400$ MeV

$T = 220$ MeV

$p_T^e > 200$ MeV

$|y_{ee}| < 0.35$
Mean free paths for vector mesons:

- $L_\rho \sim 0.8\text{fm} \ll L_{\text{fireball}} \sim 5-10\text{fm}$
- $L_\omega \sim 16\text{fm} \gg L_{\text{fireball}}$

Why it is relevant in medium?

LPB "vacuum"

≠ empty vacuum
= coherent state of vacuum mesons

Matching on $\zeta \cdot x = 0$

Thus to save energy-momentum conservation transmission must be accompanied by reflection back. Enhancement of in-medium decays of $\omega$ mesons!
A possible gauge-invariant choice,

\[- \frac{1}{4} F^{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) \zeta^\lambda x^\gamma \theta(- \zeta \cdot x) \Rightarrow \frac{1}{2} \zeta_\mu A_\nu(x) \tilde{F}^{\mu\nu}(x) \theta(- \zeta \cdot x),\]

which however associates a space-like boundary with a space-like CS vector

\[(\zeta_\mu)(x) = \zeta(0, \bar{a}) \theta(- \bar{a} \cdot \bar{x}), \quad |\bar{a}| = 1.\]

**Compact dense stars filled by axions with density degrading to their surface?!**

Another choice: time-like CS vector and space-like boundary

\[(\zeta_\mu)(x) = \zeta(\gamma(- \bar{a} \cdot \bar{x}), - \bar{a} x_0 \delta(- \bar{a} \cdot \bar{x})) \text{ gauge invariance condition} \quad \partial_\nu \zeta_\mu = \partial_\mu \zeta_\nu. \quad \text{Singular interaction on a space-like boundary!}

**Axial chemical potential for a fireball**

\[\star \star \star \star \star \star \]

**Matching on the boundary** \[\zeta \cdot x = 0\]

\[\delta(\zeta \cdot x) [ A_\mu^{\text{vacuum}}(x) - A_\mu^{\text{MCS}}(x) ] = 0,\]
Axial chemical potential in a bounded volume
⇒ looking for boundary influence on meson propagation!

\[(\zeta_{\mu})(x) = \zeta(\theta(-x_1), -x_0 \delta(x_1), 0, 0); \quad \zeta \sim \mu_5\]

The MCS dispersion laws for different polarizations (inside of fireball),

\[
\begin{align*}
k_{1L} &= k_{10} = \sqrt{\omega^2 - m^2 - k_{\perp}^2} \\
\frac{k_{1-}}{} &= \sqrt{\omega^2 - m^2 - k_{\perp}^2 + \frac{\zeta^2}{2} + \zeta \sqrt{\omega^2 - m^2 + \frac{\zeta^2}{4}}} \\
\frac{k_{1+}}{} &= \sqrt{\omega^2 - m^2 - k_{\perp}^2 - \frac{\zeta^2}{2} - \zeta \sqrt{\omega^2 - m^2 + \frac{\zeta^2}{4}}}
\end{align*}
\]

- \(k_{1*}\) are momentum components orthogonal to boundary;
- \(k_{10}\) is outside the P-breaking medium;
- \(k_{1A}\), \(A = L, +, -\) are in the MCS vacuum;
- \(\vec{k}_{\perp}\) is parallel to the boundary;
- \(\omega\) is the energy.
In terms of effective mass $M^2 = k_\mu k^\mu$
adapted to describe dilepton decay channels!

\[
k_{1\pm} = \left(\frac{M^2 - m^2}{\zeta^2}\right)^2 - k_\perp
\]

\[
k_{10} = \left(\frac{M^2 - m^2}{\zeta^2}\right)^2 + (M^2 - m^2) - k_\perp
\]

\[
R = \frac{\sqrt{\frac{(M^2 - m^2)^2}{\zeta^2}} - k_\perp - \sqrt{\frac{(M^2 - m^2)^2}{\zeta^2}} + (M^2 - m^2) - k_\perp}{\sqrt{\frac{(M^2 - m^2)^2}{\zeta^2}} - k_\perp + \sqrt{\frac{(M^2 - m^2)^2}{\zeta^2}} + (M^2 - m^2) - k_\perp}
\]
Reflection from boundary depending on effective mass: negative chirality

Dependence on effective mass for zero $p_T \equiv |\vec{k}_\perp|$, CS vector $\zeta = 300$ MeV
Reflection from boundary: negative chirality, $p_T \neq 0$
Reflection from boundary: positive chirality, $p_T \neq 0$
Reflection from boundary: both chiralities, $p_T \neq 0$
Quantization of MCS in a half-space: Bogolyubov transformation

In vacuum,

\[ A_{\text{vacuum}}(x) = \int d^3 \hat{k} \theta(\omega^2 - k_\perp^2 - m^2) \sum_{r=1}^{3} \left[ a_{\hat{k},r} u_{\hat{k},r}(x) + a_{\hat{k},r}^\dagger u_{\hat{k},r}^\ast(x) \right], \]

Canonical commutation relations
\[ [a_{\hat{k},r}, a_{\hat{k}',s}^\dagger] = \delta(\hat{k} - \hat{k}') \delta_{rs} \]

In P-breaking medium \((A \in \{L, +, -\})\),

\[ A_{\text{MCS}}(x) = \int d^3 \hat{k} \theta(\omega^2 - k_\perp^2 - m^2) \sum_{A} \left[ c_{\hat{k},A} \nu_{\hat{k},A}(x) + c_{\hat{k},A}^\dagger \nu_{\hat{k},A}^\ast(x) \right] \]

Canonical commutation relations,
\[ [c_{\hat{k},A}, c_{\hat{k}',A'}^\dagger] = -g_{AA'} \delta(\hat{k} - \hat{k}') \]
Matching

\[ \delta(\zeta \cdot x) \left[ A^\mu_{\text{vacuum}}(x) - A^\mu_{\text{MCS}}(x) \right] = 0, \]

Bogoliubov transformation

\[ a_{\hat{k}, r} = \sum_{A=\pm,L} \left[ \alpha_{rA}(\hat{k}) c_{\hat{k}, A} - \beta^*_{rA}(\hat{k}) c^\dagger_{\hat{k}, A} \right] \]

\[ c_{\hat{k}, A} = \sum_{r=1}^{3} \left[ \alpha^*_{rA}(\hat{k}) a_{\hat{k}, r} + \beta^*_{rA}(\hat{k}) a^\dagger_{\hat{k}, r} \right] \]

Relations between coefficients

\[ \nu_{\hat{k}, A}(\hat{x}) = \sum_{s=1}^{3} \left[ \alpha_{sA}(\hat{k}) u_{\hat{k}, s}(\hat{x}) - \beta_{sA}(\hat{k}) u^*_{\hat{k}, s}(\hat{x}) \right] \]
Two vacuums as coherent states

Two different Fock vacua

\[ a_{\vec{k}, r} |0\rangle = 0 \quad c_{\hat{k}, A} | \Omega \rangle = 0 \]

From Bogoliubov tranformation,

\[ |0\rangle = \mathcal{N} \exp \left[ \int d^3 \hat{k} \theta (\omega^2 - k_\perp^2 - m^2) \times \right. \]

\[ \left. \times \left\{ \frac{\beta_{r+}^* (\hat{k})}{2 \alpha_{r+} (\hat{k})} (c^\dagger_{\hat{k}, +})^2 + \frac{\beta_{r-}^* (\hat{k})}{2 \alpha_{r-} (\hat{k})} (c^\dagger_{\hat{k}, -})^2 + \frac{\beta_{rL}^* (\hat{k})}{2 \alpha_{rL} (\hat{k})} (c^\dagger_{\hat{k}, L})^2 \right\} \right] | \Omega \rangle \]

and inversely

\[ | \Omega \rangle = \tilde{\mathcal{N}} \exp \left[ \int d^3 \hat{k} \theta (\omega^2 - k_\perp^2 - m^2) \times \right. \]

\[ \left. \times \left\{ \frac{-\beta_{A1}^* (\hat{k})}{2 \alpha_{A1}^* (\hat{k})} (a^\dagger_{\hat{k}, 1})^2 + \frac{-\beta_{A2}^* (\hat{k})}{2 \alpha_{A2}^* (\hat{k})} (a^\dagger_{\hat{k}, 2})^2 + \frac{-\beta_{A3}^* (\hat{k})}{2 \alpha_{A3}^* (\hat{k})} (a^\dagger_{\hat{k}, 3})^2 \right\} \right] |0\rangle \]
Local (finite-volume) PB is not forbidden by any physical principle in QCD at finite temperature/density

The PB leads to unexpected modifications of the in-medium properties of vector mesons and photons, in particular, resonance splitting in polarizations ⇒ the detailed analysis is given in the next talk by Domenec Espriu.

Boundary enhancement of in-medium $\omega$ decays + LPB → broadening of $\omega$ resonance in fireballs (observed on PHENIX!)

Axion stars discovery from exotic photon spectra (boundary effects)??