Chiral Symmetry and the Thermodynamics of the Dyonic Vacuum

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Introduction
History of Calorons with non-trivial holonomy

- 1998 Kraan, van Baal and separately Lee and Lu - Discovery of calorons with non-trivial holonomy
- 2004 D. Diakonov, N. Gromov, V. Petrov, S. Slizovskiy - Quantum weight of dyons and of instantons with nontrivial holonomy
- 2007 D. Diakonov and V. Petrov argue that the so called “confining holonomy” is preferred
- 2009 V. G. Bornyakov, E.-M. Ilgenfritz, B. V. Martemyanov, M. Muller-Preussker - Quenched lattice studies of the effect of dyons
What are (SU(2)) instanton Dyons?

- Dyons are generalizations of instantons at finite $T$ (caloron).
- In addition to instanton parameters (size, position, and color orientation), they possess so-called holonomy:

$$\mathcal{P} e^{\int_0^\beta A_4 dt} = e^{iv\beta \tau^3 / 2}$$

- Once $v \neq 0$ the instanton can split into two stationary dyons.
- One says that it is trivial if $v\beta = 0$ and nontrivial if $v\beta = \pi$ (sometimes referred to as confining and non-confining).
- Hence one can view nontrivial holonomy as an appearance of a “$A_4$ condensate”.
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\[ F^2 \propto [A_4^a, A_i^b]^2 \rightarrow v^2 A_i^2 \epsilon^{abc} \tau_c \]

Gluons which have different color direction then \( A_4 \) condensate obtain an effective mass

**Monopole Solution!**

However since we can always change \( v \rightarrow v - 2\pi T \) by an anti-periodic gauge transformation (which keeps gauge fields periodic), there are actually two monopole solutions! It turns out that the second (twister) monopole has its action proportional to \( \bar{v} = 2\pi T - v \).
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Dyons in Vacuum
The Dyons in Vacuum
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- Classical
- One loop gluon - perturbative
- Fermion
- Moduli space interaction
The partition function for a single neutral molecule

Partition Function for a Dyon Molecule

\[ dZ_{mol} = dZ_{LM}dZ_{\bar{L}\bar{M}} \left[ \frac{m_f^2 + |T(r_{L\bar{L}})|^2}{\Lambda^2} \right]^{N_f} \left( C(N_f) \left( \frac{\pi^2 r_{LM}r_{\bar{L}\bar{M}}\Lambda^4}{T^2} \right) \right)^{N_f/6} e^{-V_{scr} - V_{L\bar{L}}} \]
Dyon Molecule

\[ \lambda_D = \frac{3}{\pi T (2N_c + N_f)} \]

\[ \bar{v} = 2\pi T - v \]
Three Molecular Models
Three Molecular Models

1. Random Dyon Gas Model
2. Random Molecular Model
3. Reweighed Molecular Model
The Random Dyon Gas Model
Random Dyon Gas Model

- Take N dyons and antidyons which carry zeromodes (i.e. $L, \bar{L}$ dyons for anti-periodic fermions)
- Randomly select their positions in a box of size $L$ so that density $N/V = N/L^3 = 1$
- Taking that $\mathcal{D} \approx T_{I\bar{J}}$, where $T_{I\bar{J}}$ is the Dirac operator in the basis of quasi zeromodes, i.e.

$$T_{I\bar{J}} = \int d^3x \bar{\psi}_I \mathcal{D} \psi_{\bar{J}} \approx \int d^3x \bar{\psi}_I \partial \psi_{\bar{J}}$$

- Zeromodes $\psi_{I,\bar{I}} \propto \frac{e^{-\bar{v}r}}{\sqrt{r}}$ at large $r$ therefore we take that

$$T_{I\bar{J}} = c \frac{e^{-Mr}}{\sqrt{1 + rM}}$$

where $M$ is connected with holonomy (i.e. $M = \nu/2$ or $M = \bar{\nu}/2$).
The Random Molecular Gas Model
The Random Molecular Gas Model
The Random Molecular Gas Model
The Random Molecular Gas Model
The Random Molecular Gas Model
The Spectrum

eigenvalue in units of $c$
The Random Molecular Gas Model
The Random Molecular Gas Model

- Take $N$ pairs of dyons and antidyons which carry zeromodes (i.e. $L$, $\bar{L}$ dyons for anti-periodic fermions)
- Randomly select the position of these pairs positions in a box of size $L$ so that density $N/V = N/L^3 = 1$
- The distribution of the relative distance between the members of the pair is given by

$$\text{distribution} = r^2 \left( \frac{e^{-Mr}}{\sqrt{1 + Mr}} \right)^{2N_f}$$
The Random Molecular Gas Model
The Random Molecular Gas Model
Reweighted Random Molecular Gas Model
What Exactly Happens?
What Exactly Happens?
Prediction!

\[ n N_f^3 = \text{const} \]

\[ C_1 e^{-\frac{8\pi^2}{g_1^2}} N_f^1 N_f^3 - C_2 e^{-\frac{8\pi^2}{g_2^2}} N_f^2 N_f^3 = 0 \]

\[ \frac{8\pi^2}{g_2^2} = \frac{8\pi^2}{g_1^2} - 3 \log \frac{N_f^2}{N_f^1} - \log \frac{C_2}{C_1} \]
Diakonov, Petrov proposed a guess for the self-dual metric for dyons. This consists of the determinant of attractive Coulombic terms between $L - M$ dyons and repulsive terms between $L - L$ and $M - M$ dyons. This suggests that dyons may “condense” into a crystal under certain circumstances.
Diakonov Moduli Space Metric

a)  
b)  
c)  

- L dyon  
- M dyon  
- L̅ dyon  
- M̅ dyon
Stability of the Crystal
We propose some of the following tests:

- Looking for the "dyonic molecule" at $T > T_c$
- Looking for correlations between Polyakov loop (it takes values $\pm 1$ at $M, L$ (anti)dyons)
- Looking at fermionic zeromodes with periodic and antiperiodic boundary conditions
- $n \equiv n_3^f = \text{const.}$, where $n$ is proportional to topological charge density
- Topological susceptibility measurements
- Counting fermionic zeromodes
- Crystal-like correlations
- Diakonov crystal-like next-neighbor correlations below $T_c$
- Hexagonal-densly packed "crystal" correlations for $N_f \neq 0$ between $L - \bar{L}$
Summary of Predictions

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Outlook

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- Compute free energy dependence on holonomy parameter (adjoint and fundamental)