Holographic hydrodynamics of systems with broken rotational symmetry

Johanna Erdmenger

Max-Planck-Institut für Physik, München

Based on joint work with M. Ammon, V. Grass, M. Kaminski, P. Kerner, H. T. Ngo, A. O’Bannon, H. Zeller
Gauge/Gravity Duality at finite charge density requires 5d Chern-Simons term:

Axial contribution to hydrodynamic expansion of current

\[ J_\mu = \rho u_\mu + \bar{\xi} \omega_\mu, \quad \omega_\mu = \frac{1}{2} \epsilon_{\mu \nu \sigma \rho} u^\nu \partial^\sigma u^\rho \]

Chiral vortical effect in field-theory context

Related to axial anomaly  (Son, Surowka 2009)
Motivation:

Gauge/gravity duality: New tools for strongly coupled systems

Famous result: Shear viscosity/Entropy density

\[ \frac{\eta}{s} = \frac{1}{4\pi} \frac{\hbar}{k_B} \]

Kovtun, Son, Starinets

From `Planckian time' \[ \tau_P = \frac{\hbar}{k_B T} \], Universal result

This talk:

Deviations from this result at leading order in \( \lambda \) and \( N \)
Holographic proof of universality relies on space-time isotropy

Key ingredient for changes to the universal result: Spacetime anisotropy

Rotational invariance broken

Holographic $p$-wave superfluids/superconductors

$\rho$ meson condensate breaks rotational symmetry

At finite isospin density (or in external magnetic field)
Outline

- Holographic superconductors
- Transport coefficients in anisotropic systems
- Condensates at finite magnetic field
Holographic Superfluids

- Holographic Superfluids from charged scalar in Einstein-Maxwell gravity  
  (Gubser; Hartnoll, Herzog, Horowitz)

- p-wave superfluid  
  Current dual to gauge field condensing  
  SU(2) Einstein-Yang-Mills model  
  (Gubser, Pufu)
s-wave superfluid:

\[ \mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4} F^{ab} F_{ab} - V(|\psi|) - |\nabla \psi - iqA\psi|^2 \]

**Operator** \( \mathcal{O} \) dual to scalar \( \psi \) condensing  
Herzog, Hartnoll, Horowitz 2008

p-wave superfluid:

\[ S = \frac{1}{2\kappa^2} \int d^4x \left[ R - \frac{1}{4} (F^a_{\mu\nu})^2 + \frac{6}{L^2} \right] \]

**Current** \( J^1_x \) dual to gauge field component \( A^{1x} \) condensing  
Gubser, Pufu 2008
P-wave superfluid from probe branes

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

- A holographic superconductor with field theory in 3+1 dimensions for which
- the dual field theory is explicitly known
- there is a qualitative ten-dimensional string theory picture of condensation
This is achieved in the context of adding flavour to gauge/gravity duality.

Brane probes added on gravity side $\Rightarrow$ fundamental d.o.f. in the dual field theory (quarks)

Additional D-branes within $AdS_5 \times S^5$ or deformed version thereof
Quarks within Gauge/Gravity Duality

Adding D7-Brane Probe:

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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td>D3</td>
<td>X</td>
<td>X</td>
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<td>X</td>
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Probe brane fluctuations $\Rightarrow$ Masses of mesons ($\bar{\psi}\psi$ bound states)
On gravity side:
Probe brane fluctuations described by Dirac-Born-Infeld action

\[ S_{\text{DBI}} = -T_{D7} \int d^8 \xi \text{Str} \sqrt{\det(G + 2\pi \alpha' F)} \]

On field theory side: Lagrangian explicitly known

\[ \mathcal{L} = \mathcal{L}_{\mathcal{N}=4} + \mathcal{L}(\psi^i_q, \phi^i_q) \]
Turn on finite temperature and isospin chemical potential:

**Finite temperature:** Embed D7 brane in black hole background

**Isospin chemical potential:** Probe of two coincident D7 branes

Additional symmetry $U(2) = SU(2)_I \times U(1)_B$

\[
A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha' \rho^2} \rho_H + \ldots, \quad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha' \rho^2} \rho_H + \ldots
\]

Condensate $\langle J_3 \rangle, \quad J_3 = \bar{\psi}_d \gamma_3 \psi_u + \text{bosons}$

rho meson condensation in Sakai-Sugimoto: Aharony, Peeters, Sonnenschein, Zamaklar ’07

Calculate correlators from fluctuations
Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

$G^R$ retarded Green function for fluctuation $a_2^3$

$\omega = \omega/(2\pi T')$

$T/T_c$: Black: $\infty$, Red: 1, Orange: 0.5, Brown: 0.28.

(Vanishing quark mass)

Interpretation: Frictionless motion of mesons through plasma
Effective 5d model \(\Rightarrow\) anisotropic shear viscosity

**Bottom-up: Including the backreaction**

Ammon, J.E., Graß, Kerner, O’Bannon 0912.3515

Einstein-Yang-Mills-Theory with \(SU(2)\) gauge group

\[
S = \int d^5 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (R - \Lambda) - \frac{1}{4\hat{g}^2} F_{\mu\nu}^a F^{a\mu\nu} \right]
\]

\[\alpha = \frac{\kappa_5}{\hat{g}}\]

\(\alpha^2 \propto\) number of charged d.o.f./all d.o.f.

In presence of \(SU(2)\) chemical potential, same condensation process as before
Hairy black hole solution

- **metric ansatz**

\[
ds^2 = -N(r)\sigma(r)^2 dt^2 + \frac{1}{N(r)} dr^2 + \frac{r^2}{f(r)^4} dx^2 + r^2 f(r)^2 \left( dy^2 + dz^2 \right)
\]

with

\[
N(r) = - \frac{2m(r)}{r^2} + r^2
\]

AdS boundary \( r = r_{\text{bdy}} \to \infty \) & black hole horizon \( r = r_h \)

- **gauge field ansatz**

\[
A = \phi(r)\tau^3 dt + w(r)\tau^1 dx
\]
<table>
<thead>
<tr>
<th>Field Theory</th>
<th>⇔</th>
<th>Gravity</th>
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<tbody>
<tr>
<td>chemical potential $\mu$</td>
<td>$A^3_t = \phi(r) \neq 0$</td>
<td></td>
</tr>
<tr>
<td>$SU(2) \rightarrow U(1)_3$</td>
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<td></td>
</tr>
<tr>
<td>$\langle J^x_1 \rangle \neq 0$</td>
<td>$A^1_x = w(r) \neq 0$</td>
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<tr>
<td>$U(1)_3 \rightarrow \mathbb{Z}_2$, $SO(3) \rightarrow SO(2)$</td>
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<td></td>
</tr>
</tbody>
</table>

- $w(r_{\text{bdy}}) = 0 \Rightarrow$ SSB $U(1)_3 \rightarrow \mathbb{Z}_2$ & $SO(3) \rightarrow SO(2)$

$\Rightarrow$ holographic p-wave superfluid with backreaction

- 5 fields: $\{\phi(r), w(r), \sigma(r), f(r), m(r)\}$
Variation of on-shell action at AdS boundary gives

- energy-momentum tensor

\[ \langle T_{\mu\mu} \rangle \propto T^4 \cdot \text{Func}(m_0^b, f_2^b), \text{ with: } \langle T_{yy} \rangle = \langle T_{zz} \rangle \neq \langle T_{xx} \rangle \]
\[ \langle T_{\mu\nu} \rangle = 0 \text{ for } \mu \neq \nu \]

\[ \langle T_{xx} \rangle = P + \Delta \langle J_1^x \rangle \langle J_1^x \rangle \]

- condensate

\[ \langle J_1^x \rangle \propto T^3 w_1^b \]
Phase transition becomes first order above $\alpha_{\text{crit}}$
Phase diagram

- Numerics not trustable
- RN metastable
- RN unstable
Anisotropic shear viscosity

- viscosity tensor $\eta_{\alpha\beta\gamma\delta}$
- anisotropic systems: 21 components
- isotropic systems: 1 shear viscosity
- transversely isotropic systems: 2 shear viscosities

Holographic calculation: J.E., Kerner, Zeller 1011.5912; 1110.0007
Classification of Fluctuations

- set $k_\perp = 0$

$\Rightarrow$ classification under $SO(2)$ rotational symmetry around x-axis possible:

<table>
<thead>
<tr>
<th>helicity</th>
<th>dynamical fields</th>
<th>constraints</th>
<th># physical modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>helicity 2</td>
<td>$h_{yz}, h_{yy} - h_{zz}$</td>
<td>none</td>
<td>2</td>
</tr>
<tr>
<td>helicity 1</td>
<td>$h_{ty}, h_{xy}; a^a_y$</td>
<td>$h_{yr}$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>$h_{tz}, h_{xz}; a^a_z$</td>
<td>$h_{zr}$</td>
<td></td>
</tr>
<tr>
<td>helicity 0</td>
<td>$h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt}; a^a_t, a^a_x$</td>
<td>$h_{tr}, h_{xr}, h_{rr}; a^a_r$</td>
<td>4</td>
</tr>
</tbody>
</table>

gauge choice $h_{Mr} = 0$ and $a^a_r = 0 \Rightarrow$ 14 physical modes
Transport coefficients from Green functions

One non-trivial helicity 2 mode gives well-known result \( \frac{\eta}{s} = \frac{1}{4\pi} \)

Helicity 1 modes:

- in \( \vec{k} \to 0 \) limit additional symmetry: \( \mathbb{Z}_2: x \to -x, w \to -w \)

\[ \Rightarrow \text{helicity 1 modes decouple in 2 blocks:} \]

- even parity: \( \{ \Psi_t = g^{yy} h_{t\perp}, a^3_\perp, h_{r\perp} \} \)
- odd parity: \( \{ \Psi_x = g^{yy} h_{x\perp}, a^1_\perp, a^2_\perp \} \) \( \Rightarrow \) 3 independent fields: \( \Psi_x, a^1_\perp, a^2_\perp \)

\[ \Rightarrow \text{Green's function: } 3 \times 3 \text{ matrix} \]
Linear response

- choose basis: \( a_\perp^\pm = a_\perp^1 \pm ia_\perp^2 \)

\[ \Rightarrow \text{transform in fundamental repr. of unbroken } U(1)_3 \]

- field theory:

\[
\begin{pmatrix}
\langle J^\perp_+ \rangle \\
\langle J^\perp_- \rangle \\
\langle T^{x\perp} \rangle
\end{pmatrix}
= \begin{pmatrix}
G^\perp_+,- & G^\perp_-,- & G^\perp_{x\perp}^+ \\
G^\perp_-,+ & G^\perp_+,- & G^\perp_{x\perp}^- \\
G^{x\perp}_+ & G^{x\perp}_- & -\langle T_{xx} \rangle - i\omega \eta_{x\perp}
\end{pmatrix}
\begin{pmatrix}
a_\perp^+ \\
a_\perp^- \\
h_{x\perp}
\end{pmatrix}
\]

- with

\[
\eta_{x\perp} = -\lim_{\omega \to 0} \frac{1}{\omega} \text{Im} \left( G^{x\perp,x\perp} \right)
\]
Anisotropic shear viscosity

\[ \eta_{yz}/s = 1/4\pi; \quad \eta_{xy}/s \text{ dependent on } T \text{ and on } \alpha \]

Critical behaviour: 
\[
1 - 4\pi \frac{\eta_{xy}}{s} \propto \left(1 - \frac{T}{T_c}\right)^\beta \quad \text{with} \quad \beta = 1.00 \pm 3\%, \alpha\text{-independent}
\]

Non-universal behaviour at leading order in \( \lambda \) and \( N \)

Critical exponent confirmed analytically in Basu, Oh 1109.4592
Flexoelectric Effect

Nematic phase:
A strain introduces spontaneous electrical polarization

In our case:
A strain $h_{x\perp}$ introduces an inhomogeneity in the current $J^x_1$ which introduces a current $J^\perp_\pm$

J.E., Kerner, Zeller
1110.0007
A magnetic field leads to

\( \rho \) meson condensation and superconductivity in the QCD vacuum

Effective field theory:
(Chernodub)

Gauge/gravity duality
magnetic field in black hole supergravity background
(J.E., Kerner, Strydom PLB 2011)
Condensation in magnetic field

\[ S = \int d^5x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (R - 2\Lambda) - \frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu} \right] + S_{\text{bdy}} \]

\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c \]

\[ A_3^y = xB \]

Fluctuations

\[ 0 = \partial_u^2 E_x^+ + \frac{1}{f} \partial_x^2 E_x^+ + \left( \frac{f'}{f} - \frac{1}{u} \right) \partial_u E_x^+ - \frac{2}{xf} \partial_x E_x^+ + \left( \frac{\omega^2}{f^2} - \frac{B^2 x^2}{f} \right) E_x^+ \]

cf. Chernodub; Callebaut, Dudas, Verschelde; Donos, Gauntlett, Pantelidou
Comparison to field theory calculation

Condensate M. Chernodub

Condensate Gauge/Gravity Duality

Magnetization
Gauge/Gravity Duality
Conclusions

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add backreaction in bottom-up model
- Anisotropic shear viscosity: Non-universal contribution at leading order in N and $\lambda$
- Flexoelectric effect
- Condensation at finite magnetic field
Superfluidity in imbalanced mixtures

Shin, Schunck, Schirotzek, Ketterle, Nature 2008
Imbalanced mixtures

Contain different number of spin up and spin down particles

How does an imbalance in numbers (spin polarization) affect the superfluid phase transition?
QCD at finite isospin chemical potential

He, Jin, Zhuang, PRD 2005
Inbalanced Mixtures and Quantum Phase Transition

Shin, Schunck, Schirotzek, Ketterle, Nature 2008
He, Jin, Zhuang, PRD 2005

Lithium superfluid  QCD at finite isospin density
There appears to be universal behavior.

Can we describe imbalanced mixtures in gauge/gravity duality?

Yes!

Can we obtain a similar phase diagram?

We can, in principle...
Holographic Imbalanced Mixtures

Turn on both isospin and baryon chemical potential

\[ U(2) = SU(2) \times U(1)_B \]

Condensate \( \bar{\psi}_d \gamma_3 \psi_u \) (rho meson)

Increasing \( \mu_B \) turns \( u \) into \( \bar{u} \) quarks
Inbalanced Mixtures and Quantum Phase Transition

J.E., Graß, Kerner, Ngo 1103.4145

Turn on both isospin and baryon chemical potential in D3/D7 setup

\[ \frac{T}{\mu_I} \]

\[ \langle J^1_x \rangle = 0 \]

\[ \langle J^1_x \rangle \neq 0 \]

\[ \Delta = 1 \]

CFT

\[ \frac{\mu_B}{\mu_I} \]

Phase transition second order
Quantum phase transition

Figure by Patrick Kerner
Inbalanced Mixtures and Quantum Phase Transition

Example with backreaction: $SU(2)$ Einstein-Yang-Mills Model
BKT transition in gauge/gravity duality

Jensen, Karch, Son, Thompson 2010
Evans, Gebauer, Kim, Magou 2010

Order parameter scales as \( \exp\left(-c/\sqrt{T_c - T}\right) \)

Gravity side: violation of the BF bound in the IR

IR \( \text{AdS}_2 \times S^2 \) region

Only possible when the two parameters have the same dimension
D3/D7 vs. backreacted model

**D3/D7:**

Effective IR mass of $A^1_x/r$ vanishes, independently of $\mu_B$

BF bound violated along flow, but not in IR

Flavor fields directly interact with each other

**Einstein-Yang-Mills:**

Effective IR mass depends on $\mu_B/\mu_I$

BF bound violated in IR

$AdS_2 \times S^2$ region in IR

Flavor fields interact with gluon fields
Conclusion

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add backreaction in bottom-up model
- Anisotropic shear viscosity: Non-universal contribution at leading order in $N$ and $\lambda$
- Flexoelectric effect
- Add baryon chemical potential: Imbalanced mixtures
- Quantum critical point arising from $\text{AdS}_2$ in IR