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"GENERAL SYSTEM" DEFINED FOR PREDICTIVE TECHNOLOGIES OF A-GSBT (AXIOMATIC-GENERAL SYSTEMS BEHAVIORAL THEORY)

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This article presents the foundations of A-GSBT (Axiomatic-General Systems Behavioral Theory) by providing a clear logical definition of *general system*. Such a definition is critical for devising mathematical models for predicting results in behavioral systems. The purpose of A-GSBT is to provide an *Options Set* to construct a theory with predictive technology applications to various behavioral systems. Such systems include educational systems, military systems, and terrorist network systems among others. A future report will consider the methodology for implementing the *Options Set*. A-GSBT provides, especially for educologists, a procedure for constructing scientific theory founded on an axiomatic logic and mathematics instead of relying on hypothesis-driven methodologies that result in open-ended testing with no reliable predictive results in diverse applications.

Keywords: General systems theory, behavioral theory, education theory, SIGGS, SimEd, A-GSBT.

INTRODUCTION

This report presents an extension and adaptation of theories developed by other researchers of General Systems Theory (GST). In particular, it is an adaptation of the definition of *general system* developed by Wayne Wymore. We will apply this adaptation to an extension of the SIGGS theory model of Elizabeth Steiner and George Maccia as extended by Theodore W. Frick.

There is a significant distinction between the approach used herein concerning *general system* and that used by Wymore and many other general system theorists. Frequently, *general system* definitions provide designs of models of known behaviors. That is, the intent is to derive a mathematical model of an observed behavior. A-GSBT, however, derives predicted system behavior from an applicable theory.

A-GSBT (Axiomatic-General Systems Behavioral Theory) is a logico-mathematical model for analyzing certain types of behavioral systems to obtain predictive results for these systems. The behavioral systems of concern are those that are goal-directed or intentional, as opposed to whimsical, chaotic or capricious. Therefore, it is clear that we are not concerned with predicting the behavior of individual human beings in their day-to-day life.

Whether or not A-GSBT applies to other types of behavioral systems, initially the systems of concern are those comprised of a relatively large number of human beings. Such systems are those commonly described as education systems, terrorist network systems, employment hiring systems, military training systems, etc.

Since the introduction of the concept of *general systems theory* (GST) by Ludwig von Bertalanffy in a report in 1950 (Bertalanffy, 1950), there have been extensive contributions by others in the development of GST as a logical and mathematical theory, as well as those who have contributed to its extension by presenting well-developed descriptive theories. The SIGGS theory model (Maccia and Maccia, 1966), while not an axiomatic model, did provide the first extensive formalization of a model for educational theorizing.

In 1972, Bertalanffy presented an overview of GST, "The History and Status of General Systems Theory" (Bertalanffy, 1972). Certain assertions of that overview are worth recognizing as they set the foundations for this report.

> The goal obviously is to develop general systems theory in mathematical terms (a "logico-mathematical field") because mathematics is the exact language permitting rigorous deductions and confirmation (or refusal) of theory. (p. 30)
>
> One approach or group of investigations may, somewhat loosely, be circumscribed as *axiomatic*, inasmuch as the focus of interest is a rigorous definition of system and the derivation, by modern methods of mathematics and logic, of its implications. Among other examples are the system descriptions by Mesarović, Maccia [Steiner] and Maccia, …. (p. 31)

It is the intent of this research to develop *A-GSBT* as an axiomatic, logico-mathematical theory model. Thompson worked with Steiner and Maccia from 1965 to 1966 as a Research Assistant along with James F. Andris in the Education Theory Center, The Ohio State University, conducting the mathematical research for the report *Development of Educational Theory Derived from Three Educational Models* (Maccia and Maccia, 1966) that produced the SIGGS Theory Model.

In 1994, Theodore W. Frick extended the work of Maccia and Maccia by classifying the system properties into *Basic*, *Structural*, and *Dynamic Properties* (Frick, 1994). After reading the work by Frick, Thompson recognized that the *Structural Properties* essentially defined the *System Topology* (Thompson, 2004). This development came a long way from the *Topological Field Theory* proposed by Kurt Lewin in 1936 (Lewin, 1936). Although Lewin introduced mathematical terminology to study human behavior, and even recognized "systems of behavior" (p. 15) with no further elaboration, he did not utilize the power of mathematical topology. Although he attempted to present the image of a mathematical topological theory, in fact it was not. Lewin's theory was a descriptive theory that utilized topological concepts to describe "psychological regions," which he used as set-theoretic concepts and did not utilize topology. On the other hand, SIGGS, with the extension by Frick, and now by Thompson has the promise of actually being able to present a behavioral theory that is logically and mathematically sound. While logical analyses of A-GSBT have resulted in minimal predictive outcomes and have promise for much more, we cannot yet determine if mathematical topology, and especially vector topology, is applicable to the development of this theory.

In addition to the classification of the SIGGS properties, Frick has also conducted extensive research in education, including his report: "SIGGS as a theory model for understanding systemic change in education" (Frick, 1994). His report is the first to introduce *SimEd*, which is a simulation program for predicting outcomes of education systems.

In addition to (Elizabeth S. Steiner) Maccia and (George S.) Maccia, and Frick, Thompson has relied on the work by Mihajlo D. Mesarović (Mesarović, 1964, 1972), and A. Wayne Wymore (Wymore, 1967).

The work of Wymore, although developed for engineering models, is applicable here, as the definition of *general system* specifies all relevant parameters. Such a design for the definition of *general system* allows for the precision and rigorous treatment required to fully explicate an axiomatic, logico-mathematical theory envisioned by Bertalanffy.

Joseph V. Cornacchio provides an excellent overview of Wymore's theory in "Topological Concepts in the Mathematical Theory of General Systems" (Cornacchio, 1972). More important, this work by Cornacchio introduces topology as a possibility for analysis in an axiomatic general systems theory as presented by Wymore and now extended in A-GSBT. Cornacchio states:

> Although a formal systems model can, in an ad hoc manner, incorporate a topological structure by simply postulating that the sets of interest carry a classical topology, that is, that they are to be a priori considered topological spaces, we have taken the approach that it is of more fundamental value to inquire as to the existence of a natural topology determined by the intrinsic algebraic and set-theoretic properties of the structure itself. … [Such an approach can yield] a topological structure intrinsically determined by the theory.

In the development of A-GSBT, every *affect relation* defines a topology on the system. While topological considerations are beyond the scope of this report, that *affect relations* define system topologies suggests that such topological analyses are a means of determining predictive behaviors for systems. Thompson contemplates that such analyses in conjunction with logical analyses will be required to obtain the real-time predictive results desired, in particular, for identifying terrorist threats and targets from analyses of terrorist network systems. However, this report is restricted to explicating only the definition of *general system*.

While working with Steiner and Maccia in the 1960's, much effort went into identifying numerous properties for SIGGS; however, time constraints precluded developing a consistent nomenclature, or developing consistent definitions of the properties. The development of A-GSBT has corrected these two problems. In addition, the development of A-GSBT has expanded the list of properties to more than twice that initially provided in SIGGS. Such an expansion is logically necessary in order to define *general system*, and to provide the properties required to analyze the behavioral systems of concern.

Thompson has also considered the numerous hypotheses listed in SIGGS, 201 of them, and has brought them into A-GSBT as axioms, as was the intended interpretation in SIGGS. With the number of axioms, it was probable that some of them were actually theorems, and some may be inconsistent. Thompson has determined that many of them are in fact theorems and has discovered three pairs to be inconsistent. On the other hand, research conducted by graduate students in a course at Indiana University offered by Frick during Autumn Semester 2004, *Research Methods in Instructional Systems Technology*, validated 12 of the theorems.

THE FORMAL THEORY

Primitive Terms

The construction of a formal theory begins with the designation of the *primitive terms*. The *primitive terms* used in *A-GSBT* are the following: *set*, *element*/*component*/*object*, *contained in*, *ordered pair*, *universe of discourse*, *characterization*, *occurrences*, *parameters*, *connection*, *relation*, *affect relation* (only for the Steiner-Maccia information system), *event*, and *sequence*.

We will define *affect relation* under general system theory.

Element, *component*, and *object* but represent different names for the same concept. Normally *element* is used in mathematics, *component* is used in the social sciences, and *object* may be used in both. *A-GSBT* uses these terms interchangeably.

System

There are various definitions of *system* in the literature. Some of the definitions are required due to mathematical concerns. Others are very imprecise, descriptive arguments rather than developed with logical or mathematical precision. The initial definition used here follows the convention of a system, $\mathbb{S}$, being an ordered pair consisting of an object-set, S, and a relation set, $\Re$.

DEFINITION: $\mathbb{S} = (S, \Re)$.

Maccia and Maccia, in SIGGS, followed this convention and defined *system* as follows:

DEFINITION: System, $\mathbb{S}$, $=_{df}$ A *group* with at least one *affect relation* that has *information*.

$$\mathbb{S} =_{df} (S, \Re)$$

> A **system** is an ordered pair defined by a *group* (that is, a set with at least two components), and an *affect-relation* (that is, a connection between at least two components) that has *information*.

In this definition, *affect relation* is a primitive term that replaces the more general concept *relation*. This definition also incorporates the primitive concept of *information* as a qualifier of affect *relation* that is not part of the more general concept of *relation*.

This definition can be read more generally, without *information*, as: "A **system** is an ordered pair defined by a non-empty *object-set* with at least two components, and a non-empty *relation-set*." To avoid confusion with a *Steiner-Maccia information system*, we will define *system* by this more general interpretation, and replace the nomenclature with '$\mathbb{S}_{o}$' and '$\mathbb{S}_{\phi}$', respectively, for S and $\mathfrak{R}$. We will identify these, respectively, as the *object-set* and *relation-set*, and use the following definition of *system*:

DEFINITION: System, $\mathbb{S}$, $=_{df}$ An ordered pair identified as the object-set and relation-set.

$$\mathbb{S} =_{df} (\mathbb{S}_{o}, \mathbb{S}_{\phi})$$

Basic Properties

The ***Basic Properties*** of a system define the initial attributes required to identify and analyze a system. They are basic to the concept of *system*, $\mathbb{S}$. Before defining *general system*, we will make explicit the formal definitions of *system object-set*, $\mathbb{S}_{o}$, and *system relation-set*, $\mathbb{S}_{\phi}$.

DEFINITION: System object-set, $\mathbb{S}_{o}$, $=_{df}$ A set with at least two components within the universe of discourse.

$$\mathbb{S}_{o} =_{df} \{x \mid x \in S \in \mathcal{U}\} \wedge |S| > 1$$

In this definition, '$=_{df}$' is to be read "is defined as," '$\mathcal{U}$' is the universe of discourse, 'S' is an object-set of $\mathcal{U}$, and '$|S|$' is the set-cardinality function.

For example, for educational systems, $\mathbb{S}_{o}$ consists of *students*, *teachers*, *administrators*, *instructional materials*, *volunteer personnel*, *community support personnel*, and any other community-based or school-based personnel or materials required for the educational system.

The above list indicates the great complexity of a school system, and other systems with which we are concerned. Yet, such systems are manageable. It is the purpose of general systems theory to describe how to manage a system by predicting what will happen under varying conditions.

DEFINITION: System relation-set, $\mathbb{S}_{\phi}$, $=_{df}$ A non-empty set of ordered pairs of components from the object-set.

$$\mathbb{S}_{\phi} =_{df} \{(x,y) \mid \exists x,y(x \in \mathbb{S}_{ox} \wedge y \in \mathbb{S}_{oy})\}$$

'$\mathbb{S}_{ox}$' and '$\mathbb{S}_{oy}$' identify the specific *object-sets* of $\mathcal{U}$ that contain x and y, respectively. $\mathbb{S}_{ox}$ and $\mathbb{S}_{oy}$ are not necessarily disjoint.

For example, for educational systems, $\mathbb{S}_{\phi}$ consists of *students learn from textbooks*, *teachers instruct students*, *administrators control student-enrollment*, *school security personnel protect students*, and any other community-based or school-based affect relations required for the educational system.

The following diagram of a *system*, $\mathbb{S}_{o}$, and its *negasystem*, $\mathbb{S}'_{o}$, within a universe of discourse will help to put the components of a system and their relations into perspective. As this report is restricted to those considerations required to define *general system*, it will be restricted to defining and considering the various concepts indicated in the diagram below.

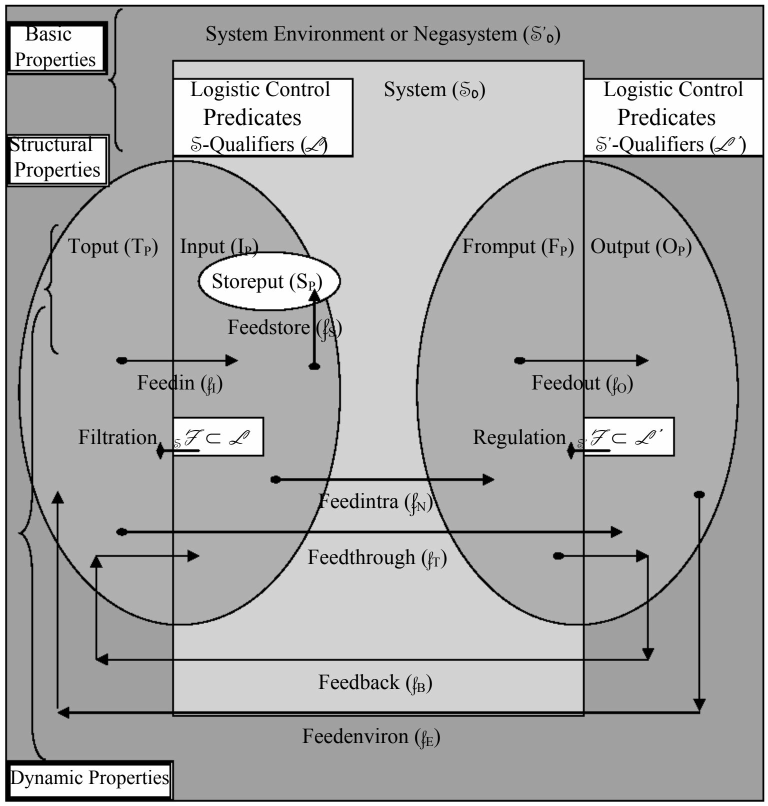
The light-gray area rectangle contains the components of the *system*, while the outer dark-gray rectangular area contains the components of the *negasystem*, both of which combined represent the universe, $\mathcal{U}$.

General System

Now we can define *general system*, $\mathcal{G}$, as an adaptation of that proposed by Wymore (Wymore, 1967). We will define a *general system* within a *Universe of Discourse*, $\mathcal{U}$. $\mathcal{G}$ consists of the following parameters: *Object partitioning set*, $\mathcal{P}$; *Family of affect relations set*, $\mathcal{A}$; *Transition function set*, $\mathcal{T}$;

Diagram of System Properties

Universe of Discourse ($\mathcal{U}$) $\mathcal{G} =_{df} (\mathcal{P}, \mathcal{A}, \mathscr{T}, \mathcal{T}, \sigma)$[1,2]



[1] $\mathcal{G}$ is the *General System*, $\mathcal{P}$ is the *Object Partitioning Set*, $\mathcal{A}$ is the *Family of Affect Relations Set*, $\mathscr{T}$ is the *Linearly Ordered Time Set*, $\mathcal{T}$ is the *Transition Function Set*, and σ is the *System State Transition Function*. $_{\mathbb{S}}\mathscr{T} \subset \mathscr{L}$ is System Filtration, and $_{\mathbb{S}'}\mathscr{T} \subset \mathscr{L}'$ is Negasystem Filtration (Regulation).

[2] $T_P, I_P, F_P, O_P, S_P, \mathscr{L}, \mathscr{L}', \mathbb{S}_{BX}, \mathbb{S}'_{BY} \in \mathcal{P}$; $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \in \mathcal{A}$; $t_1, t_2, \ldots, t_k \in \mathscr{T}$, $f_I, f_O, f_N, f_T, f_B, f_E, f_S \in \mathcal{T}$; where '$T_P$' represents *toput*, '$I_P$' represents *input*, '$F_P$' represents *fromput*, 'O_P' represents *output*, 'S_P' represents *storeput*, '$\mathscr{L}$' represents *system qualifiers*, '$\mathscr{L}'$' represents *negasystem qualifiers*, '$\mathbb{S}_{BX}$' represents *system background components*, and '$\mathbb{S}'_{BY}$' represents *negasystem background components*; and 'f_I' represents *feedin*, 'f_O' represents *feedout*, 'f_T' represents *feedthrough*, 'f_B' represents *feedback*, 'f_S' represents *feedstore*, 'f_E' represents *feedintra*, and 'f_N' represents *feedenviron*.

Linearly ordered time set, $\mathscr{T}$, and *System state-transition function*, σ. Since $\mathcal{G}$ is a system, its object-set and relation-set initially identify it. '$\mathcal{G}_o$' and '$\mathcal{G}_\phi$' identify these two sets, respectively; such that, $\mathcal{G}_o = \mathbb{S}_o$ and $\mathcal{G}_\phi = \mathbb{S}_\phi$. With this convention, the definition of *general system* is:

DEFINITION: General system, $\mathcal{G}$, $=_{df}$ an ordered sequence of five parameters: *object-partitioning set* ($\mathcal{P}$), *affect relations set* ($\mathcal{A}$), *transition functions set* ($\mathcal{T}$), *time set* ($\mathscr{T}$), and *system state-transition function* (σ).

$$\mathcal{G} =_{df} (\mathcal{P}, \mathcal{T}, \mathcal{A}, \mathscr{T}, \sigma)$$

Defined below are the parameters of *general system*.

DEFINITION: Object-partitioning set, $\mathcal{P}$, $=_{df}$ a family of disjoint *object-sets* of the *general system object-set*, $\mathcal{G}_0$.

$$\mathcal{P} = \{\mathbb{S}_{0i} \mid \exists i \in \mathcal{I} \forall \mathbb{S}_{0i}(\mathbb{S}_{0i} \subset \mathcal{G}_0 \wedge (i \neq j \supset \mathbb{S}_{0i} \cap \mathbb{S}_{0j} = \emptyset))\};$$

where $\mathcal{I}$ is the set of positive integers.

Whereas all sets of $\mathcal{P}$ are disjoint, $\mathcal{G}_0$ contains any object-set of $\mathcal{U}$, not all of which have to be disjoint. By this definition, *general system* is a restriction or qualification of *system*. We expect this, since *general system* is a construct designed to give us the tools by which we explicate empirical observations. Contrary to our normal expectations of *general*, in this case, *system* contains much more than the construct *general system* is prepared to explicate.

Intuitively, the set $\mathcal{P}$ contains all of the "things" within a system and its negasystem. In an education-system, the partitioned sets may be comprised of *students*, *teachers*, *administrators*, *parents*, *textbooks*, *community support personnel*, etc. Some of these sets are in the system some are in the negasystem, but they are all disjoint sets. For example, while an individual may be a *student* at one time and a *teacher* at another time, that individual is in only one partitioned set at any one time.

DEFINITION: Affect relation set, $\mathcal{A}$, $=_{df}$ a family of *relation-sets* of the *general system relation-set*, $\mathcal{G}_\phi$, defined by *qualifier predicates*, $\mathcal{L}$.

$$\mathcal{A} =_{df} \{\mathbb{S}_{\phi i} \mid \exists i \in \mathcal{I} \forall \mathbb{S}_{\phi i}(\mathbb{S}_{\phi i} = \{(x,y) \mid P(x,y) \in \mathcal{L}\} \subset \mathcal{G}_\phi \};$$

where $\mathcal{I}$ is the set of positive integers, and 'x' and 'y' are *extensions* of the predicate 'P'.

'$\mathcal{L}$' is the set of *qualifying predicates* with elements $P(x,y)$, such that $P(x,y)$ is a statement that has x and y as variables. The elements of $\mathbb{S}_{\phi i}$ have the following form, $(x,y) = \{\{x\},\{x, y\}\}$:

$$\mathbb{S}_{\phi i} =_{df} \{ \{\{x_i\},\{x_i, y_i\}\} \mid \forall x_i,y_i \, (x_i \in \mathbb{S}_{0xi} \wedge y_i \in \mathbb{S}_{0yi})\}]\}.$$

We will then say that $\mathbb{S}_{\phi i} = \mathcal{A}_i \in \mathcal{A}$.

Intuitively, the set $\mathcal{A}$ contains all of the *relations* or *connections* between elements of $\mathbb{S}_\phi$. In an education-system, *teacher-instruction-of-student*, *student-learning-from-textbook*, *parent-to-teacher*, etc., define such relations.

Affect relations determine the structure of the system by the connectedness of the components. This is important since *structure* provides the basis for predicting *system behavior*. Further, it is important to recognize that prior states do not predict future behavior—developing structure does.

$\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are the affect relation-sets of $\mathcal{G}_\phi$. These sets are elements of the family of affect relations, $\mathcal{A}$. As shown below, these sets define each component-partition of $\mathcal{G}_0$.

DEFINITION: Transition function set, $\mathcal{T}$, $=_{df}$ the set of "feed-" functions.

Transition functions give the system dynamics. These are the functions that are operated on by the *system state-transition function*, σ, to change the system structure and thereby the *behavior* of the system. *System behavior* is a sequence of *system states*. A consistent pattern of system states defines *system dispositional behavior*. While *system behavior* and *system dispositional behavior* do not determine predictive results, they do provide a base against which to analyze new data. However, the introduction of new data that changes system structure that is divergent from that base changes the predictive outcomes. A basic tenant of A-GSBT is that predictive outcomes are dependent on current structure and not prior states that thereby produces non-intuitive results. This is important; for example, when one is trying to destroy a terrorist network system. A-GSBT produces non-obvious tactics that result in the strategic paralysis of the terrorist network system. While this tenant is obvious for intuitive outcomes; for example, that blocking the financial resources of a terrorist cell will disrupt its ability to pursue terrorist targets, that similar results are obtained for non-intuitive outcomes is part of the research yet to be conducted.

The transition functions required for state-transition analysis are f_I, f_O, f_T, f_B, f_S, f_E, and f_N. The *transition function set*, $\mathcal{T}$, is necessary in order to "move" objects about the system. Without transition functions, nothing moves. In an education-system, *applicants-become-students*, *students-become-graduates*, *teachers-become-administrators*, etc. represent the feed-functions. The definition of transition function allows for temporal analysis of the system. However, no time parameter is required for the definitions since the transition is dependent only on system conditions at a specific time. Time parameters will be required, for example, when determining the dispositional behavior of a system, but not for determining the fact that a transition has occurred. Regardless of when it is determined; for example, that a *toput* component has a value of $\top$, such value results in an immediate recognition of *input*.

DEFINITION: Feed-function schema: The "feed-" functions, f_X; that is, f_I, f_O, f_T, f_B, f_S, f_E, and f_N, are state transition functions between two disjoint sets, X_p and Y_p, defined as follows:

$$\sigma(\mathfrak{x}_{X_p})(f_F \circ g \circ f) \in Y_p \mid \sigma(\mathfrak{x}_{X_p}) = \mathfrak{x}_{Y_p}\text{; where } f\text{: } X_p \times {}_{X_p}\mathcal{L}_C \rightarrow \{\bot, \top\}\text{, and}$$

'${}_{X_p}\mathcal{L}_C$' designates the "X_p logistic-control qualifier."

$$g(\mathfrak{x}_{X_p}) = \begin{cases} \emptyset, \text{ if } f = \bot \\ \mathfrak{x}_{X_p}, \text{ if } f = \top \quad \text{and} \end{cases}$$

$$f_F\text{: } W \subset X_p \rightarrow Y_p \mid g(\mathfrak{x}_{X_p}) \neq \emptyset \wedge g(\mathfrak{x}_{X_p}) = \mathfrak{x}_{X_p} \in W \wedge f_F(\mathfrak{x}_{X_p}) = \mathfrak{x}_{Y_p}$$

For example, the transition of *toput* to *input* results in the following *feedin* state transition:

$$\sigma\,(f_I \circ g \circ f) \in I_p \mid \sigma(\mathfrak{x}_{T_p}) = \mathfrak{x}_{I_p}\text{; where } f\text{: } T_p \times {}_{T_p}\mathcal{L}_C \rightarrow \{\bot, \top\}$$

$$g f(T_p \times {}_{T_p}\mathcal{L}_C) = \mathfrak{x}_{T_p} \mid f(T_p \times {}_{T_p}\mathcal{L}_C) = \top\text{; and } f_I\text{: } T_p \rightarrow I_p \mid f_I(\mathfrak{x}_{T_p}) = \mathfrak{x}_{I_p}$$

The feedin function is a type of component-identity function where the '$\mathfrak{x}$' from one set that has transitioned to another set is identified by subscripts; for example, $f_I(\mathfrak{x}_{T_p}) = \mathfrak{x}_{I_p}$.

We will now define the "feed-" functions and "-put" properties identified in the above definition. The "feed-" transition functions move components from one subsystem to another, while the "-put" properties identify the component-partitioned sets.

DEFINITION: Feedin, f_I, $=_{df}$ Transmission of *toput* from a negasystem to *input* in a system.

$$\sigma(\mathfrak{x}_{T_p})(f_I \circ g \circ f) \in I_p \mid \sigma(\mathfrak{x}_{T_p}) = \mathfrak{x}_{I_p}$$

DEFINITION: Feedout, f_O, $=_{df}$ Transmission of *fromput* from a system to *output* in a negasystem.

$$\sigma(\mathfrak{x}_{F_p})(f_O \circ g \circ f) \in O_p \mid \sigma(\mathfrak{x}_{F_p}) = \mathfrak{x}_{O_p}$$

DEFINITION: Feedintra, f_N, $=_{df}$ Transmission of *input* from a system to *fromput* of a system.

$$\sigma(\mathfrak{x}_{I_p})(f_N \circ g \circ f) \in F_p \mid \sigma(\mathfrak{x}_{I_p}) = \mathfrak{x}_{F_p}$$

DEFINITION: Feedenviron, f_E, $=_{df}$ Transmission of *output* from an environment to *toput* of an environment.

$$\sigma(\mathfrak{x}_{O_p})(f_E \circ g \circ f) \in T_p \mid \sigma(\mathfrak{x}_{O_p}) = \mathfrak{x}_{T_p}$$

DEFINITION: Feedthrough, f_T, $=_{df}$ Transmission of *toput* from a negasystem through a system to *output* of a negasystem.

$$\sigma(\mathfrak{x}_{T_p})(f_T \circ g \circ f) \in O_p \mid \sigma(\mathfrak{x}_{T_p}) = \mathfrak{x}_{O_p}$$

Feedthrough is *feedback* with respect to the negasystem.

Positive and **negative feedthrough** definitions are as follows:

$$f^{+}_{T} =_{df} \mathcal{A}(f_T)_{t(1)} < \mathcal{A}(f_T)_{t(2)} \qquad \bar{f}_{T} =_{df} \mathcal{A}(f_T)_{t(1)} > \mathcal{A}(f_T)_{t(2)}$$

APT (Analysis of Patterns in Time), $\mathcal{A}$, analyses measure *positive* and *negative feedthrough*. APT analyses determine measures of *system state* and a comparison of these measures before and after feedthrough determines positive or negative feedthrough. Frick developed APT (Frick, 1990) as "a method for gathering information about observable phenomena such that probabilities of temporal patterns of events can be estimated empirically [and] temporal patterns can be predicted from APT results" (p. 180).

DEFINITION: Feedback, $\not{f}_B$, $=_{df}$ Transmission of *fromput* from a system through a negasystem to *input* of a system.

$$\sigma(x_{F_{\mathcal{P}}})(\not{f}_B \circ g \circ f) \in O_{\mathcal{P}} \mid \sigma(x_{F_{\mathcal{P}}}) = x_{I_{\mathcal{P}}}$$

Positive and **negative feedback** definitions are as follows:

$$\overset{+}{\not{f}}_B =_{df} \mathcal{A}(\not{f}_B)_{t(1)} < \mathcal{A}(\not{f}_B)_{t(2)} \qquad \overset{-}{\not{f}}_B =_{df} \mathcal{A}(\not{f}_B)_{t(1)} > \mathcal{A}(\not{f}_B)_{t(2)}$$

APT (Analysis of Patterns in Time), $\mathcal{A}$, analyses measure *positive* and *negative feedback*. APT analyses determine measures of *system state*, and a comparison of these measures before and after feedback determines positive or negative feedback.

DEFINITION: Toput, T_P, $=_{df}$ *Negasystem* components for which *system toput control qualifiers* are "true."

$$T_P =_{df} \{x \mid x \in \mathcal{G}_{O_{\mathbb{S}'}} \wedge \exists P(x) \in {}_{T_{\mathcal{P}}}\mathcal{L}_C\, [f(x)(T_{\mathcal{P}} \times {}_{T_{\mathcal{P}}}\mathcal{L}_C) = \mathsf{T}]\}.$$

DEFINITION: Input, I_P, $=_{df}$ Resulting transmission of *toput* components; that is, system components for which *system input control qualifiers* of *toput* components are "true."

$$\mathbf{I_P} =_{df} \{x_{I_{\mathcal{P}}} \mid x_{I_{\mathcal{P}}} \in \mathcal{G}_{O_{\mathbb{S}}} \wedge \exists \sigma(\sigma(x_{T_{\mathcal{P}}} \in T_{\mathcal{P}}) = x_{I_{\mathcal{P}}})\}.$$

Input does not comprise all of the system; *input* is *toput*-transmitted components.

DEFINITION: Fromput, F_P, $=_{df}$ S*ystem* components for which *negasystem fromput control qualifiers* are "true."

$$F_P =_{df} \{x \mid x \in \mathcal{G}_{O_{\mathbb{S}}} \wedge \exists P(x) \in {}_{F_{\mathcal{P}}}\mathcal{L}'_C\, [f(x)(F_{\mathcal{P}} \times {}_{F_{\mathcal{P}}}\mathcal{L}'_C) = \mathsf{T}]\}.$$

DEFINITION: Output, O_P, $=_{df}$ Resulting transmission of *fromput* components; that is, negasystem components for which *negasystem output-control qualifiers* of *fromput* components are "true."

$$O_P =_{df} \{x_{O_{\mathcal{P}}} \mid x_{O_{\mathcal{P}}} \in \mathcal{G}_{O_{\mathbb{S}'}} \wedge \exists \sigma(\sigma(x_{F_{\mathcal{P}}} \in F_{\mathcal{P}}) = x_{O_{\mathcal{P}}})\}.$$

DEFINITION: Storeput, S_P, $=_{df}$ *System input components* for which *system fromput control qualifiers* are "false."

$$\mathbf{S_P} =_{df} \{x_{S_{\mathcal{P}}} \mid x_{S_{\mathcal{P}}} \in \mathcal{G}_{O_{\mathbb{S}}} \wedge \exists P(x) \in {}_{F_{\mathcal{P}}}\mathcal{L}_C\, \exists \sigma[f(x_{S_{\mathcal{P}}})(F_{\mathcal{P}} \times {}_{F_{\mathcal{P}}}\mathcal{L}_C) = \bot \wedge \sigma(x_{I_{\mathcal{P}}} \in I_{\mathcal{P}}) = x_{S_{\mathcal{P}}})]\}.$$

We will now define the final two parameters that define general system, the *linearly ordered time set* and *system state-transition function*.

DEFINITION: Linearly ordered time set, $\mathcal{T}$, $=_{df}$ a linearly ordered set.

$\mathcal{T}$, the *linearly ordered time set*, is required in order to give the system a "dynamic" property and may be the reals (set of real numbers) or any subset thereof. This set helps to keep the system organized by assigning an appropriate "time" that the event occurs. Without this set, there would be no "order" or "sequence" to the events of a system. In an education-system, the reals determine the sequence of events; for example, tax levies result in school programs and improvements, students completing homework results in graduates, etc.

DEFINITION: System state-transition function, σ, $=_{df}$ the function that maps a current system state onto a subsequent system state.

σ, the *system state-transition function*, is required in order to alter the "state" of a system. Whereas $\mathcal{T}$, the *Transition Function Set*, moves objects about the system, σ changes the state of the system due to new *affect relations* defined by the move. Both $\mathcal{T}$ and σ produce a change in the system, but each is required in order to define the changed system. In an education-system, the graduation of a student produces a new dynamic for the community to influence the policies and programs of the school; and the introduction of a new major business in the community produces a new tax-base dynamic for school improvement.

As the intent of this research is to be able to analyze behavioral structures with a multitude of components related in a variety of ways, an effective process must identify those components. In the following definition, $\bar{I}_B$ is the *information base*. With respect to a specific system, $\bar{I}_B$ consists of all known *affect relations* between the components of the system, the initial $\mathcal{G}_0$.

DEFINITION: General System Object-Set, $\mathcal{G}_0$, Construction Decision Procedure

1) Every *information base*, $\bar{I}_B$, defines affect relations, $\mathcal{A}_n \in \mathcal{A}$, by the unary- and binary-component-derived sets from the $\bar{I}_B$.

That is, the components of $\mathcal{A}_n$ are of the form: $\{\{x_i\},\{x_i, y_i\}\} \in \mathcal{A}_i \in \mathcal{A}$; such that an *affect relation* exists from x_i to y_i. The following functions, μ and β, define elements of a topology, τ_n,; that is, $\mu,\beta:\mathcal{A}_n \rightarrow \tau_n$, such that: $\mu\mathcal{A}_i = \{x_i\} \in \tau_n$; and $\beta\mathcal{A}_i = \{x_i, y_i\} \in \tau_n$. An additional function, φ, will also be required for certain properties, and will allow for specification of specific elements, as follows: $\varphi\mathcal{A}_i = y_i$. Hence, the elements of $\mathcal{G}_0$ can be specified by φ and $\mu \cap \beta$.

2) The set of elements of $\mathcal{G}_0$ will be defined by an existing $\bar{I}_B$ as follows:

$$\mathcal{G}_0 = \{x \mid \mathcal{A}_i \in \mathcal{A} \supset \exists i[(x \in \mu\mathcal{A}_i \cap \beta\mathcal{A}_i \vee x = \varphi\mathcal{A}_i]\}$$

3) New elements will be added to $\mathcal{G}_0$ by Rule 2) when the new element establishes a relation with an element in $\mathcal{G}_0$ so that it is an element of an $\mathcal{A}_i \in \mathcal{A}$.

4) No other objects will be considered as elements of $\mathcal{G}_0$ except as they are generated in accordance with Rules 1) to 3).

Before proceeding, we need to clearly understand the meaning and significance of the $\bar{I}_B$. $\bar{I}_B$ is the "information base" from which the affect relations are determined for a general system and thereby the components of the system. A $\bar{I}_B$ may be well defined, or we may only have an idea, a guess as to what components are actually contained in the $\bar{I}_B$. Whether we know the affect relations precisely or can only hypothecate them, such relations will produce the system components. The significance of so defining the $\bar{I}_B$ becomes apparent when such relations are the result of various data mining methodologies utilized in the analysis of terrorist network systems. While such methodologies can produce patterns only after-the-fact, new unstructured data immediately updates those patterns, analyzed within the axiomatic framework of A-GSBT, without having to wait for additional patterns to develop. By integrating data mining methodologies with A-GSBT, real-time predictions become possible.

The following definitions help to clarify the relation of various general system properties.

DEFINITION: *General system* parameter elements:

$$T_P, I_P, F_P, O_P, S_P, \mathscr{L}, \mathscr{L}', \mathbb{S}_{BX}, \mathbb{S}'_{BY} \in \mathcal{P}; \mathfrak{f}_I, \mathfrak{f}_E, \mathfrak{f}_O, \mathfrak{f}_T, \mathfrak{f}_N, \mathfrak{f}_B, \mathfrak{f}_S \in \mathcal{T};$$
$$\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \in \mathcal{A}; \text{ and } t_1, t_2, \ldots, t_k \in \mathscr{T}.$$

$\mathbb{S}_{BX}$ and $\mathbb{S}'_{BY}$ are the "background components" of the system and negasystem. These sets contain all components not otherwise found in input, toput, etc. In the analysis of a particular system, we will further identify these background component sets. For example, when analyzing an education system, we may further identify the background components as classrooms, administrative offices, etc.

As with the object-set, an effective procedure determines the elements of affect relations. The following decision procedure is such an effective procedure.

DEFINITION: Affect Relation-Set, $\mathcal{G}_{\mathcal{A}}$, Construction Decision Procedure

1) *Affect Relation-Set Predicate Schemas*, $P(x_n,y_n) = P(\mathcal{A}_n)$, are defined as required to define the family of affect-relations, $\mathcal{A}_n \in \mathcal{A}$, as extensions of the predicate schemas. The elements of $\mathcal{A}_n$ are of the form $\{\{x\},\{x,y\}\}$ that indicates that an *affect relation* has been determined to exist from x to y. '$P(\mathcal{A}_n)$' designates the predicate that defines $\mathcal{A}_n$.

2) The *Affect-Relation Transition Function*, ϕ_n, is defined by:

$$\phi_n: X \times Y \rightarrow \mathcal{A}_n \mid X,Y \subset \bar{I}_B \;.\wedge.\; \phi_n(X \times Y) = \{\{\{x_n\},\{x_n,y_n\}\} \mid P(\mathcal{A}_n) \wedge x_n \in X \wedge y_n \in Y\}.$$

3) The family of affect relations, $\mathcal{A} = \mathcal{G}_{\mathcal{A}}$, is defined recursively by applications of the function defined in 2) for all elements in $\bar{I}_B$ to each P($\mathcal{A}_n$) defined in 1).
4) New components are evaluated for each P($\mathcal{A}_n$) defined in 1) and included in the appropriate extension when the value is true.
5) No other objects will be considered as elements of $\mathcal{A}_n \in \mathcal{A} = \mathcal{G}_{\mathcal{A}}$ except as they are generated in accordance with rules 1) through 4).

Conclusion

This report has presented a definition of *general system* that can provide the means to obtain predictive results for certain types of behavioral systems. While the feasibility of obtaining such predictive results has been verified with a few limited theorems, the effectiveness of this theory will be dependent upon much more extensive research with analyses of various intentional systems. The current primary research application is with using A-GSBT as the logical basis for the development of SimEd, a program, still under development, designed by Frick (Frick, 1994) to analyze educational systems. It is believed that future applications should be considered for the Department of Defense and Homeland Security.

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