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Introduction to *ATIS* and Its Applications to SimEd

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Introduction to *ATIS* and Its Applications to *SimEd*

SimEd is a software program designed to simulate educational systems as intentional systems and thereby the behavior of such systems.

ATIS (*Axiomatic Theories of Intentional Systems*) is a theory model options-set that is designed to construct theory for intentional systems. In particular, it is used to develop predictive theories and technologies of the behavior of intentional systems. *SimEd* is one of those predictive technologies.

The systems with which *ATIS* is concerned are those that have large populations such as educational systems, military systems, large corporate systems, terrorist and criminal enterprise systems, large employee recruitment systems, etc. It is for this reason that *ATIS* is uniquely applicable to the design of *SimEd*.

What do we envision when we say that behavior is “predictive”?

Most often it is claimed that individual behavior is not predictable because we lack certainty of outcomes. While that is true, is this really what is desired when we attempt to “predict” what will happen? There are several ways that we make predictions. One of the more common ways to predict an event is to base it on the probability of its occurrence. But, do such analyses really “predict” occurrences?

Horse racing, poker, football games, and many other event outcomes are predicted with a reliance on the “odds” that the outcome will occur. The weather is forecast in a similar manner where we have a “30% chance of rain.” However, in all of these cases, whether we “win” or “lose” we cannot become too concerned since we really do not have any “control” over the outcome. These are not the type of predictions that concern us here. In fact, one of the basic problems with statistical-based predictive outcomes is the lack of control of that outcome—there is no way to change the outcome, other than “fixing” the race, which nullifies the “odds.”

There is an important exception to the above examples that use statistical-based analyses to predict outcomes. When there is an important social issue that impacts peoples’ lives, then statistical analyses take on a very important significance. For example, if, as it has been determined, there are a significant number of innocent individuals on death row in our prisons, and it is determined that 33% of those there are in fact innocent, then this analysis is in fact predictive of the number of innocent individuals still remaining on death row who have been convicted under similar circumstances. While we cannot necessarily predict with certainty which individuals are innocent, we can predict as though it is fact that 33% of those are. While this analysis remains a group prediction, it is a prediction of such value that decisions relating to such systems should rely

on the predictability of this analysis. In these type of statistical-predictive outcomes, there is one means of control—remove all prisoners from Death Row.

As will be shown in the development of *ATIS*, where, as above, a statistical analysis is of significant value, it can be brought into *ATIS* as a *qualifier* of the system. Its usefulness is in terms of providing filters and regulators for the system.

However, the type of behavior predictions that are our main concern are those that are very significant to us either in terms of greatly affecting our security, as in terrorist threats or military decisions; or our economic and social future, as in educational systems; or in the development of our economic life as in the development of a business. Within these systems there are certain conditions where behavior predictability is quite certain. These we now consider.

Predicting Extreme Behaviors

It is commonly believed that individual behavior, and even group behavior, is not predictable. Air Force Colonel John A. Warden, III, in his research *The Enemy as a System*, Air University, (Warden III, Colonel John A., “The Enemy as a System,” *Air and Space Power Chronicles*, Spring 1995, No. 1, pp. 40-55) reinforces this conclusion several times. In general, he is quite accurate when behavior is viewed as the day-to-day activity of an individual. Such intentional systems are non-predictable to the extent that people are intentional beings who can change their intentions from one minute to the next. However, to the extent that an individual is habitual, behavior may be very well defined and predictable. A similar predictor of behavior is offered by Icek Ajzen and Martin Fishbein who developed the *Theory of Reasoned Action* and the *Theory of Planned Behavior*. Their contention is that behavior is essentially derived from intention and by knowing the intention of an individual, then that individual’s behavior is predictable.

But, our concern is not with the whims of an individual, but with the planned activity of an intentional organization. In these cases, the fact that people are intentional implies that they **are** predictable. The “Stockholm Syndrome,” “mass panic,” and “crowd mentality” indicate that within certain social systems individual behaviors may be predictable. Also, it may be found that social systems created for the purpose of achieving certain goals will result in systems where individual behaviors are predictable. In such systems, the individual behavior is that which furthers the intention of the larger system. As a result, the individual behavior is predictable.

When considering the planned activity of an intentional organization, Col. Warden cites specific instances of behavior-predictive activity. He states that by inflicting severe physical damage, *strategic paralysis* can be attained; that is, conditions can be attained that will make it physically impossible for an enemy to oppose us or to maintain any predator capability. Such an outcome is predictable and does in fact predict the behavior of an intentional system. There are two extreme conditions that will result in the ability to predict behavior:

(1) We can predict with great certainty that if we annihilate the enemy, the enemy's behavior will no longer be a threat. Or, if we eliminate an individual from a learning program, the individual will not learn the program outcomes and will not directly affect the activity of the program. Or, if we destroy a disease, then the disease will no longer be a threat to our health. Polio is no longer a threat, the plague is no longer a threat, etc.

(2) We can predict with great certainty that if we do nothing, the enemy's behavior will not change and will remain a threat in the same manner that it is currently threatening, and that the threatening behavior will in fact increase in severity over time. This is exemplified by the actions of Hitler that led to WWII. Or, we can predict that if we do nothing to control disruptive learning behavior, then the individual will not learn and the behavior will continue to disrupt the ability for others to learn. Or, if we do not treat a disease that has attacked our body, and our immune system is unable to fight the assault, then the disease will overwhelm us, resulting in death.

The question, then, is not whether behavior is predictable, but what is the degree and nature of that predictability?

Predicting Behaviors

Let us first consider the assertion that individual behavior is not predictable because people are intentional beings and we cannot predict their intention from one minute to the next. Icek Ajzen and Martin Fishbein would in fact argue the contrary; that is, it is due to the very fact of the "intention" that a person is predictable, since intention determines behavior. Further, the claim that intention precludes predictability is a spurious argument. The reason is that predictability is dependent on *total system structure*. In fact, the argument that behavior is not predictable because "intentions may change" argues in favor of predictability. That is, apparently if the "intention" did not change, then our "prediction" would be accurate. The change of "intention" is a change of total system structure. The changed system structure results in a new prediction of behavior, presumably the behavior that we would have predicted upon discovering the new intention.

Once again, the question is not whether behavior is predictable, but what are the conditions for that predictability, and what standard of "accuracy" is required?

First, the systems with which we are concerned are organizations in which decisions have to be made in order to improve student instruction within an education system, or predict threats and targets of a terrorist network system, etc.

The question is not whether we can predict with 100% accuracy, the question is whether we can predict with a reasonable certainty so that appropriate decisions can be made with respect to

the target organization, although, as will be seen, certain behaviors are absolutely predictable within these systems.

So now the question reverts to the nature of that predictability. While people are intentional beings, it is here proposed that total system structure (not only system state, which is system property-derived) determines subsequent behavior, and it is because people are intentional beings that system structure does in fact determine subsequent behavior. The validity of this premise will be established by the validation of predictions that are made.

The distinction between *system state*, the only system description normally used when evaluating systems, and *total system structure* is defined as follows:

- Affect relations determine system properties.
- System properties determine *system state*.
- System state determines the system-descriptive axiom set.
- The axiom set provides logic-based predictive outcomes.
- Affect relations also determine system topological structure in that every affect relation defines a topology.
- Topological structure provides dynamic, real-time predictive outcomes.
- System-descriptive axiom set and system topological structure together determine *total system structure* and system-predictive outcomes.
- The logical analysis as dependent upon the system axioms comes first, followed by a topological analysis that establishes in fact the vectored system outcome; that is, that the system is actually taking the path indicated by the logical analysis.

We will now consider two different types of predictions—*System Axiomatic Predictions*, and *Anticipatory Predictions*.

System Axiomatic Predictions

System Axiomatic Predictions are those predictions that can be made as a direct result of the system-descriptive axiom set. These are predictions that result in complete accuracy since the alternative would be contrary to the underlying axioms. For example, imposing strategic paralysis is a *System Axiomatic Prediction*, since the outcome is absolutely certain. As will be considered later, any result obtained directly from the axioms is a *System Axiomatic Prediction*, since any alternative outcome would contradict the axioms.

In a broader context, this predictive strategy is distinctly different from the data-mining strategies that attempt to extract structure from an unstructured database. As an alternative, we have the *ATIS Axiomatic Prediction Principle*:

ATIS provides the system structure into which new data is integrated, and thereby gives rise to new structure that compels the recognition of a resulting action as determined by the new system parameters that if not produced would be inconsistent with *ATIS* axioms.

Stated another way, structure determines possible and intended system action:

System Structure compels the recognition of intended action determined by system parameters that if not produced would be inconsistent with *ATIS* axioms.

Each new introduction of affect relations into a system defines an *Induced System Structure* that compels recognition of new intended action.

Anticipatory Predictions

Another type of prediction with which we are concerned will be identified as *Anticipatory Predictions*. There are no claims that such predictions will be 100% accurate when empirically tested. But, an *Anticipatory Prediction* will be considered to have been validated if the outcome is either accurate, is accurate within acceptable well-defined tolerances, or can be explained by changes that occurred between the time the system structure was evaluated and the outcome was observed.

In fact, *Anticipatory Predictions* account for “changed intentions.” That is, an outcome that is contrary to that predicted but was obtained as the result of “changed intentions” remains an accurate outcome if, in particular, the actual outcome would have resulted had the changed intentions been known.

The premise of *ATIS* is that *total system structure* will provide the most accurate predictive capability. Further, predictability is not the result of system dispositional behavior, nor the result of any statistical inference obtained from prior behavior. Dispositional behavior and prior states provide an invariant structural base against which to analyze current data, but do not result in behavior predictability. Anything other would result in a deterministic or mechanistic type system, rather than the dynamic teleological type system here contemplated.

Further, the system structure provides a basis for analysis, but the resulting behavior prediction is a result of a logical as well as topological analysis, it is not simply the result of a single algorithm derived from a logical schema, or a single topological analysis derived from the affect relations.

ATIS for SimEd

SimEd, as designed and developed by Theodore W. Frick, Associate Professor and Web Designer, Indiana University, is a software program designed to simulate educational systems behavior. Any behavior-predictive software must be founded on a logical base of some kind. Frequently, or almost exclusively, such logics are founded on scenario-based programs that are discussed below.

As will be discussed in this report, *SimEd* is a *model-of* an education system. It is designed so that selected parameters can be evaluated to determine projected outcomes in view of these parameters. It is a computer simulation of an educational system.

As with any computer simulation, the predictive outcomes obtained from *SimEd* are dependent on the program that is used to analyze the selected input parameters. There are essentially two types of programs that can be applied—Logic-Based and Scenario-Based.

Most often, and especially for the “Sim” models, a Scenario-Based Model is used. Such models are dependent on the imagination of the designer and comprehensiveness of the data included in the program.

Scenario-Based programs are defined as programs that provide “scripts” to determine outcomes. The scripts can be narrative or quantitative. Narrative scripts characterize the qualitative parameters of a system; that is, the social, philosophical, and individual descriptions and the uncertainty of future outcomes. Quantitative scripts define the scientific facts, known or credible data, and quantitative models that are used to determine future outcomes.

However, regardless of the type of script, their content is closed; that is, there are a limited number of possible outcomes, and the scripts predetermine the outcomes. Ted Friedman recognizes this closed characteristic of Scenario-Based Models in his report on SimCity, “The Semiotics of SimCity,”¹ when he states:

Of course, however much "freedom" computer game designers grant players, any simulation will be rooted in a set of baseline assumptions. SimCity has been criticized from both the left and right for its economic model. It assumes that low taxes will encourage growth while high taxes will hasten recessions. It discourages nuclear power, while rewarding investment in mass transit. And most fundamentally, it rests on the empiricist, technophilic fantasy that the complex dynamics of city development can be abstracted, quantified, simulated, and micromanaged.

¹ “The Semiotics of SimCity,” *First Monday*, Peer-Reviewed Journal on the Internet, 1999, http://www.firstmonday.dk/issues/issue4_4/friedman/.

On the other hand, Logic-Based Models are not dependent on analyses of predetermined values, but on the logic of a theory that has been shown to be valid for the target empirical system; for example, an education system. The theory describes the empirical system in terms of its affect relations, properties, and axioms. The theory logic is then used to project outcomes founded on the theory with respect to input parameters.

Unlike Scenario-Based Models that are closed due to the limited number of scripts, Logic-Based Models essentially have an infinite number of outcomes. This is especially so with the logic designed for the *ATIS Model*. Due to the number of axioms involved, there are initially tens-of-thousands of theorems that can be obtained. However, the SCTs (Structural Construction Theorems) provide an open-ended number of additional theorems. The reason is that new properties, system-descriptive parameters, or affect relations can be inserted into the SCTs that will automatically generate thousands of additional theorems. The Logic-Based Model is not dependent on what has been initially programmed for the logic, but what is subsequently programmed as a result of new system parameters.

The strength of a Logic-Based Model will be seen in what follows, and additional analyses relating to the two types of models will be discussed.

Elizabeth Steiner has explicated concepts of ‘theory’ and ‘model’ in great detail. She has also explicated the relation between ‘theory’ and ‘model’. We will start with her analyses².

It is commonly believed that developing theory is derived from one of two logical processes—induction or deduction. Theory development, however, is not derived from either.

Induction brings together many observations from which it is claimed a generalization, or pattern, can be obtained. Such, however, is not the case. It is clear that in order to obtain the pattern, the observer must be able to recognize the pattern, and therefore has brought the generalization to the data. The data does not induce the generalization, the observer does.

This is a mode of inquiry, but it is not theorizing. Induction provides the basis for data-mining technologies that are widely used to develop structure from unstructured data. The structuring of unstructured data is not theorizing, but is neither more nor less than a classificatory methodology. Induction is a means of evaluating data so as to recognize and develop patterns,

² See *Methodology of Theory Building*, Elizabeth Steiner, Indiana University, Educology Research Associates, Sydney, 1988. This text is an extension of her earlier work that includes the following: *The Model in Theorizing and Research*, Elizabeth Steiner Maccia, Educational Theory Center, The Ohio State University, May, 1965; *Sources of Empirical Theory*, Elizabeth Steiner Maccia, The Ohio State University, May, 1964; *Theory Construction and Models*, Elizabeth Steiner Maccia, The Ohio State University, May, 1964; *The Conceptions of Model in Educational Theorizing*, Elizabeth Steiner Maccia, Center for the Construction of Theory in Education, The Ohio State University, August, 1962; *Models and the Meaning of ‘Retroduction’*, Elizabeth Steiner Maccia, Foundations Division, The Ohio State University, June, 1962; and *The Way of Educational Theorizing Through Models*, Elizabeth Steiner Maccia and George S. Maccia, Foundations Division, The Ohio State University, June, 1962.

patterns that may validate hypotheses but not devise them. Patterns are not theory. Theory is otherwise developed and then the data validates the theory, a process of induction.

Deduction is a means of explicating existing theory; that is, “to clarify and complete theory” [Steiner, *Models and the Meaning of ‘Retroduction’*, Foundations Division, The Ohio State University, June, 1962]. The theory presents the postulates or axioms upon which the theory is based. Deduction then provides the logical process by which the theory is made explicit through its deductive statements. Deduction generates conclusions of the theory in the form of theorems or hypotheses that are to be evaluated for validity.

Deduction explicates a theory into statements, and induction evaluates the statements.

What then is the logical process by which theory is developed? Retroduction.

‘Retroduction’ is a “moving backward.” From one perspective we move backward to devise another perspective.

For example, the “Holographic Paradigm” may provide a perspective for education. Considering that a holograph can be generated from any of its facets, it may suggest that a student may learn, not by focusing on the subject of concern directly, but by developing coordinated skills in a discipline not normally considered as being directly relevant. But, the “parts” of the divergent discipline may have “facets” that in fact produce the entire “hologram” in the subject of concern. For example, one may learn how to interpret historical events by taking acting lessons whereby the skill is developed that allows one to become immersed in a period thinking and lifestyle.

In terms of theory development, one theory that can be used as a model for developing another theory is a “devising theory.”

Retroduction is the process of using one theory as a model to devise another theory.

Therefore:

- Retroduction devises theory,
- Deduction explicates theory, and
- Induction evaluates theory.

‘Retroduction’ and ‘Abduction’ Confusion

While retroduction will be used in this research to develop education theory from the SIGGS Theory Model as extended by the *ATIS Model*, a distinction needs to be made concerning the use of the terms ‘retroduction’ and ‘abduction’.

It is recognized that *retroduction* is normally, if not universally, defined as *abduction*. It is suggested, however, that such is in error. First, there is a recognizable distinction between a “taking from,” *abduction*, and a “moving backward,” *retroduction*.

It is presumed that Peirce generally defines ‘abduction’ and ‘retroduction’ as the same, although a careful reading indicates that he does not. An analysis of this confusion is worth considering due to its almost universal acceptance.

The *‘retroduction’ and ‘abduction’ confusion* seems to have come from the work edited by Charles Hartshorne and Paul Weiss, *Collected Papers of Charles Sanders Peirce*.³ For example, if one goes to the index for Volume 1, the reference for ‘retroduction’ is: “see Abduction.” The implication is that they are the same. But, when we look at the first reference for ‘abduction’, §65, we find that the two are not the same at all. Peirce writes:

§10. KINDS OF REASONING

65. There are in science three fundamentally different kinds of reasoning, Deduction (called by Aristotle συναγωγή or αναγωγή), Induction (Aristotle’s and Plato’s έπαγωγή) and Retroduction (Aristotle’s άπαγωγή), but misunderstood because of corrupt text, and as misunderstood usually translated *abduction*. Besides these three, Analogy (Aristotle’s παραδειγμα) combines the characters of Induction and Retroduction.

It should be clear that ‘retroduction’ and ‘abduction’ are not the same, and that they have been equated only because of “corrupted text.”

So, what is the distinction between ‘retroduction’ and ‘abduction’? Consider the following example.

A *direct affect relation* in a behavioral topological space is defined in terms of a *mathematical vector*. That is, it is recognized that the concept of “vector” is applicable to this behavioral theory. This transition was recognized as a result of affect relations being interpreted as “force fields.”

Gravitational and electromagnetic force fields are vector fields; fluid velocity vectors, whether in the ocean or the atmosphere, are vector fields; and weather pressure gradients are vector

³ Hartshorne, Charles and Paul Weiss, *Collected Papers of Charles Sanders Peirce*, The Belknap Press of Harvard University Press, Cambridge (1960).

fields. Affect relations within a behavioral system are vector fields – they are dynamic. They exhibit both direction and magnitude. They exhibit the change and flow of any other empirical vector field.

This process of applying an interpretation to the mathematical construct “vector,” is a logical process of *abduction*. This is not a process of “moving backward,” but a process of “taking from.” The mathematical measure is simply being applied to the content of a behavioral theory. There is no theory development; there is simply an explication of the theory by mathematical means. The important concept here, as defined by Steiner, is that there is no “theory development,” but “theory explication,” the logical process of deduction. The mathematical concept of a vector field is utilized as a measure to further explicate the theory. The concept ‘*affect relation*’ was already in the theory, so it is clear that no theory development was accomplished. Was there a retroduction of the “form” as a single predicate from mathematics? No. What is being utilized here is simply the definition of a vector field. The definition of ‘vector field’ is being “taken from” mathematics in order to deductively explicate the theory of affect relations. As Thompson remarked following the classification of the SIGGS properties by Frick: “It was recognized that the Structural Properties represented the topology of the theory.” Such recognition was a deductive process and not a retroductive process. The mathematical vector field theory was not used to devise the SIGGS theory; it was used to explicate the SIGGS theory.

This research defines ‘retroduction’ and ‘abduction’ as distinct logical processes, and such that they complement the logical process of ‘deduction,’ as follows:

- **Deduction** is the logical process by which a conclusion is obtained as the implication of assumptions.
- **Retroduction** is the logical process by which a point of view is utilized to devise a conjecture or theory.
- **Abduction** is the logical process by which a theoretical construct of one theory is utilized to analyze or interpret the parameters of another theory.

While the *Deduction Theorem* is a standard part of mathematic logic, this research extends this analysis to include the *Retroduction Theorem* and *Abduction Theorem*.

The *Deduction Theorem* will be stated first. The applicable logical schema of the Statement Calculus is:

If $P \supset Q$, then $P \vdash Q$; and If $P \vdash Q$, then $P \supset Q \equiv P \vdash Q \equiv \vdash P \supset Q$

The *Deduction Theorem* is a statement of the following implication:

Deduction Theorem: $P \vdash Q \Rightarrow \vdash P \supset Q$

The statement of the *Retroduction Theorem* and *Abduction Theorem* are much more complex.

Retroduction Theorem

Steiner defines retroduction schematically as follows:

Given theories \mathcal{A} and β , theory \mathcal{A} is a devising model for theory β if there is a subset, \mathcal{D} , of \mathcal{A} such that the predicates of β are a representation in substance or form of the predicates of \mathcal{D} ; whatever is true of \mathcal{D} is true of β ; and not whatever is true of β is true of \mathcal{A} .

Initially it would appear that the following implication holds: $\mathcal{A} \supset \beta$. However, as Steiner points out, “The theory or conjecture that emerges (conclusion) contains more than the theory or point of view from which it emerges (premises). The implication, then, can only hold from the conclusion to the premise”; that is, $\beta \supset \mathcal{A}$.

It could be argued that the sentential and predicate logic do not hold in this instance. But, if not, we are left with a state of confusion when we are attempting to develop a scientific theory that relies on just such logics. Therefore, it must be assumed that the logic holds and we need to take a closer look at just what is required.

Taking retroduction, as it is conceptually defined; all we have is that theory \mathcal{A} is a devising model for theory β . By this is meant that the predicates for theory β are derived from a subset, \mathcal{D} , of the predicates of theory \mathcal{A} in such a manner that they imply predicates of theory \mathcal{A} ; that is, $P(\hat{h}) \in \beta \supset P(h) \in \mathcal{D} \subset \mathcal{A}$, but $P(\hat{h}) \notin \mathcal{A} \setminus \mathcal{D}$. More explicitly: $\exists P(\hat{h}) \in \beta [\sim (P(\hat{h}) \in \beta \cong P(h) \in \mathcal{A})]$. This meets the final requirement by Steiner. (Where, ‘ \cong ’ =_{df} “is isostruct to”; and isostructism is a mapping of one entity to another to which it is isomorphic or isosubstantive.)

Further, it is required that the predicates of “ β are a representation in substance or form of the predicates of \mathcal{D} .” This requirement is formalized as follows:

$$P(h) \in \mathcal{D} \cong P(\hat{h}) \in \beta =_{df} P(h) \in \mathcal{D} \cong P(\hat{h}) \in \beta : \forall P(h) \in \mathcal{D} \cong P(\hat{h}) \in \beta;$$

Where, ‘ \cong ’ =_{df} “is isostruct to”; ‘ \cong ’ =_{df} “is isomorphic to” and ‘ \cong ’ =_{df} “is isosubstantive to”.

An isostructism is a mapping of one entity to another to which it is isomorphic or isosubstantive. An isomorphism is a mapping of one entity into another having the same elemental structure, whereby the behaviors of the two entities are identically describable by their affect relations. An isosubstantism is a mapping of one entity into another having similar predicate descriptors.

Therefore, all of Steiner’s stipulations have been met. As a result, the *Retroduction Theorem* is formalized as follows:

Retroduction Theorem:

$$P(h) \in \mathcal{D} \subset \mathcal{A} \cong P(\hat{h}) \in \beta, \exists P(\hat{h}) \in \beta [\sim(P(\hat{h}) \in \beta \cong P(h) \in \mathcal{A})] \vdash$$

$$P(\hat{h}) \in \beta \supset P(h) \in \mathcal{D} \subset \mathcal{A}.$$

Proof of Retroduction Theorem:

For the purposes of this proof, since the conclusion is simply the result of the assumptions by definition, all that needs to be argued is that the *Predicate Calculus* applies to theory β . To apply, theories \mathcal{A} and β must be isostruct with respect to \mathcal{D} . By assumption, they are.

Since the theories are isostruct, any proof in \mathcal{D} is applicable to a corresponding proof in β , since they will have corresponding axioms and assumptions. Further, any predicate in β not in \mathcal{A} can be taken as an assumption or axiom from which resulting theorems can be derived by the *Sentential* and *Predicate Calculi*.

The value of this theorem is that it establishes that the logic of the *Axiomatic Sentential* and *Predicate Calculi* apply to theory β .

As has been shown above, there is a distinction between *retroduction* and *abduction*. The *Abduction Theorem* is given below.

Abduction Theorem:

Given theories \mathcal{A} and β , theory \mathcal{A} is a formal model-of theory β if there is a subset, \mathcal{D} , of \mathcal{A} such that the predicates of β are an equivalent representation in form of the predicates of \mathcal{D} ; whatever is true of \mathcal{D} is true of β ; and whatever is true of β is true of \mathcal{D} .

The formal statement of the **Abduction Theorem** is:

Abduction Theorem:

$$h \cong \hat{h} \vdash P(h) \in \mathcal{D} \subset \mathcal{A} \text{ :=: } P(\hat{h}) \in \beta$$

Proof of Abduction Theorem:

- | | | |
|-----|---|----------------------|
| (1) | $h \cong \hat{h}$ | Assumption |
| (2) | $P(h_1), P(h_2), \dots, P(h_n) \in \mathcal{D} \subset \mathcal{A}$ | Assumption |
| (3) | $P(\hat{h}_1), P(\hat{h}_2), \dots, P(\hat{h}_n) \in \beta$ | Substitution, 1 in 2 |
| (4) | $\therefore P(h_1), P(h_2), \dots, P(h_n) \in \mathcal{D} \vdash P(\hat{h}_1), P(\hat{h}_2), \dots, P(\hat{h}_n) \in \beta$ | 3 from 1 and 2 |
| (5) | $P(\hat{h}_1), P(\hat{h}_2), \dots, P(\hat{h}_n) \in \beta$ | Assumption |
| (6) | $P(h_1), P(h_2), \dots, P(h_n) \in \mathcal{D} \subset \mathcal{A}$ | Substitution, 1 in 5 |
| (7) | $\therefore P(\hat{h}_1), P(\hat{h}_2), \dots, P(\hat{h}_n) \in \beta \vdash P(h_1), P(h_2), \dots, P(h_n) \in \mathcal{D} \subset \mathcal{A}$ | 6 from 1 and 5 |
| (8) | $h \cong \hat{h} \vdash P(h_1), \dots, P(h_n) \in \mathcal{D} \subset \mathcal{A} \text{ :=: } P(\hat{h}_1), \dots, P(\hat{h}_n) \in \beta$ | Definition, 4 & 7 |
- Q.E.D.

The significance of this theorem is that formal predicates of a given theory that are isomorphic to formal predicates of another theory, define the properties of the second theory.

Types of Models

Steiner presents the concept of ‘model’ as a dichotomy: ‘model-of’ and ‘model-for’. Intuitively, ‘model-of’ corresponds to the familiar type of construction models—model cars, model planes, etc. Also, intuitively, ‘model-for’ corresponds to the familiar type of exemplary models—professional models who exemplify appearances or role models who exemplify behaviors.

From these examples, it is seen that a ‘model-of’ is a representation of an object, possibly a “scale model”; and a ‘model-for’ is the object that is being represented in an ideal, possibly a “super model.”

Model-of is a scaled version of the intended object.

Model-for is a paradigm that can be used to describe ideal structures.

These models are designated, respectively, *first-order model* and *second-order model*.

Models that are used to devise theory by retrodution are second-order models. They provide the perspective desired to develop the new theory; it is an exemplary model of the new theory. Thus Steiner defines the relation between the types of logic and the types of models as follows:

Retroductive Logic =_{df} Devising of theory from a second-order model.

Deductive Logic =_{df} Explicating a theory for clarification or completeness.

Inductive Logic =_{df} Evaluating a theory to delineate the range of defined objects.

A theory may be further delineated by the referents of the theory; that is, if the theory is about actually existing objects, then the theory will be called an ‘empirical theory’. For example, theorizing about social referents is an attempt to characterize actually existing objects falling within the domain of some social context or process. Education theorizing is such a theory; it is empirical theorizing. In an empirical theory, the statements not only express the nature of the objects, but also the way in which the objects are interrelated.

In view of the preceding, in this research *ATIS* is a *model-for* education theorizing. *SimEd*, on the other hand, is a *model-of* an educational system.

Now that the type of theorizing has been established, the *ATIS* model will be further explicated. *ATIS* is a logico-mathematical model; that is, it is a *formal model*.

In mathematics, model theory is defined as a branch of logic that studies mathematical structure, and, in particular, the structures of axiomatic set theory. *ATIS* is a generalization of mathematical model theory.

Axiomatic set theory is set theory founded on axioms with no empirical content. As set theory is closely associated with mathematical logic, there is an integration of the *Sentential* and *Predicate Calculi* in *ATIS* that results in a formal theory that provides the rigor of deduction and proof.

While the properties and axioms of *ATIS* are initially framed in the context of an empirical theory, those properties and axioms are transformed into a formal logico-mathematical theory that allows for the analysis of *ATIS* as a formal theory.

Others have developed mathematical models for general systems theory. One in particular, Mihajlo D. Mesarović, has developed this area extensively.

Mihajlo D. Mesarović, in “A Mathematical Theory of General Systems,”⁴ has developed measures for system properties. In his work, Mesarović restricts the measures to “General Systems Theory of Hierarchical Systems.”⁵ The mathematical measures developed by Thompson in this research are a generalization of the Mesarović measures as extended by Yi Lin.⁶

However, Mesarović also introduces a “coordination strategy” that will not be applied to *ATIS* measures. This strategy was designed by Mesarović to “adjust” the theoretical projections with actual observations. As described, it appears to simply classify two sets of systems, those that can be “adjusted” and those that cannot. Such a dichotomy is not appropriate for the type of systems here being considered. For *ATIS*, the criteria for validation are with respect to the theorems of the theory without adjustment.

The distinction between the Mesarović approach and that proposed here is that Mesarović relies on models that are “scientifically” developed yet closed, whereas the approach here is founded on the logic of system’s theory; such logic being applied to open systems that provides an open-ended number of outcomes. As described earlier, the proposed model for this research can be tailored to the specific needs of an empirical system without having to modify the initial program, as would have to be done in a Scenario-Based Model.

While Mesarović has contributed greatly to the mathematical development of general systems theory, his system models do not have a basis founded on theory. One such model is WIM (World Integrated Model) that was developed with 49 subroutines. It was quite refined in that it utilized about 21,000 numbers to describe the state of the global system at any one time. (WIM can be found online by searching for “World Integrated Model”.)

The direction being taken by this research is distinct from that of most, if not all, other social models—this SimEd Model will rely on a Logic-Based Model for its projections. It is believed that the parameters are too numerous and the possible outcomes are so extensive that anything less would result in a model that could end up with the same shortcomings as that recognized by Friedman concerning SimCity.

SimEd will rely on a Logic-Based Model.

⁴ Mesarovic, Mihajlo D. (1972), “A Mathematical Theory of General Systems,” *Trends in General Systems Theory*, Ed., George J. Klir, Wiley-Interscience, p. 251.

⁵ Ibid., p. 264.

⁶ Lin, Yi (1999), *General Systems Theory: A Mathematical Approach*, Kluwer Academic/Plenum Publishers, NY.

Research Organization

The purpose of this Proffitt research is to develop the “Configuration” aspect of APT&C (Analysis of Patterns in Time and Configuration). The relation between SimEd and *ATIS* was stated on page 1 as:

SimEd is a software program designed to simulate educational systems behavior.

ATIS (Axiomatic Theories of Intentional Systems) is a theory model options-set that is designed to construct theory for certain types of behavioral systems. In particular, it is used to develop behavioral predictive theories and technologies. SimEd is one of those behavioral predictive technologies.

Thompson recognized that APT could be used to analyze systems in a manner that real-time system parameters could be recognized. With APT, it was suggested that a system analysis was possible. Frick then recognized that in order to properly accomplish this analysis, APT would have to be revised so that system configurations could be properly analyzed. From this exchange, APT was revised to include APT&C.

Preliminary definitions have been provided for certain structural properties that provide measures for these properties. However, a more thorough development of these measures is required and additional measures developed for all structural properties. Further, these measures must be consistent so that they can be related through the axioms to obtain outcomes that can be empirically validated.

It is the purpose of this Proffitt Grant research to obtain those measures for APT&C.

In order to accomplish the purpose of this research, additional basic theory development is required to provide the foundation for the property measures. Therefore, in addition to the research required for Proffitt, additional research will be provided, although the copyright to all such material remains with Kenneth R. Thompson. While this additional research as stated in the proposal will be provided for Proffitt, it is research that goes beyond that intended for Proffitt. Such research while being part of separate research endeavors will be provided for Proffitt to provide a comprehensive presentation of the theory upon which the “Configuration” research relies.

To clearly identify the extended research, all such reports will be numbered sequentially within Proffitt, but will also be identified by an “-ER” suffix. For example, the second report may be identified as: “Report 2-ER.” If the tenth report relates directly to the Proffitt “Configuration” research, then it would be identified as: “Report 10.”