Overview

- Pseudoscalar QED-QCD couplings
- CME phenomenology
- Results

M. Asakawa, A. Majumder & BM, PRC 81 (2010) 064912
BM & A. Schäfer, PRC 82 (2010) 057902
QED-QCD pseudoscalar sector

Gluon $E^a \cdot B^a$ interacts with $E \cdot B$ of the electromagnetic field via a quark loop:

Pseudoscalar mesons interact with electromagnetic $E \cdot B$ via a triangular quark loop:

$$\mathcal{L}_{\text{QED}} = \kappa \alpha \alpha_s (E^a \cdot B^a) (E \cdot B)$$

Effective interaction from $\eta, \eta'$ exchange:

$$\mathcal{L}'_{\text{QED}} = \sum_{i=\pi^0, \eta, \eta'} \frac{\alpha}{\pi f_i} \phi_i (E \cdot B)$$

$\kappa \approx \frac{1.46}{\pi^2 f_\eta^2 m_{\eta'}}$

$fi$ is the nonperturbative "meson decay constant"
Anomalous current

Maxwell’s equations dictate that the effective interaction
\[ \mathcal{L}_{\text{QED}} = \mathcal{P} \mathbf{E} \cdot \mathbf{B} \]
gives rise to an anomalous current
\[ \mathbf{j}_{\text{an}} = (\partial_t \mathcal{P}) \mathbf{B} = \sigma_x \mathbf{B} \quad \text{(D’Hoker & Goldstone - 1985).} \]

If \((\partial_t \mathcal{P})\) has a nonzero expectation value, this implies a new kind of conductivity \(\sigma_x\) (the chiral conductivity), which “violates” parity, because \(\mathbf{B}\) is parity-even and \(\mathbf{j}\) is parity-odd.

But even if \((\partial_t \mathcal{P})\) has a vanishing expectation value, nonvanishing fluctuations can exist:
\[ \langle j_i(x) j_k(x') \rangle = \langle \partial_t \mathcal{P}(x) \partial_t \mathcal{P}(x') \rangle B_i(x) B_k(x') \]

Strong \(B\)-fields exist in non-central relativistic heavy ion collisions; maybe the fluctuations of the anomalous current can be observed!

(Kharzeev, McLerran, Warringa ‘07)
Hadronic anomalous current

Vector meson dominance (VMD) relates the electromagnetic hadronic current to the neutral rho-meson field:

\[ j^\mu = -\frac{e m^2}{g_{\rho \pi \pi}} \rho^\mu \]

Thus, to generate an “anomalous” current, the B-field needs to convert a pseudoscalar meson into a rho-meson (Asakawa, Majumder, BM ’10):

\[(\gamma \pi \pi \pi \pi \text{ coupling})\]

\[ j(x) \otimes \rho \rightarrow \pi \]

\[ B \]

The relevant interactions are well known from radiative decays:

\[ \Gamma_{\rho^0 \rightarrow \pi^0 \gamma} = 3\alpha g_{\rho \pi \gamma}^2 \frac{p_{\text{cm}}^3}{m_\rho^2} \approx 90 \pm 12 \text{ keV} \]

\[ \Gamma_{\eta' \rightarrow \rho \gamma} = \alpha^2 g_{\rho \eta' \gamma}^2 \frac{p_{\text{cm}}^3}{m_{\eta'}^2} \approx 60 \pm 5 \text{ keV} \]

Suppressed, if \( B \) is constant, but not if \( B \) is strongly time dependent.
CME effective action

True CME = Anomalous current induced by winding number change in B-field:

\[ \tilde{j} = \sigma \chi \tilde{B} \quad \text{with} \quad \sigma \chi = \frac{3e^2 n_5}{2\pi^2 T^2} = -\sum_f \frac{3e_f^2 g^2}{16\pi^4 T^2} \int dt \left( \tilde{E}^a \cdot \tilde{B}^a \right) \]

Compare with Ohm’s law: \( \tilde{j} = \sigma \tilde{E} \)

\[ L^{(Ohm)}_{\text{eff}} = -\int \langle \tilde{j} \rangle \cdot d\tilde{A} = \frac{1}{2} \sigma \int_{-\infty}^{t} d\tilde{t} \tilde{E}(\tilde{t})^2 \quad \Leftrightarrow \quad \tilde{L}_{\text{eff}} = \frac{i\sigma}{2\omega} |\tilde{E}(\omega)|^2 \]

Nonlocal (in time) effective action for CME:

\[ L^{(CME)}_{\text{eff}} = -\sum_f \frac{e_f^2 g^2}{8\pi^4 T^2} \int_{-\infty}^{t} dt' \left( \tilde{E} \cdot \tilde{B} \right) \int_{-\infty}^{t'} dt'' \left( \tilde{E}^a \cdot \tilde{B}^a \right) \]
CME phenomenology
Requirements

- **Strong B-field:**
  - Available in heavy ion collisions
  - What are the relevant time and length scales?

- **Pseudoscalar QCD source:**
  - Chern-Simons number fluctuation
  - 2-gluon scattering in the pseudoscalar channel
  - Pseudoscalar meson condensate domain
  - Pseudoscalar meson excitation
  - What are the relevant length and time scales?
Mechanisms

- **CGC mechanism:** Two gluons from the initial nuclei fuse in the pseudoscalar channel and generate an anomalous current in the strong magnetic field;

- **Glasma mechanism:** Gluons in the “glasma” generate an anomalous current in the strong magnetic field via a winding number fluctuation;

- **QGP mechanism:** Gluons in the equilibrated quark-gluon plasma generate an anomalous current in the strong magnetic field via a winding number fluctuation (“sphaleron”);

- **Hadronic mechanism:** A $\pi^0$ in the hadronic gas phase generates an anomalous current by converting into a $\rho$-meson in the strong B-field. Effect can be amplified by local $\pi^0$ or $\eta$ ($\eta'$) condensate;

- **Corona mechanism:** A $\pi^0$ in the hadronic corona generates an anomalous current by converting into a $\rho$-meson in the strong B-field.
Time scales

Life-time of the strong magnetic field: \( \tau_B \approx 2R/\gamma \approx 0.1 - 0.2 \text{ fm/c.} \)

Time scale for winding number transitions: \( \tau_{\text{sph}} \approx 1/T \approx 0.5 - 1 \text{ fm/c.} \)

Time scale of hadronic pseudoscalar interactions: \( \tau_{\text{had}} \approx 1/m_{\eta'} \approx 0.2 \text{ fm/c.} \)

Weak, long lived B-field component due to baryon stopping and “frozen” B-fields
Length scales (B-field)

Lumpy nuclear charge distributions in individual events $\implies$ nonzero but randomly oriented B-fields even in central collisions.

Magnetic field domains are larger than coherence length of pseudoscalar QCD observables $\implies$ “homogeneous” B-field

A. Bzdak & V. Skokov, PLB 710 (2012) 171
Vector boson condensation?

\[ E^2 = k_z^2 + (2n + 1 - 2S_z)eB + m^2 \]

in homogeneous B-field

\[ E^2 < 0 \text{ for } S_z = 1, \ n = 0 \text{ and } eB > m^2 + k_z^2. \rightarrow \rho, \ W \text{ condensation?} \]

Ambjorn & Olesen, PLB 257 (1991) 201

But strong B-field is highly localized: \( \Delta z, \Delta t \approx R/\gamma \):

\[ \int Bdz \approx \int Bdt \approx \text{indep of } \gamma. \]

B-field can be idealized as \( b_0\delta(z) \) “magnetic sheet”

Exact Green function can be obtained: no vector boson condensation!
Reason: Landau level only partially interacts with magnetic field.

The anomalous current is not directly observable. What is observed is the final charged particle distribution and its asymmetry with respect to the reaction plane:

$$\Delta Q = \int d^3p \int_{\Sigma_f} \frac{d\sigma_{\mu}p^\mu}{E} \sum_i e_i f_i(x, p) \text{sgn}(p_z)\theta(\sigma_{\mu}p^\mu)$$

Transformation of spatial charge asymmetry into a momentum space asymmetry requires either **collective flow** or **opacity** during freeze-out (or both):

Charge asymmetry is created early; it must be transported to the freeze-out surface. Locally separated charged be depleted by diffusion processes.

Altogether, it is a complex transport problem, but one describable with current technology (viscous hydro + diffusion & hadron cascade).
Two ways to proceed:

1. For isochronous freeze-out, a position-momentum correlation requires collective flow.

\[ \Delta Q = \int d^3x \int d^3p \sum_i e_i f_i(x, p; \tau_f) \text{sgn}(p_z) \]

where

\[ f_i(x, p; \tau_f) = \exp \left[ -u_\mu(x)p^\mu/T_f + e_i \mu_Q(x)/T_f \right] \]

For weak flow, expand \( f(x,p) \) in first order in \( \mathbf{v} \). For charged pions only one finds the simple result:

\[ \Delta Q \approx \frac{3}{2} \int d^3x \rho(x, \tau_f) v_z(x) \]

then use the continuity equation \( \partial_t \rho = -\nabla \cdot \mathbf{j}_{\text{an}} \) to obtain:

\[ \langle (\Delta Q)^2 \rangle \approx \frac{9}{4} \int_0^{\tau_f} dt \int dt' \int d^3x \int d^3x' \]

\[ \times \nabla v_z(x, \tau_f) \cdot \langle \mathbf{j}_{\text{an}}(x, t) \mathbf{j}_{\text{an}}(x', t') \rangle \cdot \nabla' v_z(x', \tau_f). \]
2. In the geometric approximation, one simply assumes that all charges in the upper hemisphere are emitted upwards, and vice versa:

\[
\Delta Q = \int_{z>0} d^3x \, \rho(x) = \int d^4x \, \delta(z) j_z(x)
\]

Predictions using the two approaches differ by a factor \(\sim 30\) (#1 is smaller)!

The weak flow approximation differs from the geometric approximation by a factor

\[
\Theta = \frac{\langle (\Delta Q)^2 \rangle_{\text{flow}}}{\langle (\Delta Q)^2 \rangle_{\text{geo}}} \approx v_f^2 \xi_j / R
\]

where \(\xi_j\) is the anomalous current correlation length.

For \(\xi_j \approx 1\) fm, \(v_f \approx 0.5\) and \(R \approx 7\) fm, one has \(\Theta \approx 0.035 \Rightarrow \text{large uncertainty}\).
Results
This mechanism can be enhanced if the $\eta'$ mass is lowered by medium interactions.

Because CGC color fields are nearly transverse, the fields in $\mathbf{E}^a \cdot \mathbf{B}^a$ must come from different nuclei. The gluon matrix element can be expressed in terms of the nuclear gluon distribution:

$$\langle j^3(x) j^3(x') \rangle \approx e^2 C \langle (\mathbf{E}^a \cdot \mathbf{B}^a)(x)(\mathbf{E}^b \cdot \mathbf{B}^b)(x') \rangle$$

with

$$C = \frac{g_{\rho\eta'\gamma}^2}{g_{\rho\pi\pi}^2} \frac{3(Z\alpha)^2\alpha_s^2 \cos^2 \theta}{(2\pi f_\eta)^2 m_\eta^2 m_\rho^2} \frac{b^2 \gamma^2}{R^6}$$

$$\langle (\mathbf{E}^a \cdot \mathbf{B}^a)(\mathbf{E}^b \cdot \mathbf{B}^b) \rangle \propto [x_0 G(x_0)]^2 T_{AA}(\vec{x}_\perp; b)$$

This mechanism can be enhanced if the $\eta'$ mass is lowered by medium interactions.
The nuclear gluon density can be related to the CGC saturation scale $Q_s$ by:

$$A[\xi_0 G(\xi_0)] = \frac{(N_c^2 - 1) R^2 Q_s^2}{8\pi^2 \alpha_s}$$

To calculate the up-down charge asymmetry fluctuations, we calculate the current through the reaction plane and assume that all charges above are emitted upwards and all charges below are emitted downwards (an overestimate!). After a lengthy calculation involving various additional “reasonable” approximations one finds:

$$\langle (\Delta N_{\text{ch}})^2 \rangle = \begin{cases} 
C \frac{9(N_c^2 - 1)v_f^2}{(8\pi)^3 \alpha_s^2} Q_s^2 f(b) \approx 5 \times 10^{-5} v_f^2 \frac{b^2}{R^2} f(b) & \text{(weak flow)} \\
C \frac{3(N_c^2 - 1)}{32\pi^4 \alpha_s^2} Q_s^3 R f(b) \approx 1.7 \times 10^{-3} \frac{b^2}{R^2} f(b) & \text{(geometric)}
\end{cases}$$

with $f(b) \approx 1 - (b^2/R^2)(1 - b/4R)^2$
CME effective action

\[ L_{\text{eff}}^{(\text{CME})} = - \sum_f \frac{e_f^2 g^2}{8\pi^4 T^2} \int_0^{\tau_B} dt' (\vec{E} \cdot \vec{B}) \int_0^{\tau_{\text{sph}}} dt'' (\vec{E}^a \cdot \vec{B}^a) \]

Minimize action:

\[ L = \frac{1}{2} f(B) E^2 + L_{\text{eff}}^{(\text{CME})} \Rightarrow eE_{\text{min}} \]

Implies momentum shift:

\[ \Delta p = eE_{\text{min}} \tau_B \]

Thermal spectrum with transverse flow:

\[ E \frac{d^3 N}{dp^3} \sim \exp \left( -\gamma [E - \vec{v} \cdot (\vec{p} - \Delta \vec{p})] / T_f \right) \]

Asymmetry:

\[ \frac{dN^{(+)}}{dN^{(-)}} \approx e^{2 \sqrt{\tau_B eE_{\text{min}}/T_f}} \Rightarrow \Delta^\pm = \frac{dN^{(+)}/dT_f - dN^{(-)}}{dN^{(+)}/dT_f + dN^{(-)}} \approx \frac{\tau_B \sqrt{eE_{\text{min}}}}{T_f \sqrt{N \text{ domains}}} \]
CME estimate

Using \( \tau_B |eB| \approx \frac{2.3Z\alpha b}{R^2} \) gives \( |eE_{\text{min}}| \approx \frac{18.4Z\alpha^2 b \left| Q_5 \right|}{\pi f(B)T^2 R^2 V} \)

Assume: \( |Q_5|/V = \rho^{-3} \approx (0.5 \text{ fm})^{-3} \), \( T = 350 \text{ MeV} \), \( Z = 79 \), \( f(B) = 1 \):

\[
|eE_{\text{min}}| \approx (20 \text{ MeV})^2 \left( \frac{b}{R} \right)
\]

Further: \( \tau_B = 2R/\gamma_{\text{cm}} \approx 0.15 \text{ fm/c} \), \( v = 0.5 \text{ c} \), \( T_f \approx 150 \text{ MeV} \)

Number of domains: \( N = \pi R^2/\rho^2 \approx 600 \)

Final result: \( \Delta^\pm \approx 3.5 \times 10^{-6} \frac{b}{R} \) (compare with \( \Delta^\pm_{\text{exp}} \leq 6 \times 10^{-4} \))
“To Do” list

- Other experimental observables need to be studied, e.g.
  - E-by-E charge dipole.
- Beam energy dependence?
  - maximal $B$ is proportional to energy, but time-integrated $B$ is constant;
- System size dependence?
  - Central U+U can have $v_2 > 0$, but $B = 0$.
- Comprehensive overview of all experimental investigations needed
- If CME is much smaller than STAR effect, how small an effect could be seen with the most sensitive observable?

- Theoretical studies:
  - Realistic calculations of all mechanisms and backgrounds, or at least improved estimates, are needed.
  - Technically feasible, but challenging.